# TRAPPED MODES STUDY AND BBU ANALYSIS IN THE5-CELL 650 MHZ CAVITY

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*Abstract*

eRHIC project is a future electron-hadron collider proposed at BNL. The proposed electron accelerator will generate upto 21 GeV polarized electrons which will collides with proton beams with energy up to 250 GeV. The proposed collider will deliver electron-nucleon luminosity of $10^{33}$- $10^{34}$/cm$^{-2}$/s. A superconducting RF (SRF) 5-cell elliptical cavity will be utilized in electron accelerator. This paper presents a study of higher-order modes (HOM) for this 650 MHz SRF cavity. Different types of HOM modes and their BBU instabilities were investigated for frequencies up to 3.2 GHz. Threshold current values of beam breakup are estimated by GBBU code. Further improvement on this threshold current has been explored and discussed.

## INTRODUCTION

eRHIC project requires to build an electron linac ring on top of the existing RHIC ring. BNL proposed a ERL based design to accelerate electrons by multiple turns and extract their energy after collision with proton. This machine would have different operation modes. The beam parameters of different operation setup of the eRHIC project are shown in the table 1. The electron current of each turn is useful to the collision and will determine the max collision luminosity. The electron collision energy is required to be larger than 15GeV, and BNL could not afford to build a long linac to accelerate electrons up to this energy level. In this sense, a multiple turns FFAG plan is proposed to accelerate electrons by receptively using a short linac. Therefore, the multiple electron bunches with different energy will co-linearly pass the linac. The electron current in the SRF is the collision current times N while N is the turn number of FFAG. In our ultimate operation case, there would be 12 turns in the FFAG rings and electron current is 50mA in each turn AS shown in the table 1.

A 5 cell SRF cavity is optimized for various electromagnetic specification and the design mainly focus loss factor minimization. In this report, we will search the resonating frequency and the Qe on this cavity by using Eigen solver, and electromagnetic field patterns of the trapped modes will be found and addressed in the future SRF cavity design.

Its HOM damping scheme of the SRF cavity address both monopole and dipole components of resonating modes. [1] The monopole modes BBU threshold current usually very large, thus we would concern the HOM power instead. On the other hand, the BBU threshold current is limited by the dipole components of the HOM modes. In this report, we will determine the appropriate Qe of HOM to sustain the required current without BBU. By doing this, we can also obtain the impedance budget of the cavity design from the BBU perspective.

Table 1: The Different Operation Setup of eRHIC Project

| | LR Nominal design | | LR Ultimate design | |
|---|---|---|---|---|
| | e | p | e | p |
| Energy [GeV] | 10 | 250 | 8.3 | 250 |
| CM energy [GeV] | 100 | | 91 | |
| Bunch frequency [MHz] | 9.4 | | 9.4 | |
| Bunch intensity [$10^{10}$] | 1.7 | 20 | 3.3 | 30 |
| Beam current [mA] | 26 | 277 | 50 | 415 |
| rms norm.emittance h/v[um] | 36.7/36.7 | 0.5/0.5 | 16.5/16.5 | 0.27/0.27 |
| rms emittance h/v [nm] | 1.9/1.9 | 1.9/1.9 | 1.0/1.0 | 1.0/1.0 |
| beta*, h/v [cm] | 12.5/12.5 | 12.5/12.5 | 7/7 | 7/7 |
| IP rms beam size h/v [um] | 15.3/15.3 | | 8.4/8.4 | |
| IP rms ang. .spread h/v [urad] | 120/120 | 120/120 | 120/120 | 120/120 |
| max beam-beam parameter | 1.2 | 0.004 | 4.1 | 0.015 |
| e-beam disruption parameter | 20 | | 36 | |
| max space charge parameter | 1.4e-4 | 0.006 | 8.6e-4 | 0.058 |
| rms bunch length [cm] | 0.3 | 16.5 | 0.3 | 5 |
| Polarization [%] | 80 | 70 | 80 | 70 |
| Peak luminosity [$10^{33}$cm$^{-2}$s$^{-1}$] | 1.0 | | 14.4 | |

## TRAPPED MODES

This SRF cavity has a fundamental mode frequency of 647.5MHz. Current design of SRF cavity has two enlarged beam pipes on each side. In figure 1, we focus on the end group on one side of the cavity.

