# **OPTIMIZING CHROMATIC COUPLING MEASUREMENT IN THE LHC**

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### Abstract

Chromatic coupling introduces a dependency of transverse coupling with energy. LHC is equipped with skew sextupoles to compensate the possible adverse effects of chromatic coupling. In 2012 a beam-based correction was calculated and applied successfully for the fist time. However, the method used to reconstruct the chromatic coupling was dependent on stable tunes and equal chromaticities between the horizontal and vertical planes. In this article an improved method to calculate the chromatic coupling without these constraints is presented.

#### **INTRODUCTION**

Chromatic coupling is generated by sextupolar errors in magnets, in combination with dispersion. In regions of vertical dispersion an off-momentum particle passing through a normal sextupole will experience a skew quadrupole field. In case of horizontal dispersion, the off-momentum particle passing through a skew sextupole will also observe a skew quadrupole field. Since there is larger horizontal dispersion in the LHC, skew sextupolar components are the dominant source of chromatic coupling [1]. Higher-order effects, such as the chromatic coupling, have in general an effect on dynamic aperture and beam lifetime.

In order to correct the chromatic coupling, the LHC is equipped with skew sextupoles in regions with dispersion. The phase advance is such that it is favorable to correct the difference resonance but not the sum resonance.

The correction of the the chromatic coupling was already demonstrated in 2012 in the LHC [2]. The results were also compared to the predictions from the magnetic model. However, the reconstruction of the chromatic coupling required certain conditions in order to be valid. In this article we discuss these conditions and how to extend the method to be more robust. We also present measurements and corrections of the chromatic coupling from 2015 and explain why the correction was only partially successful.

#### **METHOD**

The principle to measure the chromatic coupling is to change the momentum of the particles and observe how the difference resonance coupling term  $(f_{1001})$  changes. The method used to measure and correct the chromatic coupling is described in detail in [2, 3].

However, there are certain effects that are not taken into account in this method. In this article we are only concerned about the fractional tunes and for simplicity we label them  $Q_x$  and  $Q_y$ . The amplitude of  $f_{1001}$  is strongly ISBN 978-3-95450-147-2 dependent on the fractional tune split  $Q_x - Q_y$ . A difference between horizontal and vertical chromaticity leads to a change in tune split which has a direct impact on the amplitude of the  $f_{1001}$ . In the following, we outline a simplified model to understand how linear coupling together with differences in tunes change the measurement. It is worth noting that no assumption about the origin of the tune change is needed. This means that the source may come from differences in chromaticity but may also derive from a tune drift, change in orbit that feeds down to tune, etc.

We start with the well-known relation between the Hamiltonian terms and the Resonance Driving Terms (RDTs), which is written

$$f_{1001} = \frac{h_{1001}}{1 - e^{2i\pi(Q_x - Q_y)}},\tag{1}$$

where  $h_{1001}$  is the Hamiltonian term and  $Q_x, Q_y$  are the horizontal and vertical tunes respectively.

In case of no energy dependency of the  $f_{1001}$  (no chromatic coupling), the difference will only depend on the change of tune. If we use a linear approximation and assume a tune split close to zero we can write down the following relation

$$f_{1001}^{(+)} = \frac{f_{1001}^{(0)} \Delta Q^{(0)}}{\Delta Q^{(+)}},\tag{2}$$

where  $f_{1001}^{(+)}$  is the difference resonance term for the positive change of momentum,  $f_{1001}^{(0)}$  represents on momentum,  $\Delta Q^{(+)}$  is the fractional tune split for the positive change of momentum and  $\Delta Q^{(0)}$  is the fractional tune split on momentum. Following the same logic for the negative momentum gives

$$f_{1001}^{(-)} = \frac{f_{1001}^{(0)} \Delta Q^{(0)}}{\Delta Q^{(-)}}.$$
(3)

Taking the difference between the positive and negative offmomentum RDT gives

$$f_{1001}^{(+)} - f_{1001}^{(-)} = \frac{f_{1001}^{(0)} \Delta Q^{(0)}}{\Delta Q^{(+)}} - \frac{f_{1001}^{(0)} \Delta Q^{(0)}}{\Delta Q^{(-)}}.$$
 (4)

Rewriting the equation and dividing by  $2\Delta\delta$ , which is the total momentum change during the measurement of chromatic coupling, we obtain the following equation

$$\frac{f_{1001}^{(+)} - f_{1001}^{(-)}}{2\Delta\delta} = \frac{f_{1001}^{(0)}}{2\Delta\delta} \Big(\frac{\Delta Q^{(0)}}{\Delta Q^{(+)}} - \frac{\Delta Q^{(0)}}{\Delta Q^{(-)}}\Big).$$
 (5)

01 Circular and Linear Colliders A01 Hadron Colliders This shows how linear coupling disturbs the measurement of chromatic coupling in the presence of tune shifts during the measurement. Note that in case  $\Delta Q^{(+)} = \Delta Q^{(-)}$ there is no effect from the linear coupling on the chromatic coupling measurement.

If we assume a realistic case where the difference between the positive and negative tune split is deriving from a difference in chromaticity between the two planes of 3 units. If we use realistic numbers in such a scenario where the on-momentum tune split is 0.01 and  $\Delta \delta = 4 \times 10^{-4}$  and the  $f_{1001}^{(0)} = 0.1$ . Putting all this into the equation gives  $\frac{f_{1001}^{(+)} - f_{1001}^{(-)}}{2\Delta \delta} \approx 30$ . This is to compare to the measured values of the chromatic coupling which normally is in the range 50-80. This shows that linear coupling in combination with a tune change can mimic chromatic coupling and hence affect the results.

