APPLICATION OF DIFFERENTIAL EVOLUTION ALGORITHM IN FUTURE COLLIDER OPTIMIZATION *

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Abstract

The dynamic aperture is very limited due to the very small beta at IP in the SuperKEKB. It is similar in the future storage ring based e+e- colliders. The design of CEPC is still in process. And the construction of SuperKEKB is nearly finished, but there still exist some problem which could reduce the performance. There are a few hundred parameters to be varied in the future colliders. The global optimization may be a good way to enlarge the dynamic aperture. Differential Evolution is a very simple population based, stochastic function minimizer which is very powerful at the same time. In this paper we show some application fo the algorithm in the two machines. It has the potential to help us optimize the machine.

INTRODUCTION

There is a lot of successful multi-objective optimization in the design of storage ring based light source, such as APS [1], where they did the direct optimization fo dynamic aperture and Touschek lifetime. The experimental tests validate the method, which bring significant improvements to APS operations.

In NSLS-II [2], they demonstrate a correlation between dynamic aperture and low-order nonlinear driving terms, and using both numerical tracking results and analytical estimates of the driving terms, which resulted in faster convergence.

Genetic algorithm is very popular and Huang tried particle swarm algorithm during the nonlinear dynamics optimization of a low emittance upgrade lattice of SPEAR3 [3]. The performance of the two algorithms are compared. The result shows the particle swarm algorithm converges significantly faster to similar or better solutions than the genetic algorithm and it does not require seeding of good solutions in the initial population.

In photoinjector design, there is also growing interest in using multi-objective beam dynamics optimization to minimize the final transverse emittances and to maximize the final peak current of the beam. Most previous studies in this area were based on genetic algorithms. J. Qiang propose a new parallel multi-objective optimizer based on the differential evolution algorithm for photoinjector beam dynamics optimization [4].

The author learned K. Oide's optics design for FCC-ee [5] in 2015, where he optimized the dynamic aperture with hundreds of sextupole families. The work give us the first excitation, since there are totally 18 sextupole families in BEPCII

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and 56 sextupole families in SuperKEKB. It is maybe a good idea to use hundreds of sextupole families in future e+e- storage ring collider, such as CEPC and FCC.

DIFFERENTIAL EVOLUTION ALGORITHM

In 1995, Price and Storn proposed a new floating point encoded evolutionary algorithm for global optimization and named it DE [6] owing to a special kind of differential operator, which they invoked to create new offspring from parent chromosomes instead of classical crossover or mutation. Easy methods of implementation and negligible parameter tuning made the algorithm quite popular very soon.

DE is a very simple population based, stochastic function minimizer which is very powerful at the same time. The crucial idea behind DE is a scheme for generating trial parameter vectors. In fact there are a few strategies, and we choose "rand-to-best" which attempts a balance between robustness and fast convergence. A perturbed vector v_i is generated according to

$$v_{i,j} = \begin{cases} x_{i,j} & \text{if rand(j) > CR} \\ x_{i,j} + F \times [x_{b,j} - x_{i,j}] + F \times [x_{r1,j} - x_{r2,j}] & \text{else} \end{cases}$$

$$\text{(1)}$$

where rand() is a uniform random number between 0 and 1. As we see in the above equation, there are two setting parameter F and CR here. Another parameter is the population size NP, which is usually ten or twenty times the parameter number. In our simulation, CR is chosen between [0.8,1.0], and F is randomly chosen between 0 and 1 for each generation. If the new trial solution produces a better objective function value comparing its parent, it will be put into the next generation population, otherwise the old one is kept unchanged.

Most problems in nature have several objectives to be satisfied. Many of these problems are frequently treated as single-objective optimization problems by transforming all but one objective into constraints. This is the method used in MAD [7], where the single objective function is defined as the sum of all the constraints with different weight. But the weight is hard to determine, especially when the objective are possibly conflicting.

A true multi-objective optimization means finding such a solution which would give the values of all the objective functions acceptable to the decision maker. In mathematical language, a solution x_1 is said to dominate another solution x_2, if both the following conditions are true: (1) the solution

x_1 is no worse than x_2 in all objectives. (2) the solution x_1 is strictly better than x_2 in at least one objective. Among a set of solutions, the non-dominated set of solutions are those that are not dominated by any member of the set. The Kung's algorithm [8] is used to find the non-dominated set. Attention is paid to Pareto optimal solutions, which cannot be improved in any of the objectives without degrading at least one of the other objectives. The set of Pareto optimal outcomes is often called the Pareto front.

