CALCULATION OF TRANSVERSE COUPLED BUNCH INSTABILITIES IN ELECTRON STORAGE RINGS DRIVEN BY QUADRUPOLE HIGHER ORDER MODES*

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Abstract

This paper presents a formula that estimates the growth rate of a transverse coupled bunch instability driven by quadrupole higher order modes (HOMs) in electron storage rings. Thus far, quadrupole HOMs are usually ignored in HOM driven instability studies for electron storage rings due to their weak nature compared to the lower orders. However, they may become relevant when high gradient SC multi-cell cavities with their potentially strong impedance spectrum are operated at high currents in a third generation or future synchrotron light source. An example is BESSY VSR, a scheme where 1.7 ps and 15 ps long bunches (rms) can be stored simultaneously in the BESSY II storage ring [1]. With the presented formula, instability thresholds are discussed for a recent BESSY VSR cavity model and different beam parameters.

INTRODUCTION

Coupled bunch instabilities (CBIs) driven by higher order modes (HOMs) of RF cavities or other narrow-band impedances are a classic type of instability encountered in many storage rings. An extended study of CBIs relating to BESSY VSR, the upgrade scheme of BESSY II to obtain short and long bunches simultaneously [1, 2], has recently been published [3]. This paper presents findings related to quadrupole HOMs.

The consequence of a CBI is either beam loss, usually in the transverse plane, or a saturation of the beam oscillation at very large amplitudes if it is occurring in the longitudinal plane. In both cases, the instability must be avoided to ensure the high beam quality promised by all low emittance storage rings. Common methods to suppress CBIs include the usage of an active bunch-by-bunch feedback (BBFB) in each plane that tracks the dipole motion, i.e., center of mass motion, of the beam and applies kicks such that the oscillation amplitude is reduced.

Typically, studies of HOM driven CBIs consider only two special cases, namely the dipole motion in the longitudinal or transverse plane caused by the interaction with monopole or dipole HOMs respectively. Formulas to estimate the growth rate of these instabilities based on the machine parameters and the impedance spectrum are used frequently in literature, see for example [3] and references therein. If the growth rates are compared to a damping rate, e.g. the damping rate of the BBFB, a threshold impedance is found. The threshold impedance for the longitudinal and transverse dipole CBIs driven by monopole and dipole HOMs is given by [4]

$$Z^{\parallel}_{0,\mathrm{th}}(\omega, \tau_\mathrm{d}^{-1}) = \frac{\tau_\mathrm{d}^{-1}\omega_\mathrm{s}}{\omega\alpha}\frac{4\pi E/e}{\omega_\mathrm{rev} I}, \tag{1}$$

$$Z^{\perp}_{1,\mathrm{th}}(\tau_\mathrm{d}^{-1}) = \frac{\tau_\mathrm{d}^{-1}}{\beta}\frac{4\pi E/e}{\omega_\mathrm{rev} I}, \tag{2}$$

respectively, where the first index of Z represents the order m of the impedance and ω the angular frequency at which the impedance is sampled, ω_rev the angular revolution frequency, ω_s the angular synchrotron frequency, τ_d^{-1} the damping rate, α the momentum compaction factor, E the beam energy, e the elementary charge, β the betatron function at the position of the cavity. As an example, Fig. 1 shows the impedance of the 1.5 GHz cavity model of the BESSY VSR project [5] compared to the threshold impedance based on the BESSY II machine parameters and the present BBFB damping performance [1–3].

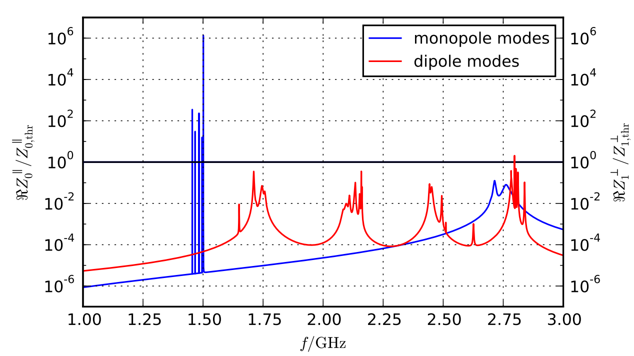

Figure 1: Ratio of real part of monopole and dipole impedance of 1.5 GHz cavity model of the BESSY VSR project [5] to the longitudinal $m = 0$ and transverse $m = 1$ CBI threshold with BESSY II machine parameters and damping provided by a BBFB system [1–3].

