# WAVE PARTICLE CHERENKOV INTERACTIONS MEDIATED VIA NOVEL MATERIALS

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#### Abstract

Currently there is an increasing interest in dielectric wall accelerators. These work by slowing the speed of an EM wave to match the velocity of a particle beam, allowing wave-beam interactions, accelerating the beam. However conventional dielectric materials have limited interaction regions, so wave-beam energy transfer is minimal.

In this paper we consider Artificial Materials (AMs), as slow wave structures, in the presence of charged particle beams to engineer Inverse-Cherenkov acceleration. AMs are periodic constructs whose properties depend on their subwavelength geometry rather than their material composition, and can be engineered to give an arbitrary dispersion relation. We show that Metamaterials, one example of an AM, can mediate an Inverse-Cherenkov interaction, but break down in high power environments due to high absorption. We consider AMs with low constitutive parameters and show they can exhibit low absorption whilst maintaining the ability to have a user defined dispersion relation, and mediate a wavebeam interaction leading to Inverse-Cherenkov acceleration.

#### **INTRODUCTION**

Artificial Materials (AMs) are periodic structures whose characteristic properties are determined by their subwavelength unit-cell structure rather than their material composition. This results in the material appearing homogeneous over certain wavelengths (adhering to the effective medium condition) allowing the use of the abstract bulk properties of permittivity ( $\epsilon$ ) and permeability ( $\mu$ ), defined from the constitutive relations,

$$\mathbf{D} = \boldsymbol{\epsilon} \mathbf{E} \tag{1}$$

$$\mathbf{B} = \mu \mathbf{H} \tag{2}$$

where D and B represent the electric and magnetic flux densities, and E and H represent the electric and magnetic fields.

By manipulating the specific geometry of the unit-cells we can engineer the dispersion relation of the material to exhibit novel electromagnetic properties, such as engineering an arbitrary phase shift of an incident EM wave.

By inserting AMs into slow wave structures such as Folded Waveguide Travelling Wave Tubes (FWTWTs), (see Fig. 1), the AM can be optimised to maximise regions of energy transfer between a charged particle beam and an incident EM wave.

This is achieved by matching the velocity of the wave at the interaction points to the speed of the charged particle beam, called the synchronicity condition. When this occurs

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Figure 1: A Folded Waveguide Traveling Wave Tube loaded with the Artificial Material. The incident wave enters from the bottom of the waveguide and travels along the serpentine structure, whereas the charged particle beam passes through the centre of the structure.

energy can transfer between the wave and beam, with the power transferred determined from Madey's Theorem [1] as

$$\Delta P = \frac{\omega^2 \mu}{\beta_0} \frac{L^3}{2ab} Z^2 \frac{d}{dx} \left( \frac{\sin^2(X)}{X^2} \right) \frac{c}{\gamma^3} \frac{1}{v_e^3} (mc^2 I_b/e) \quad (3)$$
where
$$Z = \frac{e}{mc^2} \frac{b}{p} sinc(\beta'_n \frac{b}{2}),$$

$$\gamma = 1 + V_{acc}/m_0 c^2,$$

$$X = \left( \frac{\omega}{v} - \beta'_n \right) \frac{L}{2}.$$

 $\beta_0$  is the TE<sub>01</sub> rectangular waveguide propagation constant,  $v_e$  is the velocity of the charged particle beam,  $I_b$  is the beam current, *m* is the mass of the particle,  $\beta'_n$  is the loaded propagation constant and  $V_{acc}$  is the acceleration voltage of the beam.

A plot of Eq. 3 with varying  $V_{acc}$  of an electron beam can be seen in Fig. 2, where positive power gain shows where the device will operate as a wave amplifier, and negative power gain shows where energy will be imparted into the beam, achieving Inverse-Cherenkov acceleration.

Unfortunately there are limitations with using the existing AM designs, as discussed in the next section. We continue on to discuss a new design aimed at reducing these loss issues whilst still enabling interactions between wave and beam, before concluding with some remarks and future aims.

## **PREVIOUS WORK**

Simulations were conducted to determine the thermal characteristics of an existing AM design [2]: a unit-cell consisting of two copper square Split-Ring Resonators (SRRs) on an FR4 PCB backplate with a copper strip-line along the reverse. This AM was simulated when loaded in a waveguide.

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Figure 2: The change in power for the incident wave when interacting with an electron beam, for varying accelerating voltages [1].

Using the simulation packages HFSS and ANSYS [3,4], the time averaged losses inherent to the split rings and dielectric backplate could be determined. These losses are caused by energy being stored in the SRR structure, resulting in heating.

