# RADIATION OF CHARGED PARTICLE FLYING INTO CHIRAL ISOTROPIC MEDIUM*

Sergey N. Galyamin†, Andrey V. Tyukhtin, Anton A. Peshkov
St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034 Russia

*Abstract*

In recent years, the interest to radiation of moving charged particles in media with chiral properties is connected with relatively new and prospective method for diagnostics of biological objects which uses the Cherenkov radiation – Cherenkov luminescence imaging [1]. Optical activity (chirality, gyrotropy) is typical or biological matter and is caused by mirrorless structure of molecules. Contrary to such gyrotropic medium as magnetized ionospheric plasma, aforementioned media are isotropic. One distributed model describing the frequency dispersion of isotropic chiral media is Condon model. In this report, we continue the investigation performed in our previous paper [2] where we dealt with the field produced by uniformly moving charge in infinite chiral isotropic medium. Moreover, we perform generalization of early paper [3], where the problem with half-space was considered in the specific case of slow charge motion. We present typical radiation patterns in vacuum area and corresponding ellipses of polarization which allows determination of the chiral parameter of the medium.

## THEORY AND ANALYTICAL RESULTS

Chiral isotropic medium can be described by the following symmetrized material relations [2]:

$$\vec{D}_\omega = \varepsilon \vec{E}_\omega - i\kappa \vec{H}_\omega, \qquad \vec{B}_\omega = \mu \vec{H}_\omega + i\kappa \vec{E}_\omega \tag{1}$$

connecting Fourier transforms (both 1-fold, as given by formula (1), and four-fold with respect to time and tree spatial coordinates) of field components. Condon model [4] dictates the following frequency dependence for $\varepsilon$ and $\kappa$ ($\mu = 1$):

$$\varepsilon(\omega) = 1 + \omega_p^2 \left[\omega_r^2 - \omega^2\right]^{-1}, \quad \kappa(\omega) = \omega_0 \omega \left[\omega_r^2 - \omega^2\right]^{-1}, \tag{2}$$

where $\omega_r$ is resonant frequency, $\omega_p$ is a "plasma" frequency and $\omega_0$ is a chirality parameter. Introducing "potentials" $\vec{E}_\omega^+$ and $\vec{E}_\omega^-$ [2], so that

$$\vec{E}_\omega = \vec{E}_\omega^+ + \vec{E}_\omega^-, \quad \vec{H}_\omega = \vec{H}_\omega^+ + \vec{H}_\omega^- = i\frac{\sqrt{\varepsilon\mu}}{\mu}\left(\vec{E}_\omega^+ - \vec{E}_\omega^-\right), \tag{3}$$

one can obtain the following independent equations:

$$\begin{aligned} &\Delta \vec{E}_\omega^\pm + \omega^2 c^{-1} n_\pm^2 \vec{E}_\omega^\pm = \\ &= 2\pi\mu n^{-1}\left(-i\omega c^{-2} n_\pm \vec{j}_\omega + n_\pm^{-1}\nabla\rho_\omega \pm i c^{-1}\operatorname{rot}\vec{j}_\omega\right), \end{aligned} \tag{4}$$

with $n = \sqrt{\varepsilon\mu}$, $n_\pm = \sqrt{\varepsilon\mu} \pm \kappa$, $\operatorname{Re}\sqrt{\ } > 0$. Fundamental solutions of (4) are two circularly polarized plane waves,

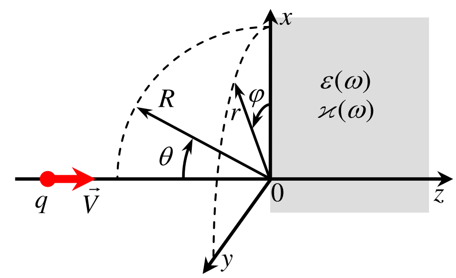


Figure 1: Geometry of the problem.

with vectors $\vec{E}^+$ and $\vec{H}^+$ rotating clockwise (right-hand polarization, "+") while vectors $\vec{E}^-$ and $\vec{H}^-$ rotating counterclockwise (left-hand polarization, "−") in the plane orthogonal to propagation direction.