With the technological advance of high frequency SC multi-cell cavities and the possibility to deploy them in present or future high current storage rings, the question of HOM interactions must be addressed again. Due to the superconductivity, the large number of cells and high frequency, strong HOMs may be present, which necessitates a study including the next higher mode order, the quadrupole HOMs.

In the following, a study of CBIs driven quadrupole HOMs is presented together with a formula that approximates the

* Work supported by German Bundesministerium für Bildung und Forschung, partly under contract 05K13PEB, and Land Berlin.
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ISBN 978-3-95450-147-2

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growth rate in a similar fashion as the well known formulas for the lower order cases. The findings are then applied to the case of BESSY VSR.

EQUATION OF MOTION

The following formulas of the transverse particle dynamics under influence of a quadrupole HOM, i.e. a transverse $m = 2$ impedance, are in analogy to the dipole $m = 1$ case described in [4, Ch. 4.2].

For simplicity, only the zeroth coupled bunch mode (CBM) and a purely horizontal motion (x coordinate) is considered. The coupling to the longitudinal motion, which necessarily arises is ignored as it is a small disturbance. Furthermore, the beam is reduced to a single one-particle bunch of charge q which performs horizontal betatron oscillations and possesses an instantaneous quadrupole moment in the horizontal direction, $qM_2 = qx(s)^2$, with q the bunch charge and $M_2 = \langle x^2 - y^2 \rangle$ the (normal) transverse quadrupole moment.

The horizontal wake force that a probing particle with charge e sees at a distance d behind the beam is given by the wake potential $c\Delta p_m^\perp$ [4, Tab. 2.2]:

$$c\Delta p_m^\perp = -2qex^2(s)W_2(-d)\bar{x}(s), \tag{3}$$

with the horizontal coordinate of the beam $x(s)$, the horizontal coordinate of the probing particle $\bar{x}(s)$ and the $m = 2$ wake function $W_2(s)$ as a function of the longitudinal coordinate s defined as

$$W_2(s) = \frac{cR_{s,2}}{Q} e^{\frac{\omega_r s}{2Qc}} \sin\frac{\omega_r s}{c}, \tag{4}$$

with $R_{s,2}$ the quadrupole shunt impedance in circuit definition[1], Q the quality factor, ω_r the angular resonance frequency, and c the speed of light.

The equation of motion is then given by

$$x''(s) + \left(\frac{\omega_\beta}{c}\right)^2 x(s) = -\frac{2q}{CE/e}x(s)\sum_{k=1}^{\infty} x^2(s-kC)W_2(-kC) \tag{5}$$

with the circumference C, ω_β the horizontal angular betatron frequency, $x'(s) = \mathrm{d}x(s)/\mathrm{d}s$ and the summation over k takes into account the contributions of the wake function from all previous passages. Unlike the equations of motion in lower order cases $m \leq 1$, Eq. (5) is a non-linear differential equation, whose solution is cumbersome. However, the problem can be solved numerically. In the following, a tracking algorithm is presented that describes the particle dynamics.

TRACKING

The tracking algorithm presented includes the same simplifications as before. It is convenient to separate Eq. (3) into a part that depends on the momentary properties of the field in the quadrupole HOM and a part dependent on the coordinates of the probing particle. This can be achieved by defining a complex phasor V with the dimension V/m^2 that represents the field in the quadrupole HOM [3]. Each bunch passage adds a field to the phasor,

$$V_{\mathrm{after}}(t) = V_{\mathrm{before}}(t) - (x(t) - x_{\mathrm{offset}})^2 q\frac{\omega_r R_{s,2}}{Q}, \tag{6}$$

with a squared dependence on the transverse coordinate x and an optional transverse offset of the reference orbit w.r.t. the electromagnetic center of the resonator mode x_{offset}. Note that a time dependent form of the wake function was used here. According to Eq. (3), the transverse kick voltage on a probing particle is then be given by

$$V_{\mathrm{acc}}^\perp(x(t)) = 2x(t)\frac{c}{\omega_r}\,\mathrm{Im}\,V(t), \tag{7}$$

with linear dependence in its transverse coordinate x.

