OPTIMIZING THE DESIGN OF LINEAR NON-SCALING FIXED FIELD ALTERNATING GRADIENT ARCS FOR THE ELECTRON RINGS OF eRHIC*

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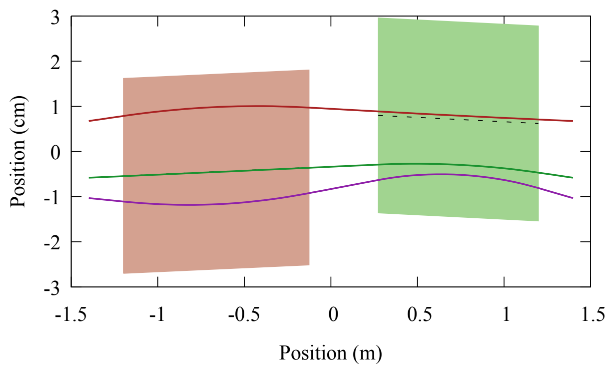

Figure 1: Trajectories in the LER cell for the design energies. Magnet locations (red for F, green for D) are also shown.

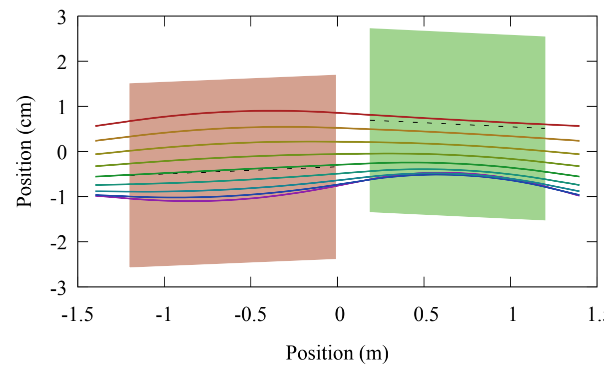

Figure 2: Trajectories in the HER cell for the design energies. Magnet locations (red for F, green for D) are also shown.

Abstract

I describe a process for producing optimal linear non-scaling fixed field alternating gradient (FFAG) arc designs for the electron rings of eRHIC, an electron-ion collider in the RHIC tunnel at Brookhaven National Laboratory. The electrons are accelerated in two FFAG rings (low and high energy), which in addition to the arcs optimized here, contain straight sections, splitter/combiner sections, and a linac shared between the rings. The optimization process I use has two layers, an inner one meeting constraints and an outer optimization that minimizes a target function. The target function is an approximation to the FFAG arc cost, for which I give the function used and the basis for that choice. While reducing synchrotron radiation is important, I show that optimizing for synchrotron radiation alone leads to significant cost an performance penalties for the rest of the machine design for very little reduction in synchrotron radiation. I describe important constraints on the design, in particular minimum drift lengths, maximum and minimum tunes, and clearance from the beam to the beam pipe. Finally, I present possible eRHIC FFAG parameters resulting from this optimization.

INTRODUCTION

In the FFAG-based linac-ring design for eRHIC [1], we will accelerate electron beams from 20 MeV to 20 GeV in two linear non-scaling FFAGs. An FFAG is a single beamline that accepts a very large energy range. To keep the orbit excursions small, the lattice cells are very short and use alternating gradient focusing. A simple cell (a doublet in our case) is repeated without variation through the entire lattice to avoid driving resonances and thereby to keep the energy acceptance large. The orbits in the eRHIC FFAG cell design described here are shown in Fig. 1 for the low energy ring (LER) and Fig. 2 for the high energy ring (HER).

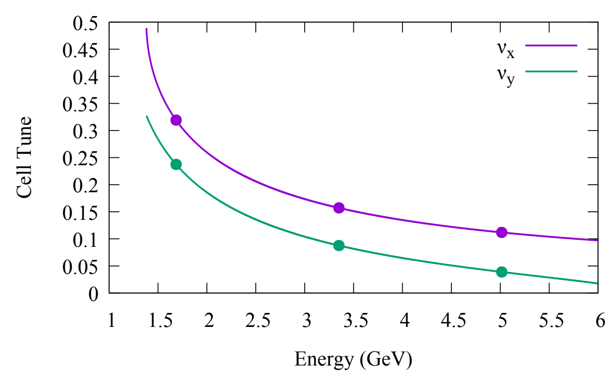

Figure 3: Tune as a function of energy in the LER arc cell. Points are the values for the design energies.

