SYMPLECTIC TRACKING OF MULTI-ISOTOPIC HEAVY-ION BEAMS IN SIXTRACK*

P. D. Hermes†, CERN, Geneva, Switzerland and University of Münster, Germany
R. Bruce, R. De Maria, CERN, Geneva, Switzerland

Abstract

The software SixTrack provides symplectic proton tracking over a large number of turns. The code is used for the tracking of beam halo particles and the simulation of their interaction with the collimators to study the efficiency of the LHC collimation system. Tracking simulations for heavy-ion beams require taking into account the mass to charge ratio of each particle because heavy ions can be subject to fragmentation at their passage through the collimators. In this paper we present the derivation of a Hamiltonian for multi-isotopic heavy-ion beams and symplectic tracking maps derived from it. The resulting tracking maps were implemented in the tracking software SixTrack. With this modification, SixTrack can be used to natively track heavy-ion beams of multiple isotopes through a magnetic accelerator lattice.

INTRODUCTION

The magnetic lattice of a heavy-ion collider, such as the CERN Large Hadron Collider [1], is matched to guide and store beams of a specific particle type. The LHC collimation system [2] is less efficient for heavy-ion beams than for proton beams, because a wide range of ion fragments escapes the collimators and is lost in the aperture of the superconducting magnets. The collimation losses can quench the cold LHC magnets and could soon become a limiting factor for the achievable heavy-ion beam intensity [3]. Accurate simulations of the cleaning efficiency are required to determine the best collimator settings and ensure the safe operation of the machine [4]. These studies are carried out by means of tracking simulations of the beam halo particles and the ion fragments generated in the collimators. The particle tracks are compared to the aperture to determine precisely their loss location. The former standard tool for heavy-ion collimation simulation, ICOSIM [5], used tracking maps taking into account dispersive effects in linear approximation. This formalism can be inaccurate for particles with large momentum offsets [6].

Higher accuracy can be expected from full symplectic tracking of the different ions through the magnetic lattice. The tracking must include dispersion from chromatic effects (momentum dispersion) and the isotopic dispersion due to the mass to charge ratio of the different isotopes. A simple manner to provide this is the usage of a symplectic mono-isotopic tool and simulate particles of the reference species with rigidities equivalent to the heavy-ion to be tracked [6].

* Work supported by the German Wolfgang Gentner Programme of the Bundesministerium für Bildung und Forschung.
† phermes@cern.ch

Native tracking of different heavy-ion species, however, requires an underlying mathematical formalism which takes into account the mass and charge of the main beam and the secondary isotopes, as well as their physical momenta and ion species.

In this article, we describe a generalized Hamiltonian for multi-isotopic particle beams which is exploited to derive thin-lens symplectic tracking maps for the implementation in the particle tracking code SixTrack [7].

THE ACCELERATOR HAMILTONIAN

Coordinate System and Basic Definitions

Figure 1 shows the curvlinear coordinate system used in SixTrack. The trajectory of the reference particle is parametrized by $s = s(t)$, the length of the ideal trajectory measured from a reference point. The particle trajectory is defined by the coordinates (x, y, z) of a right-handed orthogonal system, whose origin moves with the reference particle in s [8]. The ideal trajectory may be bent by a bending radius $\rho_0 = \frac{1}{h_x}$. Consider the trajectory of an arbitrary particle of

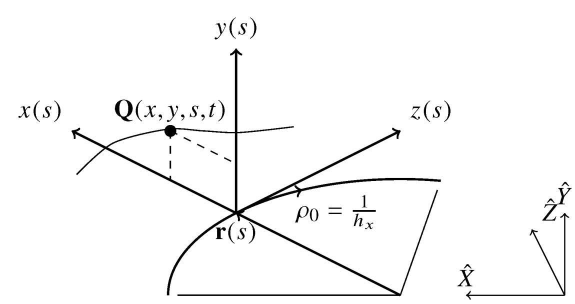

Figure 1: Accelerator coordinate system (x, y, z) parametrized by s. The radius of the bent trajectory is $\rho_0 = \frac{1}{h_x}$. Figure taken from [9].

rest mass m and charge Ze (with the charge multiplicity Z and elementary charge e) moving at the normalized speed $\beta = \frac{v}{c}$ through a magnetic field B. The trajectory is bent by a bending radius ρ, which is related to the magnetic field and the particle momentum and charge as:

$$B\rho = \frac{P}{Ze}\,. \tag{1}$$

The particle momentum can be written as $P = m\beta c\gamma$ with the relativistic $\gamma = (1 - \beta^2)^{-1/2}$. The bending radius ρ_0 of the reference particle, with its physical properties defined by the parameters m_0, Z_0, β_0, is related to ρ as follows:

$$\frac{\rho}{\rho_0} = \frac{(1+\delta)}{\chi}, \quad \chi = \frac{m_0}{m}\frac{Z}{Z_0}, \quad (1+\delta) = \frac{\beta\gamma}{\beta_0\gamma_0}\,. \tag{2}$$

The quantity χ defines the mass to charge ratio of the ion relative to the reference particle.