The differential evolution algorithm for multi-objective optimization is referencing Qiang's work [4]. The algorithm in each generation can be summarized in the following steps:

1. Generate the offspring population using the above differential evolution algorithm.

2. Find the non-dominated population set, which are treated as the best solutions in DE to generate offspring in the next generation.

3. Sorting all the population, select the best NP solution as the parents in the next generation.

4. Return to step 1, if stopping condition is not met.

In the original single objective DE, the comparison is done between the new trial vector solution and its corresponding parent and the better is kept for next generation. In the multi-objective DE, the comparison is done between all the population set, the better is kept. We do not study the effect of the change.

APPLICATION IN CEPC & SuperKEKB

CEPC

After the Higgs discovery, it is believed that a circular e+e- collider could serve as a Higgs factory. The high energy physics community in China launched a study of a 50-100 km ring collider. A Preliminary Conceptual Design Report (Pre-CDR) has been published in early 2015 [9]. This report is based on a 54-km ring design. The main lattice parameters are listed in Table. 1.

Table 1: Lattice Parameters of CEPC(Pre-CDR)

Circumference	54 km
Number of IP	2
Momentum compaction factor	4.15e-5
Horizontal emittance	6.12e-9 mrad
Emittance coupling	0.003
Energy Spread(SR)	0.0013
$\beta^*_{x/y}$	0.8/0.0012 m

The beam-beam simulation shows shat increase of β^*_y from 1.2 mm to 3 mm will not reduce luminosity performance and bring better lifetime due to the hourglass effect [10]. As the first step, the lattice team design and optimize the interaction region with $\beta_y = 3$mm. The dynamic

aperture is not good enough, we try to optimize DA by tuning the sextupole strength in the arc here. The lattice design work is evolving continuously [11], we still use an old version here as a test.

The beam-beam study [10] shows that the dynamic aperture should be larger than $20\sigma_x \times 40 \times \sigma_y 0.02$. The $x-z$ aperture of the orginal lattice is shown in Fig. 1, where the coupling is 0.3 %.
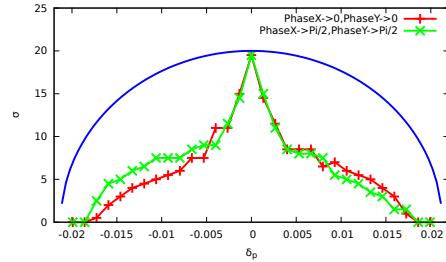

Figure 1: Dynamic aperture of CEPC before optimization.

There are only one familiy for SF/SD in the arc of the original lattice. Since the arc cell consists of $60°/60°$ FODO lattice, we set the sextupoles interleaved $180°$ one pair and there are totally 240 sextupole pairs used in the optimization. The objectives are listed in the following:

1. The tune Q_x is in the range of [0.05,0.31] and Q_y in [0.10,0.31] for $\delta \in [-0.02, 0.02]$.

2. X-Z aperture objective is defined as an ellipse $\frac{x^2}{20^2} + \frac{z^2}{16^2} = 1$, where x is the transverse amplitude in unit of RMS size with 0.3% coupling, and z is unit of RMS energy spread.

3. X-Y-Z aperture objective is defined as an elliptical ball $\frac{x^2}{20^2} + \frac{y^2}{50^2} + \frac{z^2}{16^2} = 1$.

The optimized solution seems enlarge the dynamic aperture significantly, as shown in Fig. 2.
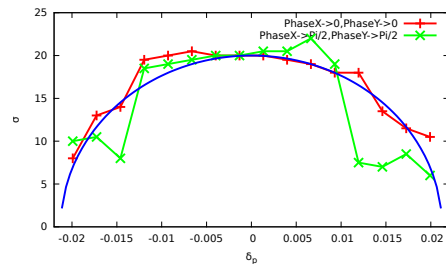

Figure 2: Dynamic aperture of CEPC after optimization.

Since there exist strong synchrotron radiation in the higgs factory, the radiation effect on DA should be also studied. The tracking shows that the damping really helps especially for large momentum offset particle, but the quantum fluctuation may reduce the DA for small momentum offset particle.

SuperKEKB

The SuperKEKB project requires a positron and electron collider with a peak luminosity $8 \times 10^{35} \text{cm}^{-2}\text{s}^{-1}$. The luminosity is 40 times that of the KEKB B-factory. An extremely small beta function at the IP and a low emittance are necessary. In addition, in order to achieve the target luminosity, a large horizontal crossing angle between two colliding beam is adopted, as is a bunch length much longer than the beta function at the IP [12].