Since linear magnets are used [2, 3], the tunes will vary with energy, as shown in Figs. 3 and 4. The focusing must be weak enough to avoid the half integer resonance at low energy, but must be strong enough to avoid loss of stability at the high energy. Furthermore, very high tunes lead to large chromaticities, which make correction of orbit errors more difficult: energy spread in the beam leads to signal decoherence when the beam is off of the closed orbit. Higher horizontal tunes are desirable to reduce the horizontal orbit excursion, while vertical tunes tend to be lower to reduce the demands on the magnet strength. As the energy gain

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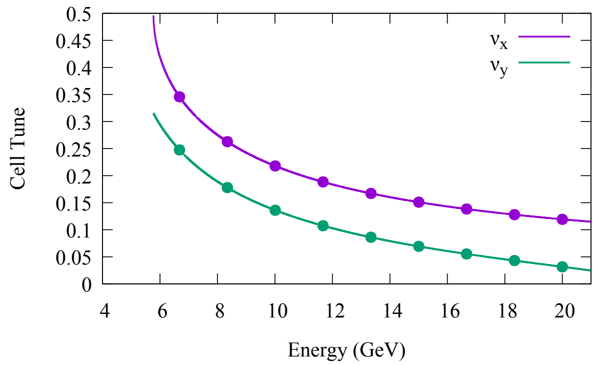

Figure 4: Tune as a function of energy in the HER arc cell. Points are the values for the design energies.

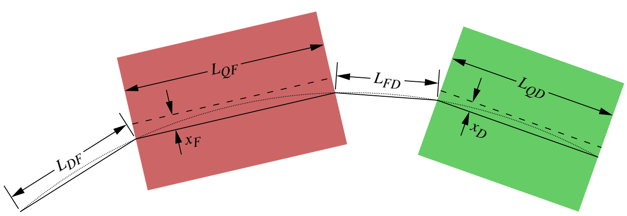

Figure 5: Layout of a lattice cell, showing meaning of dimensions.

factor increases, these demands become more challenging and costly to meet. This motivated, at least in part, a choice for 12 accelerating passes through the linac, which leads to each FFAG having just below a factor of 3 in energy gain. More turns would require one of the FFAGs to have a larger factor of energy gain, and fewer turns would increase the linac cost while still requiring one FFAG to have a factor of 3 energy gain.

Figure 5 shows the layout of the lattice cell. There are two drifts, and they should be kept as small as possible to keep the orbits compact. The short drift is set to 20 cm in the HER to provide space for magnet hardware and a BPM. The long drift is 40 cm to allow additional space for other hardware. The magnets are quadrupoles which are offset horizontally to give bending.

OPTIMIZATION

The optimization algorithm has two layers: an inner layer which solves for constraints, and an outer layer which then minimizes an optimization target. For the HER, in the inner layer, I fit the low energy horizontal tune and the high energy vertical tune to specified values, center the beam around an arc of the design radius, and keep the maximum beam distance to the magnet axis the same for both magnets, while varying the magnet lengths, the average of the magnet offsets, and the gradient (which is identical in the two magnets). The cost is then minimized by varying the difference between the magnet offsets.

The cost function I used is

$$0.029n + 75\,\mathrm{T}^{-2}\mathrm{m}^{-3}U \tag{1}$$

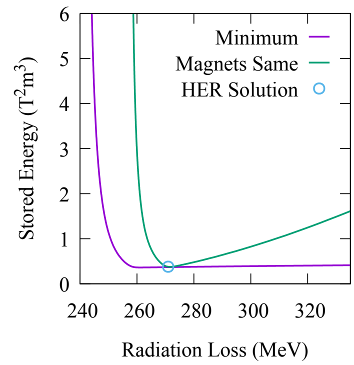

Figure 6: Boundary for minimum stored energy and radiation loss solutions. The violet line is the minimum for fixed maximum horizontal and vertical tunes. The green line is the boundary when the transverse magnet structure is constrained to be the same. The circle is our chosen solution for the HER.

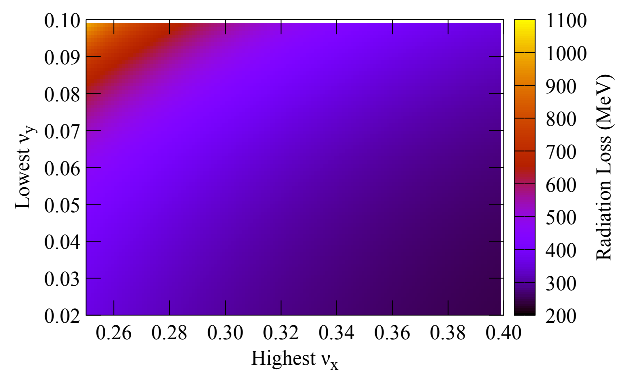

Figure 7: Synchrotron radiation loss per particle as a function of maximum horizontal and minimum vertical tune.

where n is the number of magnets and U is the magnetic stored energy (which is proportional to material cost in the magnets) for all the magnets. The magnetic stored energy for a single magnet is computed as $B_1^2 r^4 L$, where B_1 is the gradient, r is the pole radius, and L is the magnet length. The coefficients are based on an earlier eRHIC cost estimate.

Figure 6 illustrates why I do not optimize for synchrotron radiation loss in the HER. Reducing radiation much below what it would be at the minimum stored energy results in a rapid increase in stored energy and machine cost, for only a modest reduction in radiation loss.

Since earlier optimization runs ended up with the F and D magnets having similar gradients and pole radii, I chose to make the magnets have identical transverse structure (gradient and pole radius) to simplify engineering and design, and possibly produce some modest cost advantages. Figure 6 shows there is only a small penalty in synchrotron radiation loss and stored energy for doing so.