The method to compute the chromatic coupling is based on the reconstruction of the  $f_{1001}$  from AC-dipole data. This method is also very sensitive to the tune split so in case the tune split changes this also affects the reconstructed  $f_{1001}$  which in turn affects the results of the chromatic coupling. For a detailed explanation of how  $f_{1001}$  is measured with the AC-dipole, see [4].

### **IMPROVEMENTS TO THE METHOD**

Instead of computing the  $f_{1001}$  term, it would be possible to calculate the Hamiltonian term which is independent of the tune. This would be done by first measuring the  $f_{1001}$  and then use equation (1) to calculate the  $h_{1001}$ . In Fig. 1 the  $\Delta f_{1001}/\Delta \delta$  for a realistic situation with and without coupling is simulated. We observe that the difference is big with and without linear coupling. Using the chromatic coupling shown as a green line as an input for a correction would lead to an over-correction. In Fig. 2 the  $\Delta h_{1001}/\Delta \delta$  is shown with the same level of chromatic and normal coupling. We observe that in this case the effect of the normal coupling is less pronounced than in the case when we use the  $f_{1001}$ . This clearly shows the benefit of using the Hamiltonian term instead of  $f_{1001}$ .

However, that would not solve the problem with the reconstruction of the  $f_{1001}$  from the AC-dipole. In that case it would still be necessary to account for the different tunes in the reconstruction of the  $f_{1001}$  from the AC-dipole measurement. However, this does not provide a fundamental obstacle but is something that has to be taken into account throughout the reconstruction.

The other option is to adjust the tune back to the nominal value for each  $\Delta\delta$  step. That is, however, time consuming and also changes the settings of the MQTs (quadrupoles used to change the tune in the LHC) and hence the optics. This is not a main concern for the measurement of the chromatic coupling but would affect the measurement of the Montague functions [5].



Figure 1: Reconstructed chromatic coupling from  $f_{1001}$  with and without normal coupling added. The onmomentum fractional tunes were  $Q_x = 0.31$ ,  $Q_y = 0.32$  and the chromaticity was  $Q'_x = 1$ ,  $Q'_y = 4$ .



Figure 2: Reconstructed chromatic coupling from  $h_{1001}$  with and without normal coupling added. The onmomentum fractional tunes were  $Q_x = 0.31$ ,  $Q_y = 0.32$  and the chromaticity was  $Q'_x = 1$ ,  $Q'_y = 4$ .

# MEASURED CHROMATIC COUPLING

During the commissioning of the  $\beta^* = 80$  cm optics in 2015 a correction was calculated and applied. The method used was based on measuring the  $f_{1001}$  for the different momentum. Figure 3 shows the absolute values of the chromatic coupling and Figure 4 shows the real and imaginary part. Observing the absolute values it is clear that the correction was able to reduce the chromatic coupling, however, looking at the real and imaginary parts we observe that the correction was slightly too strong.

We also note that the phase of the correction is well matched since if it was wrong we would not have observed a change in the phase of the chromatic coupling. Unfor-

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Table 1: Table showing the tunes during the measurement of beam 1 from which the correction was based on.  $\Delta Q$  is the natural tune split ( $Q_x - Q_y$ ) and  $\Delta Q_{AC}$  is the driven tune split ( $Q_{ACx} - Q_{ACy}$ ).

 		1				
$\Delta\delta$	$Q_x$	$Q_y$	$Q_{ACx}$	$Q_{ACy}$	$\Delta Q$	$\Delta Q_{\rm AC}$
$-4 \times 10^{-4}$	0.320	0.309	0.297	0.329	0.011	0.032
0	0.321	0.3102	0.298	0.332	0.0106	0.034
$4 \times 10^{-4}$	0.321	0.311	0.299	0.332	0.010	0.033



Figure 3: The amplitude of the chromatic coupling, for Beam 1, before and after the correction.



Figure 4: The real and the imaginary part of the chromatic coupling, for Beam 1, before and after correction.

tunately, there was no time to reduce the strength of the correction and remeasure the chromatic coupling.

# EXPLANATION OF THE OVER CORRECTION

In this section we explain the over correction observed during the measurements. In table 1 the driven and natural tunes are presented for the measurement performed before the corrections. In particular we notice that the natural tune split did not remain constant between the negative and positive  $\Delta\delta$ . The on momentum  $f_{1001}$  was around 0.12 for this measurement. If we use these numbers in equation (5) we get that the contribution from linear coupling was around 20. This alone does not explain the entire over correction seen in Fig. 4 but the driven tune split also changed which also has an impact on the reconstruction of the  $f_{1001}$ . Note also that the same effect is also present for the evaluation of the correction. Adding all these together easily accounts for the over correction observed.

### SUMMARY AND OUTLOOK

A correction of the chromatic coupling was demonstrated during the 2015 commissioning. The correction was measured to be too strong but the reason is well understood. It derived from the change of the tune split between the different momentum settings. Two different ways to overcome this problem have been suggested and demonstrated to work in simulations. The first one is based on using the Hamiltonian terms rather than the  $f_{1001}$  to measure the chromatic coupling. The second method works through adjusting the tunes back to the nominal value before each measurement.

Next step would be to compare the measured chromatic coupling measurements, which have been taken at different energies, and to compare them to the magnetic model. This is of interest to see how well the magnetic model reproduce the  $a_3$  component for the different energies.

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