SIMULATED MEASUREMENTS OF COOLING IN MUON IONIZATION COOLING EXPERIMENT*

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Abstract

The international Muon Ionization Cooling Experiment (MICE) collaboration aims to demonstrate muon-beam cooling through ionization energy loss in material. Cooled muon beams are essential to enable future neutrino factory and muon collider facilities. A figure of merit for muon cooling in MICE is the root-mean-square (RMS) emittance reduction. To measure this, the individual muon positions and momenta will be reconstructed using two scintillating-fiber tracking detectors. However, due to the existence of nonlinear effects in beam optics leading to phase-space distortion, an apparent RMS emittance growth may be observed. In this paper, we describe an alternative approach to measuring the efficacy of muon cooling, the direct measurement of phase-space density using the kernel density estimation algorithm.

INTRODUCTION

Proposed future accelerator facilities that use muon beams, such as a neutrino factory or muon collider, demand an intrinsic brilliance significantly greater than that obtained from a decay-channel muon source. The beam must therefore be cooled, i.e. undergo a forced reduction of its phase-space volume. Conventional cooling techniques, such as synchrotron radiation and stochastic cooling, are not efficient due to the short muon lifetime. Ionization cooling is the only feasible technique [1, 2]. In ionization cooling, the beam is passed through an absorbing material and its momentum and momentum spread are reduced as it loses energy to ionization of the atomic electrons of the material. The reduction in beam momentum spread leads to a reduction in the beam RMS emittance and an increase in its phase-space density. The rate of change of the normalized transverse RMS emittance ($\varepsilon_\perp$) is given by the ionization cooling equation [3]:

$$\frac{d\varepsilon_\perp}{ds} \simeq -\frac{\varepsilon_\perp}{\beta^2 E_\mu}\left\langle\frac{dE}{ds}\right\rangle + \frac{\beta_\perp (13.6\text{MeV})^2}{2\beta^3 E_\mu m_\mu X_0}, \tag{1}$$

where β, E_μ, and m_μ are the muon velocity, energy, and mass, $\frac{dE}{ds}$ the magnitude of the mean energy loss rate through ionization, X_0 the absorber radiation length, and $\beta_\perp$ the transverse beta function at the absorber.

A schematic diagram of MICE Step IV is displayed in Fig. 1. The ionization energy loss required is achieved through the use of a low-Z absorbing material, preferably lithium hydride (LiH) or liquid Hydrogen (LH_2). MICE has upstream and downstream tracking detectors. Each tracker is composed of five scintillating-fiber planar stations each with three doublet fiber layers, located in a region of constant solenoidal field within the spectrometer-solenoid module. The spectrometer solenoid is composed of five coils, three for the constant field region and two for matching. In addition, a series of particle-identification (PID) detectors, as shown in Fig. 1 are used to ensure a pure selection of muons in the MICE beam.

The input and output beam distributions are compared at the inner face of the tracker stations closest to the absorber. Tracks that pass through both upstream and downstream trackers are selected for this analysis. Trackers reconstruct positions and momenta of individual muons at tracker reference planes through a series of cluster finding, space point reconstruction and helical track fitting algorithms [4]. These position and momentum values are then used for constructing the covariance matrix thereby deriving normalized transverse RMS emittance; however, RMS emittance is sensitive to nonlinear effects. A novel phase-space density calculation based on the Kernel Density Estimation (KDE) algorithm [5] is proposed as an alternative method for quantifying cooling performance in MICE.

KERNEL DENSITY ESTIMATION

The state of a system of N particles in phase space is described in terms of the probability density function (PDF). If the particle distribution in phase space lacks a parametric model, its underlying PDF can be found using a non-parametric density estimation (DE) technique [6]. Kernel Density Estimation (KDE) is an example of a non-parametric DE technique and it relies on the use of kernel smoothing functions to approximate the PDF of a random variable [5],

$$\hat{f}(\vec{x}) = \frac{1}{h^d N}\sum_{i=1}^{N} K\left(\frac{\vec{x}-\vec{x}_i}{h}\right), \tag{2}$$

where $\hat{f}(\vec{x})$ is the estimated density of muons, h the bandwidth parameter (a definition of h is given in the paragraphs which follow), d the phase-space dimension, and