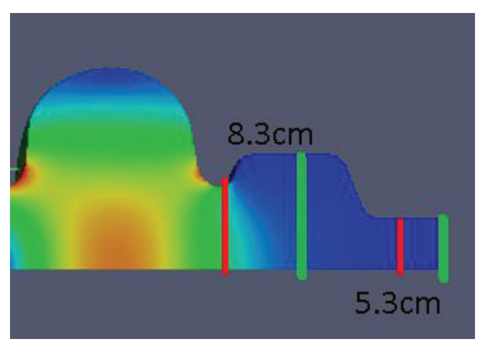


Figure 1: The enlarged beam pipe on one cavity side. Radius of both beam pipe and enlarged beam pipe are shown. Green lines are different boundary conditions that we used in the finite element eigen solver.

The propose of enlarged beam pipe is inducing the low frequency HOM out to the beam pipe and extract the HOM power there by some HOM couplers. The last iris radius and beam pipe are 8.3 cm and 5.3 cm, thus the cut-off frequencies are shown in table 2.




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Table 2: the Cut Off Frequency of Two Irises

| R=5.3 cm | | | | R=8.3 cm | | | |
|---|---|---|---|---|---|---|---|
| Frequency in GHz | | | | | | | |
| TE and TM together sorted from the lowest to the highest | | | | | | | |
| Monopole | | Dipole | | Monopole | | Dipole | |
| 2.152 | TM01 | 1.648 | TE11 | 1.374 | TM01 | 1.052 | TE11 |
| 4.940 | TE01 | 3.429 | TM11 | 2.190 | TE01 | 2.190 | TM11 |
| 7.744 | TM02 | 4.771 | TE12 | 3.154 | TM02 | 3.046 | TE12 |
| 10.552 | TE02 | 6.278 | TM12 | 4.009 | TE02 | 4.009 | TM12 |
| 13.361 | TM03 | 7.639 | TE13 | 4.945 | TM03 | 4.878 | TE13 |
| 16.171 | TE03 | 9.104 | TM13 | 5.813 | TE03 | 5.813 | TM13 |
| 18.982 | TM04 | 10.475 | TE14 | 6.738 | TM04 | 6.689 | TE14 |

To find the trapped mode, we changed the boundary conditions in the Eigen solution. In figure 1, we applied two perfect match absorbing boundaries on two green planes. Searching the resonating modes frequencies up to 3GHz, we can gather the R/Q and Qe of all the supporting modes. The Qe of the modes might change and could be an indicator because of the different boundary conditions. If the Qe of a mode is very high (>1× $10^4$) and remains unchanged, thus this mode is likely to be a trapped mode. In figure 2, we plot the Qe from two boundary conditions and found a set of modes who might be trapped modes.

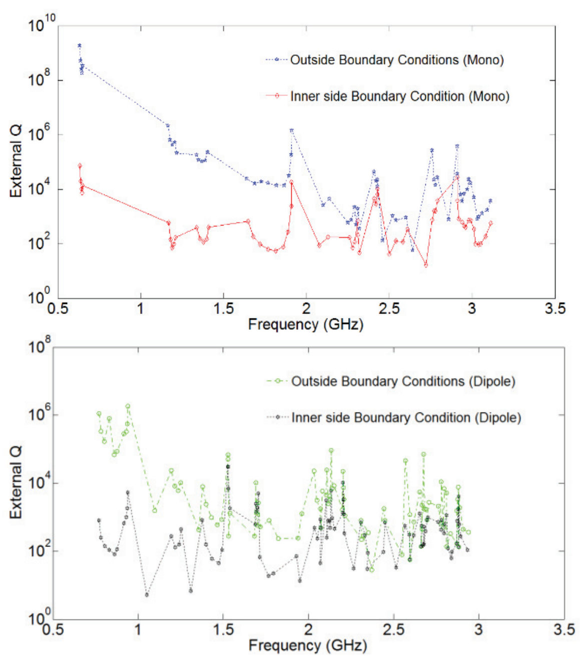


Figure 2: The external Q comparisons of monopole modes and dipole modes between two boundary conditions.