Between two bunch passes, t and t_{last}, the phasor V rotates and decays according to

$$V(t) = V(t_{\mathrm{last}})e^{i\omega_r(t-t_{\mathrm{last}})}e^{-\frac{\omega_r(t-t_{\mathrm{last}})}{2Q}}. \tag{8}$$

Particle transport around the ring between two passes of the cavity can be represented by a linear transformation,

$$X(t) = X(t_{\mathrm{last}})e^{i\omega_\beta(t-t_{\mathrm{last}})}, \tag{9}$$

where $X = x + i\tilde{x}$ represents the transverse phase space coordinates, where the transverse kick voltage is applied as

$$\tilde{x}(t) = \tilde{x}(t_{\mathrm{last}}) + V_{\mathrm{acc}}^\perp(x(t))\frac{\beta}{E/e}. \tag{10}$$

RESULTS

With the algorithm described above, a tracking code was written to study the particle dynamics. It was found that the motion is anharmonic and an instability occurs which depends on the initial conditions $(x, \tilde{x})$ and exhibits a growth faster than exponential. The momentary growth rate of the instability can be extracted from the trajectories of the tracking simulations as a function of all input parameters, as depicted in Fig. 2.

With the scalings obtained from Fig. 2, the comparison with the well known $m = 1$ case and an absolute normalization from any particular simulation run, a formula for the momentary exponential growth rate as a function of all input parameters can be written down,

$$\tau^{-1}(M_2, x_{\mathrm{offset}}) \approx \left(\frac{M_2}{2} + \frac{x_{\mathrm{offset}}^2}{4}\right)\frac{\omega_{\mathrm{rev}} I\beta}{4\pi E/e}\,\mathrm{Re}\,Z_2^\perp. \tag{11}$$

In Eq. (11), it is assumed that there is a singe resonator driven on resonance. More generally, the impedance must be summed over all harmonics,

$$Z_2^\perp = \sum_{p=-\infty}^{\infty} \mathrm{Re}\,Z_2^\perp((pM+\mu)\omega_{\mathrm{rev}} + 2\omega_\beta), \tag{12}$$

[1] $R_s = V_{\mathrm{acc}}^2/(2P_{\mathrm{diss}})$ with V_{acc} the maximum effective accelerating voltage and P_{diss} the dissipated power (m order omitted)

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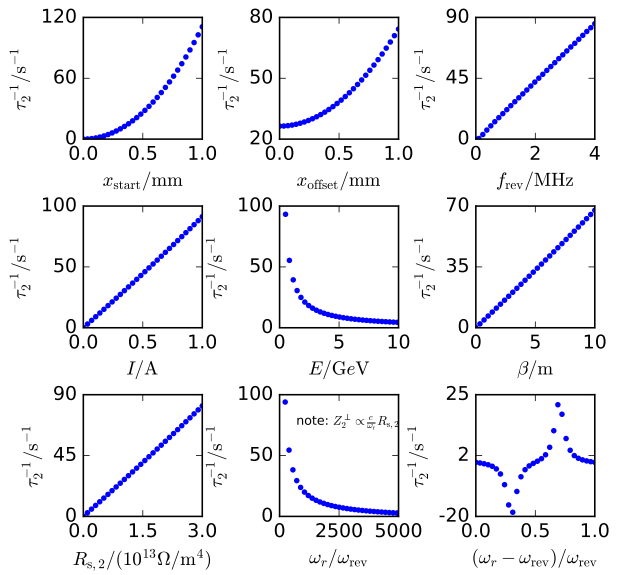

Figure 2: Instability growth rate obtained from tracking simulations as a function of the input parameters with the non-varied parameters as follows: $\omega_{\text{rev}} = 2\pi \cdot 1.25\,\text{MHz}$, $I = 300\,\text{mA}$, $E = 1.7\,\text{GeV}$, $\beta = 4\,\text{m}$, $\omega_\beta = 17.85\omega_{\text{rev}}$, $R_{\text{s},2} = 1 \times 10^{13}\,\Omega\text{m}^{-4}$, $\omega_{\text{r}} = 765\omega_{\text{rev}} + 2\omega_\beta$, $Q = 1 \times 10^4$, $x_{\text{start}} = 0.5\,\text{mm}$, $x_{\text{offset}} = V(0) = \tilde{x}(0) = 0$, maintaining the resonance condition except for the bottom right plot.