Here we consider a point charge $q$ flying from vacuum into medium (2) with constant velocity $V = \beta c$ (Fig. 1). Charge and current densities are:

$$\rho = q\delta(x)\delta(y)\delta(z - Vt), \quad \vec{j} = \vec{e}_z V \rho. \tag{5}$$

Using Fourier technique, we determined the field in both half spaces and then perform matching of tangential components for $z = 0$. On the basis of relation

$$\frac{\partial^2 E_{\omega r}^\pm}{\partial r^2} + \frac{1}{r}\frac{\partial E_{\omega r}^\pm}{\partial r} - \frac{1}{r^2}E_{\omega r}^\pm = \frac{2\pi}{n n_\pm}\frac{\partial \rho_\omega}{\partial r} - \frac{1}{2}\frac{\partial^2 E_{\omega z}^\pm}{\partial r \partial z}, \tag{6}$$

and analogous one for $E_{\omega\varphi}^\pm$, we can formulate the problem with respect to longitudinal components of potentials $E_{\omega z}^\pm$ only. As usual, the field in each half space is a sum of charge's self-field and additional (radiation) field excited by the boundary. After a series of cumbersome calculations, we obtain that radiation field in vacuum area ($z < 0$) is a sum of two waves, first of them has polarization coinciding with that of self-field ($E_{\omega r}$, $E_{\omega z}$ and $H_{\omega\varphi}$ components, co-polarization), while the second one has the orthogonal polarization ($E_{\omega\varphi}$, $H_{\omega r}$ and $H_{\omega z}$ components, cross-polarization):

$$H_{\omega\varphi}^{rad} = \frac{q k_0}{2\pi\omega}\int_0^{+\infty} dk_r J_1(k_r r) B_{co}^v \exp\left(i k_z^v |z|\right), \tag{7}$$

$$E_{\omega\varphi}^{rad} = \frac{-i k_0 q}{2\pi\omega}\int_0^{+\infty} dk_r J_1(k_r r) B_{cr}^v \exp\left(i k_z^v |z|\right), \tag{8}$$

$$\begin{aligned} B_{cr}^v = \frac{k_r^2}{\Delta_c}\Bigg[ & n\frac{n_+ k_z^{m-} - n_- k_z^{m+}}{\omega V^{-1} + k_z^v} - \\ & - \frac{k_z^{m-} + n n_- k_z^v}{\omega V^{-1} + k_z^{m+}} + \frac{k_z^{m+} + n n_+ k_z^v}{\omega V^{-1} + k_z^{m-}}\Bigg]. \end{aligned} \tag{9}$$

*Work supported by the Grant from Russian Foundation for Basic Research (No. 15-32-20985).
† s.galyamin@spbu.ru

$$B_{co}^{v}=\frac{k_r^2}{\Delta_c}\frac{\left(k_z^v n_- + nk_z^{m-}\right)\left(\omega V^{-1}\varepsilon n_+ n^{-1}-k_z^{m+}\right)}{k_r^2-s_v^2}+ \\ +\frac{k_r^2}{\Delta_c}\frac{\left(k_z^v n_+ + nk_z^{m+}\right)\left(\omega V^{-1}\varepsilon n_- n^{-1}-k_z^{m-}\right)}{k_r^2-s_v^2}- \\ -\frac{k_r^2}{\Delta_c}\left(\frac{k_z^v n_- + nk_z^{m-}}{\omega V^{-1}+k_z^{m+}}+\frac{k_z^v n_+ + nk_z^{m+}}{\omega V^{-1}+k_z^{m-}}\right), \quad (10)$$

$$2\Delta_c = n\Big[n_+\left(k_z^v n_- + nk_z^{m-}\right)\left(k_z^{m+}(n_+ n)^{-1}+k_z^v\right)+ \\ +n_-\left(k_z^v n_+ + nk_z^{m+}\right)\left(k_z^{m-}(n_- n)^{-1}+k_z^v\right)\Big], \quad (11)$$

$k_0=\omega/c$, $k_z^v=\sqrt{k_0^2-k_r^2}$, $k_z^{m\pm}=\sqrt{k_0^2 n_\pm^2-k_r^2}$, $\mathrm{Im}\sqrt{\ }>0$, $s_v^2=\omega^2 V^{-2}\left(\beta^2-1\right)$. In the case of weak chirality, $|\kappa/n|<<1$, coefficient (9) is proportional to $\kappa$.

Far-field ($k_0 R>>1$) can be determined from formulas (7) and (8) using saddle-point technique:

$$E_{\omega\theta}^{rad}=-H_{\omega\varphi}^{rad}=\frac{qk_0}{2\pi\omega}\cot\theta\, B_{co}^v\Big|_{k_r=k_0\sin\theta}\frac{\exp(ik_0R)}{R}, \quad (12)$$

$$H_{\omega\theta}^{rad}=E_{\omega\varphi}^{rad}=\frac{iqk_0}{2\pi\omega}\cot\theta\, B_{cr}^v\Big|_{k_r=k_0\sin\theta}\frac{\exp(ik_0R)}{R}, \quad (13)$$