We exam the electromagnetic patterns of these modes and illustrate them in figure 3. The high Qe mode frequencies are 1.91 and 2.99 GHz for monopole modes and 1.53GHz and 2.11GHz for dipole modes. Their Qe are all larger than 1× $10^4$. The shunt impedance can be quite large and they may increase the HOM generation and reduce BBU current threshold. Because these modes are mainly trapped in the center cell, once they are excited, we cannot effectively damp them except changing the cavity design.

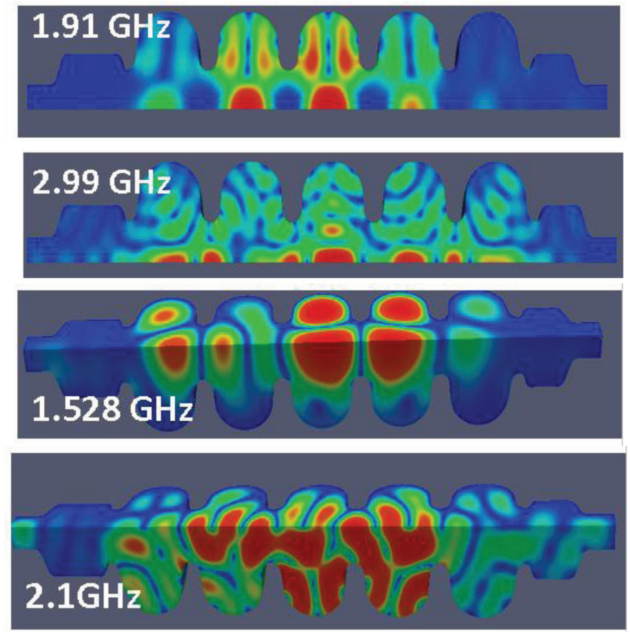


Figure 3: The Electromagnetic field pattern of high Qe modes. First two figures are monopole modes and the rest are dipole.

## BBU AND IMPEDANCE BUDGET

It is economical to design HOM damping scheme once we have the shunt impedance budget. The impedance budget may come from ring lattice setup, various apertures and beam stability. In this report, we only consider the impedance from the BBU point of view. A simple guidance for one mode and 2 pass ERL operation is given by Jefferson Lab. [2,3]

$$I_{th} = -\frac{2V_b}{(\omega / c)\left(\frac{R}{Q}\right)Q_L\, m^* \sin(\omega T_r)}$$, while

$$m^* = m_{12}\cos^2(\alpha) + (m_{14} + m_{32})\sin(\alpha)\cos(\alpha) + m_{34}\sin^2(\alpha)$$.

However, it will be more complicate for our case: multiple modes and multiple turns. GBBU code simulate the cavity stored energy in time domain when bunches pass. The stored energy includes all HOM modes numerically. when the current is smaller than the threshold, the energy will remain the same or decay exponentially. Once the current is more than the threshold, the stored HOM energy runs away in the time domain. We can ramp up the passing current to search the current threshold with any given HOM damping scheme. Before the HOM scheme is finalized, we can arbitrarily change the Qe of all the dipole modes in a fashion to let the simulated current threshold be larger than our required current. Then we will design the HOM damping mechanism to achieve those Qe of HOMs.

At this moment, we make some assumptions here, and we will address the real cases in the discussion section. 1. The resonate R/Q change little when the HOM scheme is applied. 2.

The frequency of resonating modes will remain unchanged. From the equation 1, we found the factor (f×R/Q) plays an important role in BBU threshold and this factor is intrinsic parameter which is independent to the HOM damping schemes. Thus, we start BBU simulation with the modes with highest f×R/Q factors and vary the Qe of this mode until the required threshold is reached. We add the another mode with next highest f×R/Q factors and only vary the Qe of newly added mode to match the require current. In each time, a new mode will be added into BBU simulation and its Qe will be vary and match the current threshold until all HOM modes are all included in this BBU simulation. The flow chart of BBU simulation is given in Figure 4.