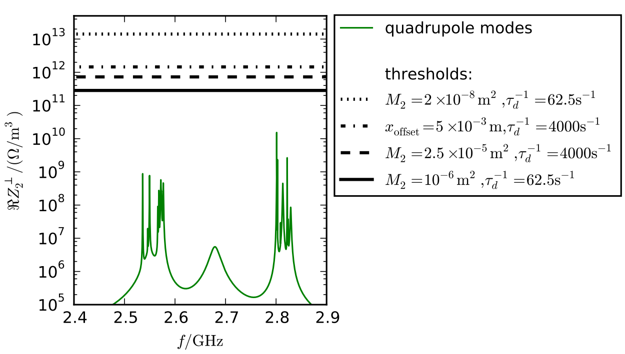

Figure 3: Real part of quadrupole impedance of 1.5 GHz cavity model of the BESSY VSR project [5] in comparison with different scenarios of the transverse $m = 2$ CBI threshold estimated by Eq. (11) with BESSY II machine parameters [2] and the parameters listed in the legend. Damping rates of $62.5\,\text{s}^{-1}$ and $4000\,\text{s}^{-1}$ correspond to radiation and BBFB damping respectively.

with the CBM μ, the maximum number of bunches in the ring M and the transverse $m = 2$ resonator impedance defined as

$$Z_2^\perp(\omega) = \frac{c}{\omega} \frac{R_{\text{s},2}}{1 + iQ\left(\frac{\omega_{\text{r}}}{\omega} - \frac{\omega}{\omega_{\text{r}}}\right)}. \qquad (13)$$

The highest growth rate appears if the resonance frequency of the quadrupole HOM equals a second betatron sideband.

As an example, Fig. 3 depicts the quadrupole modes of the 1.5 GHz cavity model of BESSY VSR mentioned above in comparison to the impedance thresholds obtained with Eq. (11) for different parameters for quadrupole moment M_2, the transverse offset x_{offset} and the damping rate τ_{d}^{-1}, according to the following realistic, albeit extreme scenarios:

- $M_2 = 2 \times 10^{-8}\,\text{m}^2, \tau_{\text{d}}^{-1} = 62.5\,\text{s}^{-1}$. A stable beam on axis w.r.t. the orbit and the HOM center. M_2 is defined by the transverse equilibrium extension of the beam, i.e. the local beta function and the horizontal emittance $M_2 \approx \beta\epsilon_x$, as the beam is flat. This configuration has no dipole moment, thus only radiation provides damping.
- $x_{\text{offset}} = 5\,\text{mm}, \tau_{\text{d}}^{-1} = 4 \times 10^3\,\text{s}^{-1}$. A stable beam on axis w.r.t. the orbit but with the HOM center shifted by x_{offset} w.r.t. the orbit. The resulting oscillation has a dipole moment, thus the BBFB can provide damping.
- $M_2 = 2.5 \times 10^{-5}\,\text{m}^2, \tau_{\text{d}}^{-1} = 4 \times 10^3\,\text{s}^{-1}$. A beam with a momentary displacement of 5 mm w.r.t. the orbit, leading to $M_2 = (5\,\text{mm})^2$. The HOM center is aligned with the orbit. As this beam has a dipole moment, the BBFB can provide damping.
- $M_2 = 1 \times 10^{-6}\,\text{m}^2, \tau_{\text{d}}^{-1} = 62.5\,\text{s}^{-1}$. A beam on axis w.r.t. the orbit and the HOM center but with momentary horizontal Gaussian extension of $\sigma = 1\,\text{mm}$. Thus $M_2 = \sigma^2$ and only radiation can provide damping.

CONCLUSION

A formula to estimate the growth rate of a transverse CBI driven by quadrupole HOMs has been presented. Most notably, this growth rate depends on the amplitude of oscillation, i.e. the quadrupole moment M_2, thus estimations require a plausible set of starting conditions. The application of the formula to the case of BESSY VSR confirms the general expectation that the instability driven by quadrupole HOMs is weaker than those driven by the lower HOM orders. For the 1.5 GHz BESSY VSR cavity model, the quadrupole impedance is about a factor of 10 below the instability threshold for the most critical scenario considered in this paper.

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