Figures 7 and 8 show that higher horizontal tunes and lower vertical tunes both lead to reduced synchrotron radiation and magnetic stored energy. I chose 0.345 (identical to

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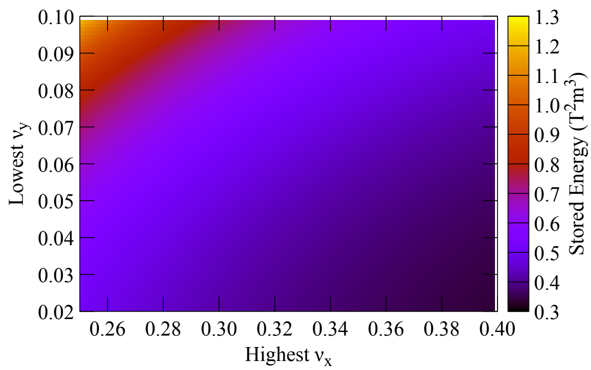

Figure 8: Magnetic stored energy as a function of maximum horizontal and minimum vertical tune.

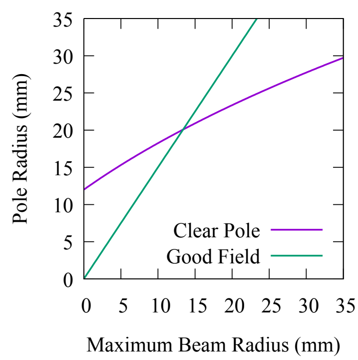

Figure 9: Magnet pole radius as a function of maximum beam radius in the midplane. The violet line is what is required to keep the beam 12 mm from the pole, and the green line is to keep the beam at 2/3 the pole radius.

an earlier baseline) for the highest horizontal tune, keeping it this low mainly to avoid introducing chromaticity, and 0.032 for the lowest vertical tune.

The stored energy increases rapidly with the magnet pole radius, so I create a model to relate the maximum beam radius in the magnet to the pole radius. The key parameter is the minimum distance from the beam to the beam pipe, and

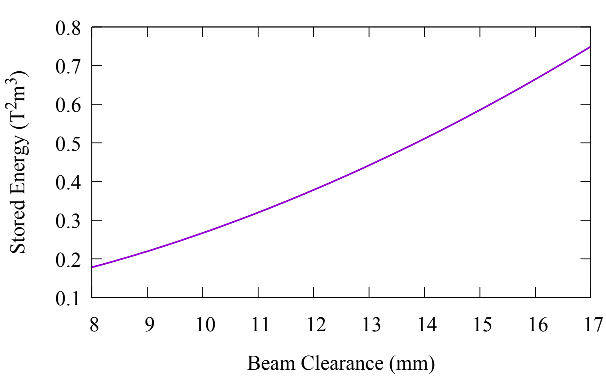

Figure 10: Magnetic stored energy as a function of the required distance to the pole.

Table 1: FFAG Lattice Parameters

	HER		LER	
Radius (m)	382.12		382.12	
Max. ν_x	0.346		0.319	
Min. ν_y	0.032		0.039	
L_{short} (m)	0.20		0.40	
L_{long} (m)	0.40		0.40	
	F	D	F	D
L_{quad} (m)	1.187	1.010	1.072	0.925
B' (T/m)	34.42	−34.42	8.64	−8.64
Quad Offset (mm)	−3.4	7.0	−3.5	8.1
Beam r_{max} (mm)	13.2	13.2	14.3	14.3
Cells/2π	≈858		≈858	

therefore (adding extra distance for the beam pipe thickness) to the pole. Since the beams are off-center in the magnet, the pole radius is larger than that distance. There are two criteria: first that the pole radius is at least 1.5 times the maximum beam radius (for field quality); and second, that the distance from the beam to a hyperbolic pole is at least the required distance. These two criteria are shown in Fig. 9. The optimum cost is almost always at the point where the two criteria are equal, at which point the beam radius is about 1.11 times the required clearance (12 mm for eRHIC). Figure 10 shows that the stored energy increases rapidly with increasing clearance, though synchrotron radiation decreases slightly with an increase in required clearance.

The stored energy for the LER is significantly less than that of the HER. The number of cells was similar enough for the LER and HER optimizations that I chose the cell lengths to be the same. Thus only a nonlinear equation solution for the tunes, orbit center, and equal maximum beam radii in the magnets was necessary, varying the magnet gradient, two horizontal offsets, and the length of one of the magnets. Both drifts were made 40 cm, since there was little penalty for doing so, and the extra space will be convenient. Furthermore, the maximum horizontal tune was reduced to 0.319 (primarily to reduce chromaticity) and the maximum vertical tune raised to 0.039 to keep further from the stability boundary.

Table 1 gives the final parameters chosen. See Fig. 5 to understand the lengths and displacements. The closed orbits at the design energies are shown in Figs. 1 and 2, and the tunes in Figs. 3 and 4.

ACKNOWLEDGEMENTS

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