MODEL-DEPENDENT ACCELERATOR LATTICE FIT BASED ON BPM DATA AND GENERATING FUNCTIONS

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Abstract

Obtaining accurate linear and nonlinear accelerator models is critical for routine accelerator operation. Here we consider a method based on BPM data and generating functions that provides fitted accelerator model. Using measurements from at least three BPMs and generating functions between them allows obtaining momenta at BPMs as the functions of model parameters and comparing them. Thus, lattice parameters can be fitted. Theoretical results are presented and the method is applied to the model examples.

INTRODUCTION

It is important to know accurate linear and nonlinear accelerator models for routine accelerator operation. Thereby, one needs to obtain fitted accelerator model. To perform this task model-dependent accelerator lattice fit method was developed. The method is based on transverse beam position measurements from BPMs and generating function computation. Transverse beam position are available from BPM measurements, but no information about beam momenta can be obtained this way. BPM data can be obtained either from turn-by-turn measurements or by scanning orbit with correctors. Knowing accelerator model between two consecutive BPMs and its dependence on parameters one can derive expressions for momentum values in monitors, e.g. by using generating functions. Generating function which depends on model parameters, can be computed with COSY INFINITY [1]. Parameters are defined as small fractional errors of the strengths of magnet elements, i.e. quadrupole, sextupole magnetic field errors and so on. BPM alignment errors can also be considered as parameters.

Then, for two successive sections confined within three BPMs one can compare any functions of phase space variables or transverse momenta, in particular. Such comparison can be used for magnetic error localization. If the model contains no errors, then beam momenta in the adjacent BPM calculated by means of the first section and the second one coincide. If there is an error, computed momenta are different and either or both sections have errors in them. Thus, carrying out momentum comparison for the whole accelerator structure one can reveal the regions with significant errors. In case momenta do not match well one can fit model parameters and obtain fitted accelerator model in BPMs. In regions with small errors one can assume that the model describes these regions accurately.

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THEORETICAL FRAMEWORK

Transfer Map

Consider accelerator section confined within two BPMs. At this section accelerator model can be described with transfer map. Transfer map allows transferring beam positions q and momenta p between BPMs:

$$\begin{pmatrix} q_f \\ p_f \end{pmatrix} = \begin{pmatrix} \mathcal{Q} \\ \mathcal{P} \end{pmatrix} \begin{pmatrix} q_i \\ p_i \end{pmatrix}$$

where $\mathcal{Q}$ and $\mathcal{P}$ denote the position and momentum parts of section transfer map.

Transfer map depends on parameters such as small fractional errors of the strengths of magnet elements. And can be represented as Taylor series expansion with respect to phase space variables with coefficients depending on model parameters:

$$q_f = \alpha_q q_i + \beta_q p_i + \gamma_q {q_i}^2 + \delta_q q_i p_i + \varepsilon_q {p_i}^2 + \ ,$$

$$p_f = \alpha_p q_i + \beta_p p_i + \gamma_p {q_i}^2 + \delta_p q_i p_i + \varepsilon_p {p_i}^2 + \ .$$

where $\alpha, \beta, \gamma, \ldots$ – coefficients of transfer map.

COSY INFINITY allows us to find these coefficients as functions of errors.

Generating Function

Let $\mathcal{I}_q$ and $\mathcal{I}_p$ denote the position and momentum parts of the identity map. Then,

$$\begin{pmatrix} q_f \\ q_i \end{pmatrix} = \begin{pmatrix} \mathcal{Q} \\ \mathcal{I}_q \end{pmatrix} \begin{pmatrix} q_i \\ p_i \end{pmatrix}$$

If the map $(\mathcal{Q}, \mathcal{I}_q)^t$ is invertible, we have the following relationship [2]:

$$\begin{pmatrix} p_f \\ p_i \end{pmatrix} = \begin{pmatrix} \mathcal{P} \\ \mathcal{I}_p \end{pmatrix} \circ \begin{pmatrix} \mathcal{Q} \\ \mathcal{I}_q \end{pmatrix}^{-1} \begin{pmatrix} q_f \\ q_i \end{pmatrix} \qquad (1)$$

which expresses initial and final momenta in terms of initial and final positions.

Right hand side of the eq. (1) represents differential of F_1-type generating function. Every symplectic map can be represented by at least one of four generating functions in mixed variables. $F_1(q_i, q_f)$ generating function of the symplectic map allows us to obtain beam momenta by knowing its positions. Generating function satisfies the following conditions:

$$\left(p_i,\ p_f\right) = \left(\nabla_{q_i} F_1,\ -\nabla_{q_f} F_1\right)$$

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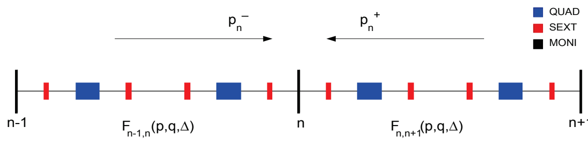


Figure 1: FODO structure.

Method Description

Consider a sequence of magnetic elements consisting of two periodic cells confined within three BPMs as shown in Fig. 1. Using measurements from at least three BPMs and generating functions between them allows obtaining momenta at BPMs as the functions of model parameters and thus comparing momenta computed for different sections. By minimizing L^2-norm of $||p_n{}^- - p_n{}^+||$ one can fit errors and thus, fitted lattice parameters such as transfer map coefficients, beta-functions, tunes and other can be obtained as well.