while $R$-component tends to zero. There are two transversal spherical waves with linear polarization. They have identical phase velocity $c$. Vectors of electric field of these waves are mutually orthogonal, $\vec{E}_{\omega}^{rad}$, $\vec{H}_{\omega}^{rad}$ and $\vec{e}_R$ form right-hand orthogonal set. If $B_{co}^v$ and $B_{cr}^v$ are purely real, then phase difference between waves (12) and (13) is equal to $\pi/2$, and the summary wave has elliptic polarization with main axes $\vec{e}_\theta$ and $\vec{e}_\varphi$. In other cases, polarization ellipse is rotated over some angle depending on phase of polarization coefficient

$$P=|P|e^{i\varphi_P}=E_{\omega\varphi}^{rad}\Big/E_{\omega\theta}^{rad}=i\,B_{cr}^v\big/B_{co}^v\Big|_{k_r=k_0\sin\theta}. \quad (14)$$

In general case, we have in the harmonic regime

$$E_\theta(t)=\mathrm{Re}\left[E_{\omega\theta}^{rad}\exp(-i\omega t)\right], \quad (15)$$

$$E_\varphi(t)=\mathrm{Re}\left[E_{\omega\varphi}^{rad}\exp(-i\omega t)\right]. \quad (16)$$

Excluding time dependence from (15) and (16), we obtain the following relation:

$$\frac{E_\varphi^2}{E_0^2|P|^2}+\frac{E_\theta^2}{E_0^2}-2\frac{E_\varphi E_\theta}{E_0^2|P|}\cos\varphi_P=\left(\sin\varphi_P\right)^2, \quad (17)$$

where

$$E_0=\frac{qk_0}{2\pi\omega}\frac{\cot\theta}{R}B_{co}^v\Big|_{k_r=k_0\sin\theta}. \quad (18)$$

Equation (17) determines the polarization ellipse, i.e. hodograph of the vector $\vec{E}=\vec{e}_\theta E_\theta+\vec{e}_\varphi E_\varphi$. As one can show from (17), main axes of this ellipse are rotated with respect to $\vec{e}_\theta$ and $\vec{e}_\varphi$ at angle $\psi_{pe}$ determined by the following relation:

$$\tan 2\psi_{pe}=2|P|\cos\varphi_P\left(|P|^2-1\right)^{-1}. \quad (19)$$

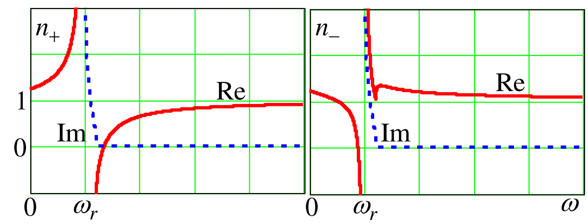

Figure 2: Typical dependencies of $n_\pm$ over $\omega$.

## NUMERICAL RESULTS

Typical behaviour of $n_\pm(\omega)$ in accordance with (2) is shown in Fig. 2. We choose 3 frequencies and calculate radiation patterns in the far-field zone using (12) and (13) for relatively small ($\beta=0.1$) and relatively large ($\beta=0.9$) charge's velocity. Results are shown in Fig. 3.

In most cases, radiation patterns have single lobe. Magnitude of the lobe for $E_{\omega\theta}^{rad}$ components is usually several orders larger compared with that for $E_{\omega\varphi}^{rad}$. However, for large frequency ($\omega=5\omega_r$) these lobes are comparable. In some cases patterns have two lobes. In particular, for $\omega=5\omega_r$ second lobe for angles close to $\pi/2$ (near the interface) is connected with lateral wave (contribution of the branch point). In fact, magnitude of this lobe is greater than that of the "ordinary" lobe.

We have also plotted polarization ellipses for angles of lobe's maximum (these angles indicated within each lobe in Fig. 3). It is supposed that corresponding wave propagates to the observer (along $\vec{e}_R$). In most cases, these ellipses are strongly prolonged along $\vec{e}_\theta$ direction, i.e. spherical waves have expressed elliptical polarization with main axes $\vec{e}_\theta$ and $\vec{e}_\varphi$. However, for $\omega=5\omega_r$ polarization ellipse is similar to circle ($\beta=0.1$), while for $\beta=0.9$ its main axes are rotated by considerable angle.

## REFERENCES


[1] Spinelli A.E. et al. *Nucl. Instr. Meth. Phys. Res. A.*, vol. 648, p. S310, 2011.
[2] Galyamin S.N., Peshkov A.A., Tyukhtin A.V. *Phys. Rev. E*, vol. 88, p. 013206, 2013.
[3] Engheta N. and Mickelson A.R., *IEEE Trans. Antennas Propag.*, vol. 30, No. 6, p. 1213, 1982.
[4] Condon E.U. *Rev. Mod. Phys.*, vol. 9, p. 432, 1937.