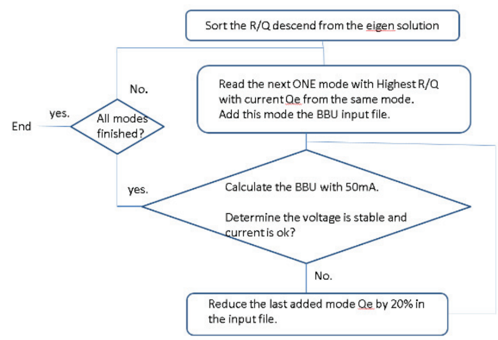


Figure 4: The simulation flow chart to search the required Qe of HOM from BBU threshold perspective. The required BBU threshold is 50Ma in this study.

The Qe of all modes are obtained, and the BBU threshold current for multiple HOM modes is the required current at this time. Since the R/Q would not change little from different HOM damping scheme, the impedance budget is given from the BBU perspective. In this report, we use the dipole modes obtained from the 5 cell design and Qe are simulated when the required BBU threshold is 50mA. The shunt impedance budget is given in figure 5.

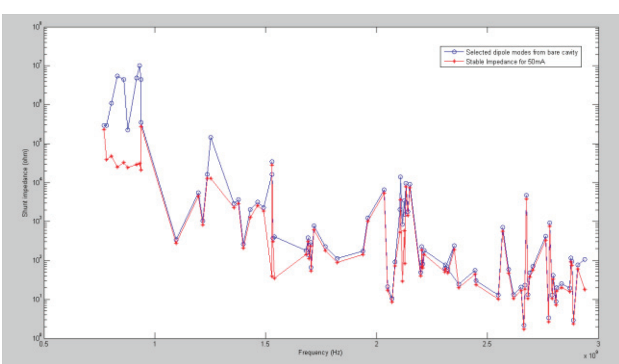

Figure 5: The blue curve is the impedances (R/Q×Qe) from Eigen solution, and the red curve are impedance with reduced Qe and the same R/Q. If shunt impedances from a given HOM damping scheme are all smaller than this red curve, the BBU threshold will be more than required current.

From figure 4, the red curve represents the impedances obtained from the BBU simulation, and they are the products of the reduced Qe and R/Q of all dipole HOMs. One can see that the low frequency HOM requires heavy HOM damping, while the high frequency HOM seems to require low level HOM damping.

## DISCUSSION

Now we discuss the scenario that our BBU assumptions might change. In our eRHIC design, the circumference revolution frequency is 78.2KHz and we will add the beam dump gap in our electron trains. Thus we need to consider that if the HOM frequencies of the SRF cavity match harmonic of 78.2KHz. Meanwhile, with HOM damping scheme applied, R/Q and frequencies of HOM modes will slightly change. We would like to presume the frequency and R/Q changes stay within ±1% and 5%, respectively. We claim that the most restrict HOM damping scheme is required when all frequencies of HOM mode match the harmonic of 78.5KHz and their R/Q increase 5%. We demonstrate the the methodology in Figure 6 and the results will be shown in the next research.

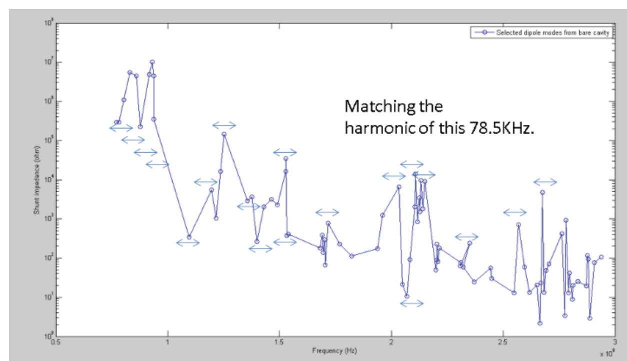


Figure 6: The blue curve is the impedances (R/Q×Qe) from Eigen solution. The horizontal arrows suggest that the resonating frequencies are shifted to the harmonic of 78.2KHz.

## CONCLUSION

In this study, we found the trapped HOM modes from current 5 cell SRF cavity design. These modes require special attention when we design the bunches trains pattern. Secondly, we setup an iterative method to determine the impedance budget from BBU view. This impedance budget will lend guidance on the HOM damping scheme optimization in the next research.

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