# EFFECT OF THE BEAM TIME STRUCTURE ON THE NEUTRONICS OF AN ACCELERATOR DRIVEN SUBCRITICAL REACTOR

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*Abstract*

When designing a high power accelerator for an ADSR, it is important to optimize the beam parameters to be compatible with the steady state character of the reactor operation and to define an adequate and safe startup procedure. In this study we investigate the impact of the beam time structure on the kinetic behavior of the sub-critical core and derive a general relationship between the time evolution of the neutron population and the proton beam.

## INTRODUCTION

The time structure of the proton beam for ADSR [1] is of major importance in sustaining the steady state of the reactor operation and to avoid transient effects. From the reactor side, the main requirement is to avoid huge fluctuations of the neutron counts. From the accelerator side, it is important to deliver the required beam power on target in a flexible and reliable way. Flexibility is required because the accelerator provides the control mechanism of the reactor.

## REACTOR POINT KINETICS EQUATIONS

We write the coupled kinetics equations for a point reactor model with one group of delayed neutrons.

$$\begin{cases} \dfrac{dn(t)}{dt} = \dfrac{\rho-\beta}{\wedge} n + \lambda.C + S \\ \dfrac{dC(t)}{dt} = \dfrac{\beta.n}{\wedge} - \lambda.C \end{cases} \tag{1}$$

where $n$ is the neutron population, $\rho$ is the sub-criticality level ($\rho = (k_{eff}-1)/k_{eff}$), $\beta$ the fraction of delayed neutrons (assuming only one group), $\wedge$ the generation time ($\wedge = l/k$), $l$ the neutron lifetime, $\lambda$ the decay constant for precursor decay, $C$ the density of the precursors and $S$ is the external source of spallation neutrons. Temperature feedback effects are ignored. More information about the latter can be found in [2].

We assume that p+1 neutron bunches are injected into the core when the accelerator beam is turned on (see Fig 1). We also assume that the time structure of the spallation neutrons is the same as that of the protons-a sum of Dirac delta-functions separated by the bunch separation time $t_b$, i.e

$$S(t) = \sum_{i=0}^{p} N_s \delta\left(t - i t_b\right) \tag{2}$$

where $N_s$ is the total number of spallation neutrons per bunch and $t$ is the time elapsed since the arrival of the first bunch of spallation neutrons into the core. Introducing $t_r$,

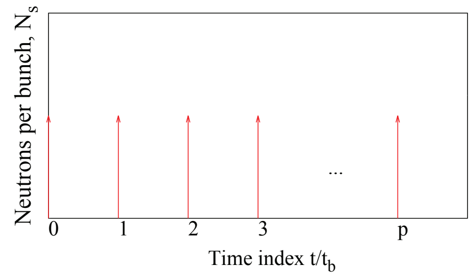


Figure 1: Beam time structure: each spallation neutron pulse is represented by a Dirac delta function.

the time elapsed since the arrival of the last bunch of neutrons indexed p, one can write,

$$\begin{cases} t = p t_b + t_r \\ 0 \leqslant t_r < t_b \\ 0 < t_b \end{cases} \implies p = E\left(\frac{t}{t_b}\right) \tag{3}$$

where $E$ is the integer part function. This initial value problem can be solved by means of Laplace transforms. After some simplifications, the solution for one bunch of neutrons is:

$$n(t_r) = A_1 \exp\left(-\frac{t_r}{\tau_1}\right) + A_2 \exp\left(-\frac{t_r}{\tau_2}\right) \tag{4}$$

where

$$\begin{aligned} \tau_1 &= \frac{\wedge}{\beta-\rho} \approx \frac{l}{1-k} \quad ; \quad \tau_2 = \frac{\rho-\beta}{\lambda\rho} \\ A_1 &= N_s \quad ; \quad A_2 = -N_s \frac{\beta}{\rho-\beta} \end{aligned} \tag{5}$$

The first term in Eq.(4) represents the contribution of the prompt neutrons and the second term represents that of the delayed neutrons. Neglecting the contribution of the latter, i.e $\beta \approx 0$ (justifying the need for a subcritical reactor, thus an external source of neutrons), one can calculate the total number of neutrons available in the core at a given time $N(t_r)$:

$$\begin{aligned} \frac{N(t_r)}{N_s} &= \exp(-\frac{t_r}{\tau_1}) + \exp(-\frac{t_r+t_b}{\tau_1}) + \ldots + \exp(-\frac{t_r+pt_b}{\tau_1}) \\ &= \exp\left(-\frac{t_r}{\tau_1}\right) \times \frac{1-\exp\left[-(p+1)\frac{t_b}{\tau_1}\right]}{1-\exp\left(-\frac{t_b}{\tau_1}\right)} \end{aligned} \tag{6}$$

Eq.(6) can also be expressed as a function of t,

$$\frac{N(t)}{N_s} = \exp\left[-\frac{t - E(\frac{t}{t_b})t_b}{\tau_1}\right] \times \frac{1 - \exp\left[-\left(E(\frac{t}{t_b}) + 1\right)\frac{t_b}{\tau_1}\right]}{1 - \exp(-\frac{t_b}{\tau_1})} \quad (7)$$

The pulsed character of the external neutron source is simply contained in the integer part function.