Elementary transformations of Eq. (2) show that δ in the multi-isotopic case is not the well-known relative momentum offset, but the relative offset of the momentum per mass unit:

$$\delta = \frac{P\,\frac{m_0}{m} - P_0}{P_0}\,. \tag{3}$$

This definition ensures also that δ is a small quantity, while, for the case of heavy ions, the relative momentum offset can be larger by two orders of magnitude. Both χ and δ quantify the dispersive offset of the particle trajectory caused by the magnets in the machine. Note that for the mono-isotopic case $m \to m_0$ and $Z \to Z_0$ the two Eqs. (2) and (3) yield the well known expressions in which δ is the relative offset of the full momentum.

The Multi-Isotopic Accelerator Hamiltonian

Consider a physical system described by the canonical coordinates $\mathbf{p}, \mathbf{q}$ with $\mathbf{p} = \{p_x, p_y, p_z\}$ and $\mathbf{q} = \{x, y, z\}$. After the transformation of the independent variable from t to $s(t)$, the accelerator Hamiltonian for the set of canonical variables $(x, p_x), (y, p_y), (-t, E)$ is given by [10]

$$\tilde{H} = -p_z = -ZeA_z - \sqrt{\frac{(E - Ze\phi)^2}{c^2} - m^2c^2 - (p_x - ZeA_x)^2 - (p_y - ZeA_y)^2}, \tag{4}$$

where ϕ is the scalar potential and A_i the electromagnetic vector potential, defining the magnetic field vector $\mathbf{B} = \nabla \times \mathbf{A}$. The canonical momenta p_i are defined as

$$p_i = m\gamma\dot{q}_i + ZeA_i\,. \tag{5}$$

In order to be capable of performing a series expansion of the right hand side of Eq. (4), we apply a normalization to obtain small quantities in the square root

$$p_i \to \tilde{p}_i = \frac{p_i}{P_0}\frac{m_0}{m} \qquad \tilde{H} \to \bar{H} = \frac{\tilde{H}}{P_0}\frac{m_0}{m}, \tag{6}$$

$$Ze\,A_i \to \chi a_i = \chi\frac{Z_0 e A_i}{P_0} \qquad E \to \tilde{E} = \frac{E}{P_0}\frac{m_0}{m}\,. \tag{7}$$

The normalization with respect to the mass is essential to fulfill the requirement of obtaining small quantities, because the masses of the different ions moving in the accelerator can differ significantly. Note that the definition of the normalized vector potential a_i is identical to the definition for the mono-isotopic case [11]. Instead of incorporating it into the definition of a_i, the magnetic rigidity change for isotopes different from the reference particle is taken into account by the additional factor $\chi = \frac{m_0}{m}\frac{Z}{Z_0}$. This allows the usage of the vector potentials well known from the derivation of the mono-isotopic tracking maps [12, 13].

Assuming that a gauge can be found, such that $\phi = 0$, the new Hamiltonian yields

$$\bar{H} = -\chi a_z \tag{8}$$

$$- \sqrt{\frac{m_0^2}{m^2}\left(\frac{E^2 - m^2c^4}{P_0^2c^2}\right) - (\tilde{p}_x - \chi a_x)^2 - (\tilde{p}_y - \chi a_y)^2}, \tag{9}$$

Using the relativistic energy-momentum relation and Eq. (3), the Hamiltonian can be written as

$$\bar{H} = -\chi a_z - \sqrt{(1+\delta)^2 - (\tilde{p}_x - \chi a_x)^2 - (\tilde{p}_y - \chi a_y)^2}\,. \tag{10}$$

The normalized canonical longitudinal momentum is still large. A more elegant description can be obtained by means of a transformation of the canonical variables

$$(x, \tilde{p}_x), (y, \tilde{p}_y), (-t, E) \to (X, P_x), (Y, P_y), (\sigma, p_\sigma)\,. \tag{11}$$

This transformation is provided by a generating function of second type [11]:

$$F_2 = x\,P_x + y\,P_y + (s - \beta_0\,c\,t)\left(p_\sigma + \frac{E_0}{\beta_0\,P_0\,c}\right)\,. \tag{12}$$

The old $(\tilde{p}_i, q_i)$ and new (P_i, Q_i) coordinates, as well as the old $(\bar{H})$ and new (K) Hamiltonian are related by the following relations:

$$\tilde{p}_i = \frac{\partial F_2}{\partial q_i} \qquad Q_i = \frac{\partial F_2}{\partial P_i} \qquad K = \bar{H} + \frac{\partial F_2}{\partial s} = \bar{H} + p_\sigma\,. \tag{13}$$