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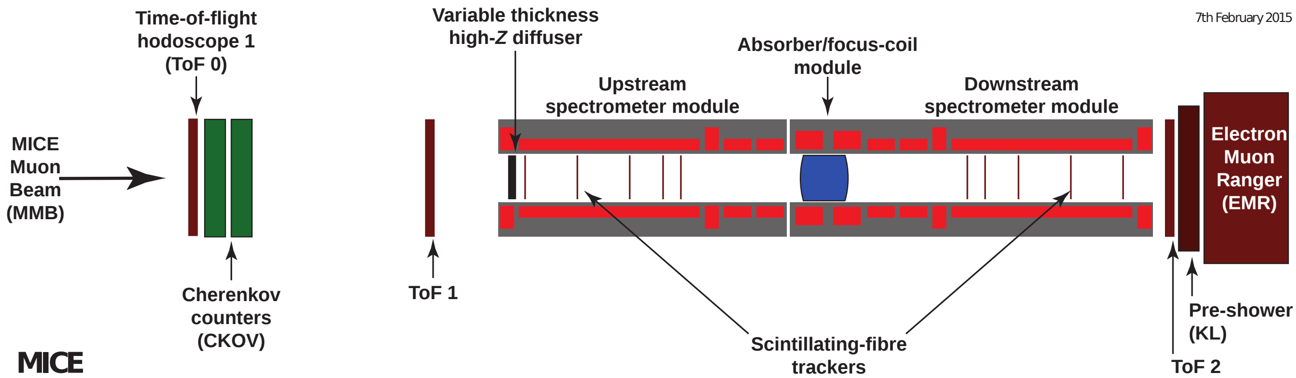

Figure 1: Schematic diagram of the Muon Ionization Cooling Experiment, in its current configuration, showing input and output spectrometers surrounding a cooling cell, and other particle ID detectors (Time--of--Flight, Cherenkov, KL, and Electron–Muon Ranger).

K the kernel function. Each muon is represented by $\vec{x}_i = (x, p_x, y, p_y)$.

To estimate properly the underlying PDF, the kernel should be an even function with similar characteristics to the PDF – non-negative and real-valued whose integral over phase space equals unity [7]. DE analysis in this paper is performed using Gaussian kernel functions,

$$K\left(\frac{\vec{x}-\vec{x}_i}{h}\right) = \frac{1}{(2\pi)^{\frac{d}{2}}} \exp\left(-\frac{|\vec{x}-\vec{x}_i|^2}{2h^2}\right). \quad (3)$$

A Gaussian kernel function is assigned with a center at x_i. The kernel is then smeared based on the bandwidth parameter h, and the individual kernel functions are summed to estimate the density. Different techniques can be used to determine h [8]. Scott's rule of thumb bandwidth selector [9] is used in this analysis.

SIMULATION RESULTS AND DISCUSSIONS

The initial particle distribution for Monte Carlo (MC) simulations is generated using MICE Analysis User Software (MAUS) [10]. The transverse profile of the initial beam is Gaussian and is matched to 4 T fields of the upstream spectrometer-solenoid modules. The emittance and momentum are set to 3π mm·rad and 200 MeV/c, respectively. The beam consists of 10k muons that are tracked, using G4beamline [11], through the MICE Step IV lattice. The currents in the spectrometer solenoids and absorber focus coils shown in Table 1, are set so that the resulting magnetic field flips sign at the center of the LiH absorber to prevent build-up of net angular momentum—the so-called "flip mode". An accurate measurement of phase-space density is ensured by selecting only those tracks that pass through both the upstream and downstream trackers. This has a selection efficiency of 99.99%.

Phase-space contour plots of upstream and downstream (x, p_x) distributions are shown in Fig. 2. The contour lines represent constant density slices of the distribution. The density is estimated using Python `scipy` package [12].