The nonlinear beam dynamics is very complicated and very hard to optimize the DA [13]. Here we only focus the positron ring SLER. One work is to try enlarge the DA. The main objective is defined as an ellipse boundary $\frac{x^2}{50^2} + \frac{z^2}{26^2} = 1$. All the sextupoles, octupoles and skew sextupoles are used, the number of variable is 68. Figure 3 shows DA optimization result.
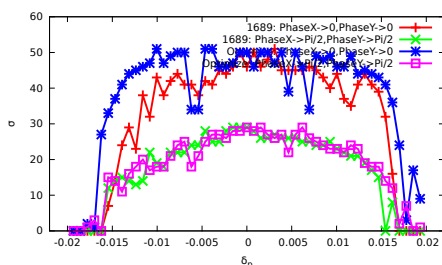

Figure 3: Dynamic aperture of sler, optimization vs v1689.

Speedup Method

The brute-force dynamic aperture tracking is very time consuming. In order to save computing time. We usually first simplify the objective, for example only track 100 turns instead of 1000 turns. In the multi-objectve optimization, some objective is very time consuming, the other may be much faster such as the linear optics calculation. We first optimize the fast objective, and then do the slow calculation when some constraints are satisfying. The method is referring to Ehrlichman's work [14].

SUMMARY

One of the most urgent task is still to speed up the convergence of the optimization. We should try to find more smart objective function by physics work to save the computing time.

The success of DE in solving a specific problem crucially depends on appropriately choosing trial vector generation strategies and their associated control parameter values. There is a lot of work on the self-adaptive DE algorithm [15], which promise us more effective, more stable. Das et al. proposed a new scheme of adjusting the velocities of the particles in PSO with a vector differential operator borrowed from the DE family. The mutual synergy of PSO with DE leading to a more powerful global search algorithm (PSO-DV) [16], which should also be tried.

ACKNOWLEDGMENT

REFERENCES

[1] M. Borland, "Parallel tracking-based optimization of dynamic aperture and lifetime with application to the APS", in *48th ICFA Beam Dynamics Workshop on Future Light Sources*, Menlo Park, California, 2010.

[2] L. Yang *et al.*, "Multiobjective optimzation of dynamic aperture", *Phys. Rev. ST Accel. Beams*, vol. 14, p. 054001, 2011.

[3] X. Huang *et al.*, "Nonlinear dynamics optimization with particle swarm and genetic algorithms for SPEAR3 emittance upgrade", *Nucl. Instr. Meth. in Phys. Research A*, vol. 757, pp. 48–53, 2014.

[4] J. Qiang *et al.*, "A parallel multi-objective differential evolution algorithm for photoinjector beam dynamics optimization", in *Proc. IPAC'13*.

[5] K. Oide, "A design of beam optics for FCC-ee", Presented at CEPC AP Meeting, Beijing, China, Sep. 2016, unpublished.

[6] Differential Evolution, `http://www1.icsi.berkeley.edu/~storn/code.html`

[7] MAD - Methoical Accelerator Design, `http://mad.web.cern.ch/mad`

[8] H. Kung *et al.*, "On finding the maxima of a set of vectors", *J. ACM* vol 22, pp. 469–476, 1975.

[9] CEPC-SPPC preliminary conceptual design report, 2015.

[10] K. Ohmi *et al.*, "Beam-beam Simulation Study for CEPC", in *Proc. IPAC'14*.

[11] Y. Wang *et al.*, "Dynamic aperture study of the CEPC main ring with interaction region", *in this proceeding*.

[12] Y. Ohnishi *et al.*, "Accelerator design at SuperKEKB", *Prog. Theor. Exp. Phys.*, 03A011, 2013.

[13] KEKB Review Committee, `http://www-kekb.kek.jp/MAC/`

[14] M. Ehrlichman, "A Genetic algorithm for chromaticity correction in diffraction limited storage rings", *arXiv:1603.02459*.

[15] A. Qin *et al.*, "Differential evolution algorithm with strategy adaptation for global numerical optimization", *IEEE Trans. on Evo. Comp.*, vol. 13, pp. 398–416, 2009.

[16] S. Das *et al.*, "Particle swarm optimization and differential evolution algorithms: technical analysis, applications and hybridization perspectives", in *Studies in Computational Intelligence*, vol. 116, pp. 1-38, 2008.