The transformed variables are then defined as follows:

$$X = x\,, \qquad Y = y\,, \qquad \sigma = s - \beta_0 ct\,, \tag{14}$$

$$P_x = \tilde{p}_x\,, \qquad P_y = \tilde{p}_y\,, \qquad p_\sigma = \frac{\frac{m_0}{m}E - E_0}{\beta_0 P_0 c}\,. \tag{15}$$

Including a last transformation for convenience $P_i \to p_i$, $K \to H$, the final multi-isotopic Hamiltonian in a straight coordinate system yields

$$H = p_\sigma - \sqrt{(1+\delta)^2 - (p_x - \chi a_x)^2 - (p_y - \chi a_y)^2} - \chi a_z\,. \tag{16}$$

In a coordinate system horizontally bent by a radius $\rho_0 = 1/h_x$, the Hamiltonian becomes

$$H = p_\sigma - (1 + h_x\,x)\cdot \cdot\left[\sqrt{(1+\delta)^2 - (p_x - \chi a_x)^2 - (p_y - \chi a_y)^2} - \chi a_s\right], \tag{17}$$

where a_s is the vector potential in the curvlinear reference coordinates, defined by:

$$p_s = \frac{m_0\,\gamma\,\dot{s}}{P_0}\,(1 + h_x x)^2 + (1 + h_x x)\,\chi a_s\,. \tag{18}$$

In the mono-isotopic limit $m \to m_0$ and $Z \to Z_0$ the multi-isotopic Hamiltonian becomes the standard Hamiltonian presented in [11, 14].

ISBN 978-3-95450-147-2

MULTI-ISOTOPIC TRACKING MAPS

With the new accelerator Hamiltonian, generic tracking maps for multi-isotopic particle beams can be derived using Hamilton's equations [12]. Complex vector potentials may require to expand the Hamiltonian in $\frac{(p_x-\chi a_x)^2+(p_y-\chi a_y)^2}{(1+\delta)^2}$ to first order, as it is done for mono-isotopic beams in [10].

Drift Space

A drift space is defined by the absence of electromagnetic fields $a_i = 0$. The ideal trajectory is not bent, thus $h_x = 0$ and the exact Hamiltonian yields

$$H = p_\sigma - \sqrt{(1+\delta)^2 - p_x^2 - p_y^2} = p_\sigma - p_z\,. \tag{19}$$

The resulting tracking maps are independent of the ion species and thus identical to the mono-isotopic case, which is discussed in detail in [15].

Dipole Magnet

Bending Dipole Using the vector potential of a bending dipole derived in [12]:

$$a_x = a_y = 0\,, \qquad a_s = k_0\left(x + \frac{h_x x^2}{2}\right), \tag{20}$$

the expanded Hamiltonian for a horizontal bending dipole magnet with the normalized strength $k_0 = \frac{B_y Z_0 e}{P_0}$ and $h_x \neq 0$ is given by

$$\begin{aligned} H \approx\; & p_\sigma - (1+h_x x)\,(1+\delta) + \\ & + \frac{1}{2}\frac{p_x^2+p_y^2}{(1+\delta)} + \chi k_0\left(x + \frac{h_x x^2}{2}\right). \end{aligned} \tag{21}$$

The tracking map for a dipole of length L in thin lens approximation $k_0 L \to 0$ can be derived using Hamilton's equations:

$$p_x \to p_x + L\,\left[h_x(1+\delta) - k_0\chi(1+h_x x)\right], \tag{22}$$
$$p_y \to p_y\,, \tag{23}$$
$$p_\sigma \to p_\sigma\,. \tag{24}$$

In the mono-isotopic limit $\chi \to 1$, the tracking map yields the standard expression derived in [13]. In the thin lens approximation, the quantities x, y, σ remain unchanged.

Kicker Dipole The magnetic kicker dipole provides a transverse magnetic field, similar to the bending dipole, but the reference orbit is not bent ($h_x = 0$). Kicker dipoles are used to control the orbit in a machine. From the Hamiltonian in Eq. (21) with $h_x = 0$, the following tracking map can be derived:

$$p_x \to p_x - k_0\,\chi\,L\,, \tag{25}$$
$$p_y \to p_y\,, \tag{26}$$
$$p_\sigma \to p_\sigma\,. \tag{27}$$

As expected, due to the isotopic dispersion, the angular kick in horizontal direction scales linearly with the relative mass to charge ratio χ.