## EFFECT OF THE REPETITION RATE ON THE NEUTRONICS

Is the best choice for an ADSR system a high repetition rate with a low peak current per bunch, or the opposite? We define the equivalence between two beams in terms of average beam current by:

$$\begin{cases} N_s \\ t_b \end{cases} \text{is equivalent to} \quad \begin{cases} fN_s \\ ft_b \end{cases} \quad (8)$$

where $f$ is a time structure scale factor: increasing f increases the number of protons per bunch and decreases the bunch frequency.

Now substituting $(N_s, t_b)$ by $(fN_s, ft_b)$ in Eq.(7), one obtains:

$$\frac{N(t)}{N_s} = f\exp\left[-\frac{t - E(\frac{t}{ft_b})ft_b}{\tau_1}\right] \times \frac{1 - \exp\left[-\left(E(\frac{t}{ft_b}) + 1\right)\frac{ft_b}{\tau_1}\right]}{1 - \exp(-\frac{ft_b}{\tau_1})} \quad (9)$$

Figs. 2 and 3 show the evolution of the neutron population $N(t)/N_s$ for various values of f.

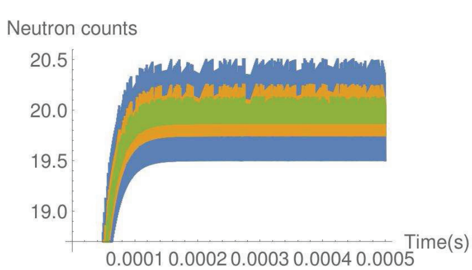


Figure 2: Time variation of the neutron population for various values of the scale factor $f$.

Increasing the bunch frequency and thus lowering $f$ is the best choice for an ADSR since it reduces the fluctuations of the neutron population. However, the average state is not affected. To calculate the average as well as the amplitude

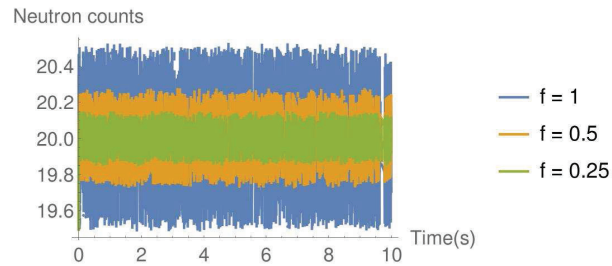


Figure 3: Time variation of the neutron population for various values of the scale factor $f$.

of the variation of the neutron population, we first introduce the equivalence (8). In the limit where $t \rightarrow \infty$, i.e when $p \rightarrow \infty$ (steady state), Eq.(6) becomes periodic in time with period $ft_b$. This yields:

$$\frac{\langle N(t_r)\rangle}{N_s} = \frac{1}{ft_bN_s}\int_0^{ft_b}\frac{fN_s\exp\left(-\frac{t}{\tau_1}\right)}{1-\exp\left(-\frac{ft_b}{\tau_1}\right)}dt = \frac{\tau_1}{t_b} = \frac{l}{t_b}\frac{1}{1-k} \quad (10)$$

$$\frac{N_{max}}{N_s} = \frac{f}{1 - \exp(-\frac{ft_b}{\tau_1})} \quad (11)$$

$$\frac{N_{min}}{N_s} = \frac{f\exp(-\frac{ft_b}{\tau_1})}{1 - \exp(-\frac{ft_b}{\tau_1})} \quad (12)$$

Thus, for equivalent beams in terms of average current, the scale factor $f$ has no influence on the average state of the reactor. However, the overall variation of the neutron population in the core can become important as shown in Fig 4.

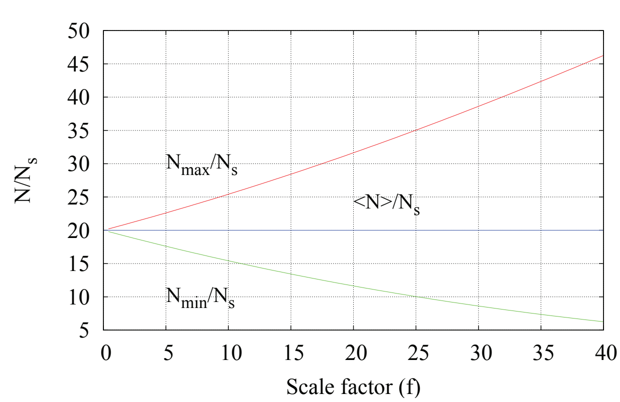


Figure 4: Domain of variation of the neutron population as a function of the scale factor $f$.