ISBN 978-3-95450-147-2

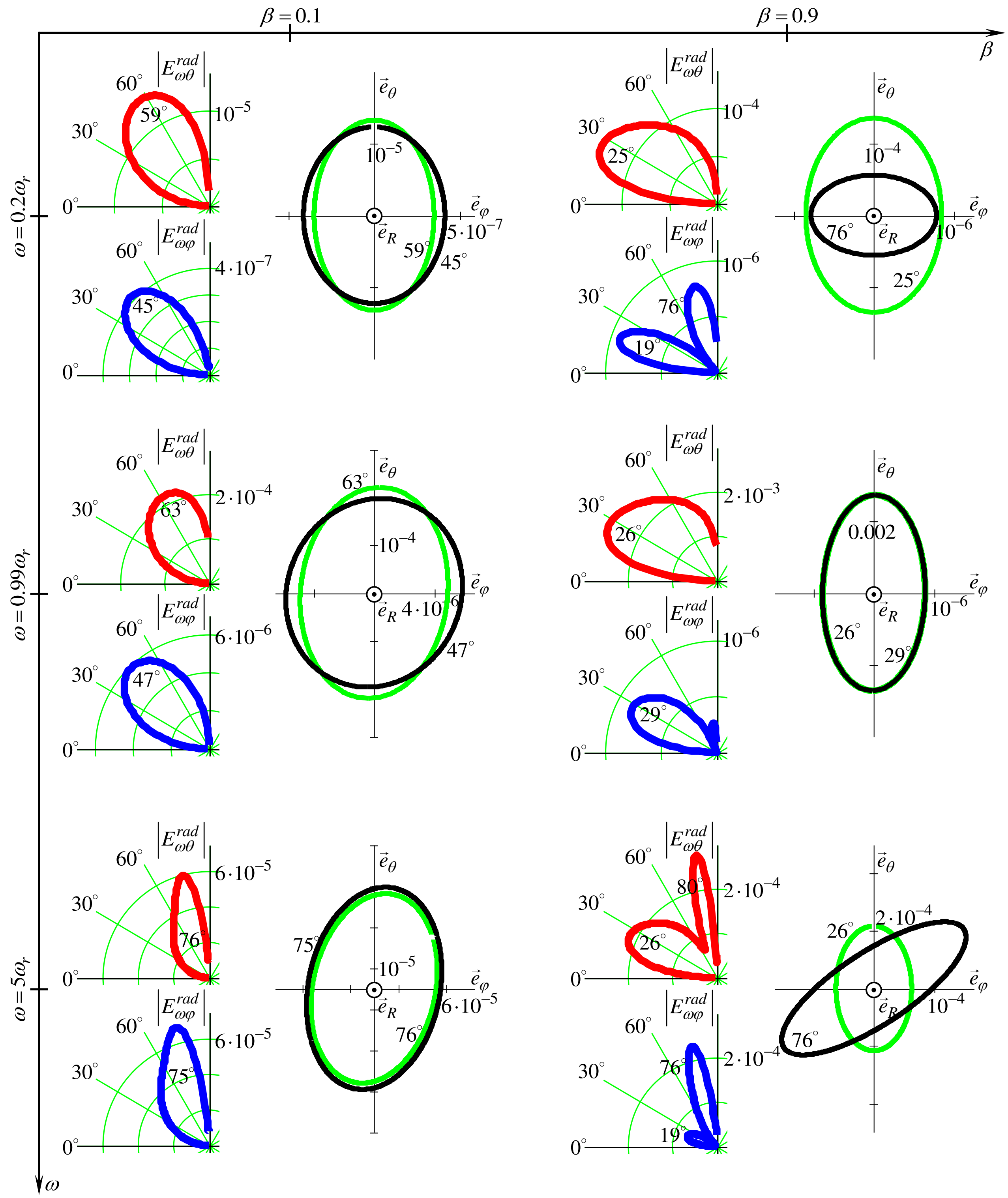


Figure 3: Radiation patterns ( $|E^{rad}_{\omega\theta,\varphi}|$ in units $q\omega_p^2/c^2$ over $\theta$ ) for $k_0R = 100$ and ellipses of polarization (plotted for angles of lobes' maxima) in vacuum area for three frequencies of Condon model (2) with $\omega_p = \omega_r$ , $\omega_0 = 0.3\omega_p$ . Waves propagate to the observer (along $\vec{e}_R$ ) and vector $\vec{E}$ rotates along polarization ellipses in $(\varphi,\theta)$ -plane.