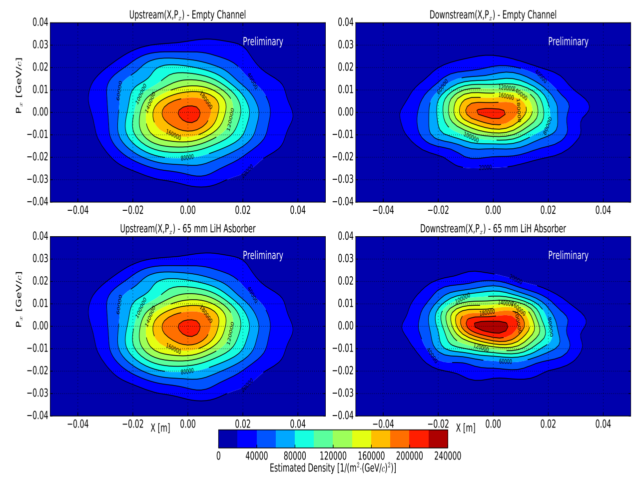

Figure 2: Phase-space contour plots in (x, p_x) coordinates and their corresponding estimated densities at the upstream (left) and downstream (right) tracker reference planes. An increase in the particle density at the beam center is observed.

Table 1: Currents of upstream spectrometer solenoid and absorber focus coils as specified in G4beamline MICE Step IV lattice. The downstream spectrometer coils have the same currents with flipped signs.

Coils	Currents A/mm^2
End1	126
Center	148
End2	133
Match2	132
Match1	133
Upstream Focus	104

In Fig. 2, the contours are colored by their corresponding estimated densities, and the bottom right plot shows an increase in estimated density from the upstream tracker reference plane to the downstream (cooling). To take the solenoidal transverse coupling into account, the KDE algorithm is applied to the four dimensional (x, p_x, y, p_y) transverse plane and Fig. 2 is an (x, p_x) slice of this transverse phase space. The estimated density increase peaks at 12.5% at the center of the beam. For comparison, the plots in the top row represent a MICE Step IV lattice without LiH absorber (empty channel). As expected, no cooling is observed in an empty cooling channel.

Figure 3 displays the change in density of the muon beam due to ionization cooling. For a four-dimensional transverse phase space surface corresponding to a constant density, the density contained within increases at the core of the beam, and decreases at the periphery of the beam. For comparison, the top row of plots in Fig. 3 shows no change in density for a channel with no material. The volumes are calculated using the Monte Carlo approach: a large number of particles are thrown at random (uniformly) inside the bounding box for the distribution, and the fraction of particles within a four-dimensional surface of interest is then calculated based on the KDE density prediction. The peak increase in density is consistent with that shown in Fig. 2 and is in agreement with the results from an independent study on direct measurements of phase-space density [13].

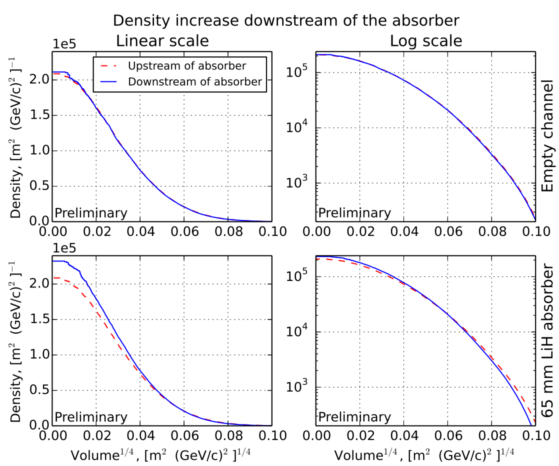

Figure 3: Change in density of the muon beam after passing through an absorber. Horizontal axis: four-dimensional volume inside the surface of constant density shown on the vertical axis. Top row: no material in the channel and no change in the density; bottom row: ionization cooling with 65 mm of LiH absorber. Left column: linear scale to emphasize the core of the beam; right column: log scale to show the details of the tail. Red line (dashed): upstream of the absorber, blue line (solid): downstream of the absorber. At the core of the beam the density increases, while at the periphery it decreases.

CONCLUSIONS

MICE Step IV lattice studied in this paper demonstrates cooling through an increase in phase-space density as estimated by KDE algorithm. The cooling performance of a symmetric MICE Step IV lattice is studied in this analysis where the problematic Match1 coil in the downstream spectrometer is assumed operable. More studies are under way in which Match1 coil is not included in the MICE Step IV lattice; the parameters of this new lattice will be taken from an on-going independent study [14]. KDE algorithm relies on approximations that are dependent on the number of muons and the choice of bandwidth parameter. Studies are under way to assess bandwidth selection and systematic errors.

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