## AVERAGE POWER OF THE CORE

The total number of fissions per second is:

$$\begin{aligned} N_f &= \frac{N_{tot}/s - N_{spa}/s}{\nu} \\ &= \frac{1}{\nu}\left[\frac{\langle N(t)\rangle}{l} - fN_s E(\frac{1}{ft_b})\right] \\ &= \frac{N_s}{\nu}\frac{1}{t_b}\frac{1}{1-k} - \frac{fN_s}{\nu}E(\frac{1}{ft_b}) \\ &\approx \frac{N_s}{\nu}\frac{1}{t_b}\frac{k}{1-k} \end{aligned} \quad (13)$$

where $\nu$ is the number of neutrons produced per fission ($\nu \sim 2.5$) and where we assume a high repetition rate ($E(1/(ft_b)) \sim 1/(ft_b)$). From that, the average power of the core is:

$$P_{th,c}[MW] = qE_f[MeV] \times N_f[Hz] + P_{dh} \quad (14)$$

where $E_f \approx 208$MeV is the energy released per fission in a fast spectrum and $P_{dh}$ is the power of the decay heat (the decay of the short-lived fission products continues even if the reactor is shut-down, here $P_{dh} \sim 0$). Injecting 13 into 14, and introducing $N_0$, the number of spallation neutrons produced per proton (the "multiplicity" of the target) yields:

$$P_{th.c}[MW] = E_f[MeV]I[A]\frac{N_0}{\nu}\frac{k}{1-k} + P_{dh} \quad (15)$$

The same result can be obtained by treating the fission neutrons as a series of functions with the same multiplication factor k. In reality, one has to make the distinction between the multiplication factor of the different generation of neutrons in the core originating from the spallation neutrons, due to the difference between the spallation and fission neutron spectra. A more detailed analysis can be found in [3]. For instance, the MYRRHA ADSR facility requires a maximum core thermal power of $P_{th,c} = 85\ MW$, a proton beam energy of 600 MeV (thus $N_0 = 13$), $k \approx 0.95$. It results that $I = 4.13$ mA which is approximately the required 4 mA beam intensity of its Linac.

## BEAM TIME REQUIREMENTS

Assuming an average beam current of 4 mA and a number of protons per bunch $N_{ppb} \sim 10^9$, this yields, $t_b = qN_{ppb}/I = 40ns$. Imposing an additional criterion of limiting the overall variation of the neutron flux over time to less than $\epsilon$. From Eqs.(10) and (11), asserting that:

$$\frac{N_{max} - \langle N\rangle}{\langle N\rangle} \leq \epsilon \quad (16)$$

yields,

$$\frac{f}{1-\exp(-ft_b/\tau_1)}\frac{t_b}{\tau_1} \leq 1 + \epsilon \quad (17)$$

Solving for $f$ with $\tau_1 = 20\ \mu s$ and $\epsilon = 0.01$ (less than 1 % fluctuation of $N$ allowed), this yields $f \leq 10$ so that the bunch repetition rate $F = 1/(ft_b) \geq 2.5\ MHz$. Thus, CW beams with a high repetition rate must be employed.
Beam interruptions can also be exploited for online monitoring of the reactivity. This needs an in-depth investigation to determine the impact of the neutron flux fluctuations on the different components of the reactor core.

## START-UP PROCEDURE

An accident occurring during start-up can be more severe than an accident during normal operation because the reactor can shoot past its nominal operating level at a high rate of power increase, reducing the time available for the action of automatic safety systems [4]. For an ADSR system, the core power control is ensured by the accelerator. Therefore, it is important to have a flexible control of the proton beam power. Ideally, the power increase should be a discrete multi-stage process, each stage governed by a relaxation time allowing the equilibrium to be reached.
For that reason, the start-up procedure may require a long time. This is also the main reason why reliability is a critical problem for ADSR.

## CONCLUSION

In this paper, we used a simplified model to describe the kinetics behavior of the reactor core in order to define the requirements in terms of beam time structure of the accelerator proton beam for ADSR. As expected, CW beams with high repetition rate must be employed to ensure the steady state of the reactor kinetics, and in a general manner, the proton bunch separation time $t_b$ must be lower than the neutron lifetime $l$ which is of the order of tens of microseconds.

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