HFSS is a 3D full-wave EM field solver with adaptive meshing, which uses the FEM method to solve wavestructure interactions. The bulk AM was simulated in HFSS to determine the complete loss density map and the results from the EM simulations were imported into ANSYS. Thermal analysis only required the loss density for single unitcells to determine that cells thermal profile. The metallic losses ( $P_{lm}$ ) and dielectric losses ( $P_{ld}$ ) are given by

$$P_{lm} = \sqrt{\frac{\omega\mu_0}{8\sigma}} \int_{s} |\mathbf{H}_t|^2 ds \tag{4}$$

$$P_{ld} = \frac{Im(\epsilon_r)\omega}{2} \int_V |\mathbf{E}|^2 dV$$
 (5)

where  $\omega$  is the angular frequency of the incident wave,  $\mu_0$  is the permeability of free space,  $\sigma$  is the conductivity,  $\mathbf{H}_t$  is the tangential magnetic field,  $\epsilon_r$  is the relative permittivity and **E** is the electric field [5].

The simulations exposed the AM structure to 1 W of power at 10 GHz, the frequency where the largest  $\mu$  response is observed, and predicted temperatures exceeding 600 °C (the combustion point of FR4), shown in Fig. 3a.



Figure 3: a) Image showing the HFSS/ANSYS predictions for the thermal heating of the unit-cell. b) Experimental results after exposing the fabricated unit-cell to 1W of RF power at 10 GHz [2].

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To test the validity of these predictions, the AM was fabricated and exposed to 1 W of RF power at 10 GHz. After 15 seconds the AM began to combust, showing that temperatures exceeding 600 °C had been reached, see Fig. 3b. By comparing the combustion pattern of the experimental unit-cell to the corresponding simulation unit-cell, we can see that the experiment does indeed validate the simulation method used.

By looking at the loss plot, (see Fig. 4), determined by  $Loss = 1 - |S_{11}^2| - |S_{21}^2|$ , we can see that the operating frequency lies directly on the peak loss (0.3) for this system, caused by energy storage due to resonance. This storage of energy increases until breakdown occurs.



Figure 4: Plot of the total losses for the Artificial Material tested, showing a peak loss of 0.3.

### RESULTS

Simulations in HFSS of an optimised geometry were run, and the scattering parameters  $S_{11}$  (related to reflection) and  $S_{21}$  (related to transmission) were extracted. Figure 5 shows the geometry of the unit-cell, where the EM wave is incident from below the unit-cell.



Figure 5: Geometry of the Optimised Unit-Cell.

By using a Nicholson-Ross-Weir based extraction technique [6,7] we can extract the relative  $\epsilon_r$  and  $\mu_r$  for the AM. These can be seen in Fig. 6, with  $\epsilon_r$  values as low as -0.02 (see inset) being predicted.

We can see that the predicted losses in the X-band region are below 0.05 (see Fig. 7), significantly less than the 0.3 observed in the previous case. This suggests that this optimised unit-cell geometry will withstand significantly higher powers than the original design.

Figure 8 shows the dispersion plot (determined using Smith's technique [8]) for this geometry. 20, 25 and 30 keV

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Figure 6:  $\epsilon$  and  $\mu$  of the Optimised Unit-Cell, showing low values of  $\epsilon_r$  are achievable.



Figure 7: Plot of the losses for the Optimised Unit-Cell, showing losses of less than 0.05 over the investigated frequency range.

electron beam lines are shown, along with the light line. Intersections between the dispersion plot and the beam lines suggest regions where energy transfer between beam and wave can occur. From this plot we observe that forward wave interactions with a 20-30 keV electron beam are possible around 9 GHz, and backward wave interactions are possible around 11GHz, suggesting the possible dual-use of this AM in the X-band region, all occurring with losses of less than 0.05.

# CONCLUSIONS

In this paper numerical simulations have shown that Artificial Materials (AMs) provide a route into producing devices with engineerable properties. This enables the user to tailor the dispersion relation to achieve beam-wave interactions, producing desired effects such as Inverse-Cherenkov acceleration. By manipulating the geometry of the constituent unitcells, the user can optimise energy transfer while keeping losses to a minimum. The material presented here suggests successful high power operation, with the possibility of both forward and backward wave operation.

To establish the feasibility of this design, further simulations characterising the thermal behaviour of this AM need

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Figure 8:  $\omega$ - $\beta$  plot for the Optimised Unit-Cell, showing interactions regions around 9 GHz (forward-wave) and 11 GHz (backward-wave).

to be produced. Particle In Cell (PIC) simulations need to be produced investigating whether the material will successfully interact with the charged particle beam, and ensuring issues like charge build-up are not a problem. Finally, the AM will need to be fabricated and tested in both low and high power environments to ensure it demonstrates successful energy transfer, and successful Inverse-Cherenkov acceleration.

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