Quadrupole Magnet

The vector potential of a quadrupole with the normalized gradient $k = \frac{Z_0 e}{P_0}\,g$ is given by

$$a_x = a_y = 0\,, \qquad a_s = -\frac{1}{2}k(y^2 - x^2)\,. \tag{28}$$

Note that the ideal particle is not subject to magnetic forces in a quadrupole, so the ideal trajectory is straight ($h_x = 0$). The resulting tracking map for a quadrupole of length L in thin lens approximation is given by:

$$p_x \to p_x - k\chi L x\,, \tag{29}$$
$$p_y \to p_y + k\chi L x\,, \tag{30}$$
$$p_\sigma \to p_\sigma\,. \tag{31}$$

The focal length of the quadrupole scales linearly with the relative mass to charge ratio.

Accelerating Cavity

The energy gain ΔE of a particle in an accelerating cavity with wave number $k = \frac{\omega}{c} = 2\pi f$ can be approximated by

$$\Delta E = ZeU\,\sin\left(\phi - k\,\frac{\sigma}{\beta_0}\right), \tag{32}$$

where U is the average voltage during the particle's passage through the cavity [11]. In the approximation of a thin cavity, the following vector potential can be derived:

$$A_x = A_y = 0 \quad A_s = -\frac{U}{\omega}\cos\left(\phi - k\frac{\sigma}{\beta_0}\right)\tilde{\delta}(s)\,, \tag{33}$$

where $\tilde{\delta}(s)$ is the Dirac function. Using the substitution $U_n = \frac{Z_0 e}{P_0 c}U$, the transfer map for p_σ can be deduced as:

$$p_\sigma \to p_\sigma + \chi U_n \sin\left(\phi - k\frac{\sigma(s)}{\beta_0}\right). \tag{34}$$

The change in p_σ is, as expected, proportional to $Ze\,\frac{m_0}{m}$.

SUMMARY AND OUTLOOK

This article presents the derivation of an accelerator Hamiltonian describing the motion of particles of different species in the same magnetic lattice. Physically accurate tracking studies of multi-isotopic heavy-ion beams require a consistent formalism including the dispersion from chromatic effects and the mass to charge ratio relative to the reference particle χ.

The Hamiltonian is employed for the derivation of symplectic transfer maps for the individual beam line elements. This allows the native tracking of ions of different species at their physical momenta instead of using equivalent momenta of the main beam particles to simulate the ion rigidities.

The tracking maps are implemented in a new version of SixTrack (Heavy Ion SixTrack [4]) in which heavy ions and secondary ion fragments scattered out of the collimation system are tracked for collimation simulations.

REFERENCES

[1] O. S. Brüning *et al.* (Eds.), "LHC design report v.1 : The LHC main ring", CERN-2004-003-V1 (2004).

[2] R. W. Assmann *et al.*, in *Proc. of EPAC'06*, Edinburgh, United Kingdom, paper TUODFI01, pp. 986–988 (2006).

[3] P. D. Hermes *et al.*, CERN-ACC-Note-2016-0031 MD, (2016).

[4] P. D. Hermes *et al.*, "Simulation of heavy-ion beam losses with the SixTrack-FLUKA active coupling", presented at IPAC'16, Busan, Korea, May 2016, paper TUPMW014, this conference.

[5] H. H. Braun *et al.*, in *Proc. EPAC'04*, Lucerne, Switzerland, paper MOPLT010, pp. 551-553, (2004).

[6] P. D. Hermes *et al.*, Nucl. Instr. Meth. Phys. Res. A 819, 73–83, (2016).

[7] F. Schmidt, CERN/SL/94–56 (AP).

[8] H. Wiedemann, "Particle Accelerator Physics", Springer Verlag, Heidelberg, (2007).

[9] R. De Maria *et al.*, SixTrack Physics Manual, `http://sixtrack.web.cern.ch/SixTrack/doc/physics_manual/sixphys.pdf`

[10] E. D. Courant and H. S. Snyder. "Theory of the Alternating-Gradient Synchrotron" Annals of Physics, 281(1-2):360–408,(2000).

[11] A. Wolski, "Beam Dynamics in High Energy Particle Accelerators", World Scientific, pp. 59-80, (2014).

[12] D. P. Barber *et al.*, "A non-linear canonical formalism for the coupled synchro-betatron motion of protons with arbitrary energy", Technical Report 87-36, DESY, (1987).

[13] K. Heinemann *et al.*, "Construction of nonlinear symplectic six-dimensional thin-lens maps by exponentiation", Technical Report 95-189, DESY, (1995).

[14] S. Turner (ed.), Proceedings of CERN Accelerator School: 5th Advanced Accelerator Physics Course, CERN-95-06, (1995).

[15] M. Fjellström, "Particle Tracking in Circular Accelerators Using the Exact Hamiltonian in SixTrack", Master's thesis, Lulea University, Sweden, (2013).

ISBN 978-3-95450-147-2