MODEL EXAMPLES

Error Localization

As stated above, the method can be used for error localization. We consider simple accelerator structure consisting of twelve FODO cells with four sextupoles in each cell and BPMs are placed between the cells (Fig. 2).

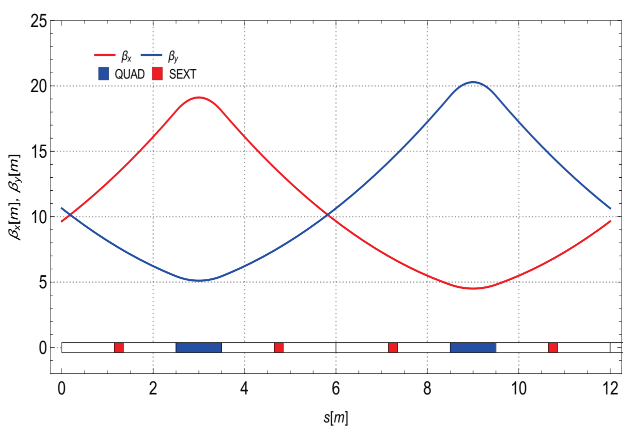


Figure 2: Beta-functions and profile for a single periodic cell.

The beam momenta in BPMs can be determined from transverse beam position in three neighboring monitors. Thus, for each BPM we can compute transverse momenta $p_n^-(q_{n-1}, q_n)$ and $p_n^+(q_n, q_{n+1})$. If there are no errors in the region placed between three monitors, then correlation between the momenta in the adjacent BPM should be equal to 1.

As an example, we set errors in the 5th, 9-11th cells. The results of momentum comparison is shown in Fig. 3, where blue dots denote momentum comparison based on two neighboring cells and momentum comparison with tracking is denoted by red dots. As we can see, the regions with errors have been localized. We use

$$\Delta_n = 1 - 1/2(corr(p^-_{x,n}, p^+_{x,n}) + corr(p^-_{y,n}, p^+_{y,n}))$$

as a comparison criterion with n being BPM number. Δ_n is equal to zero when two neighbouring cells have no errors and not equal to zero in case when either or both of them have errors. Δ_n is larger for larger errors. Hence, comparing momenta for all BPMs one can distinguish lattice regions with significant errors.

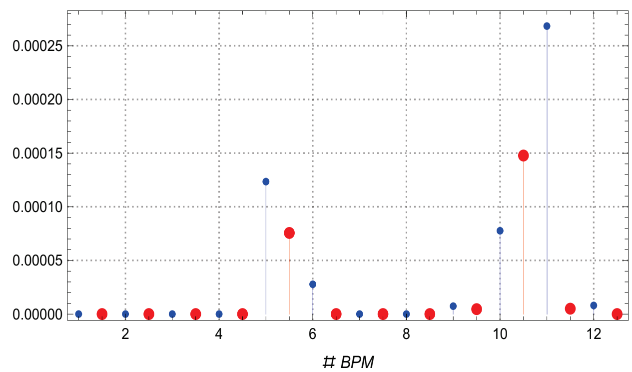


Figure 3: Error localization by comparison of momenta between the cells (blue) and momenta of each cell with ones from tracking (red).

Model Parameter Fitting

We use the above lattice for model parameter fitting. Here model parameters are small fractional errors of the strengths of quadrupoles and sextupoles. We set normally distributed random errors with ±5%. For each BPM we compute transverse momenta as functions of model parameters. Then we fit the parameters to minimize $||p_n{}^- - p_n{}^+||$.

Three model examples have been considered:

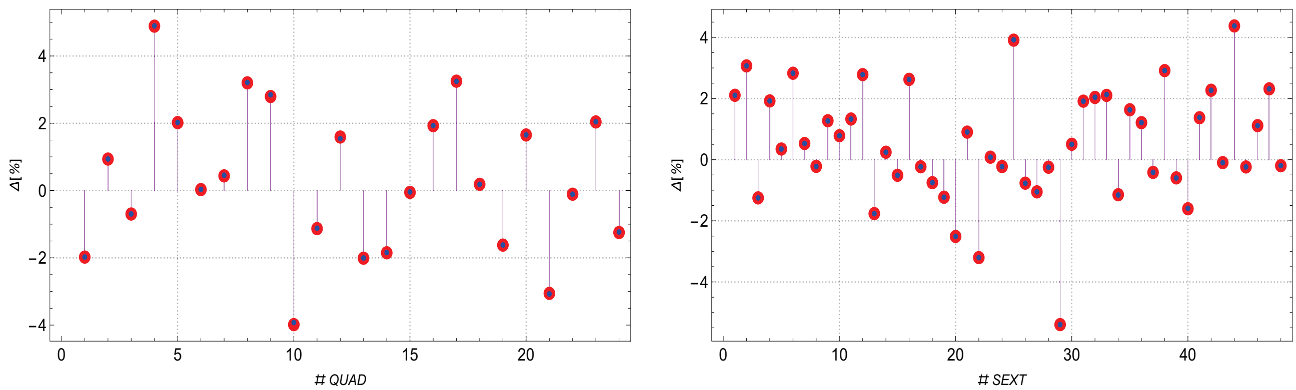
- quadrupole errors
- sextupole errors
- quadrupole and sextupole errors

In case of a set of quadrupole errors or sextupole ones the method determines model parameters with great accuracy as shown in Fig. 4.

In case when both types of errors are set model parameters are not determined exactly, but their fitted values provide good approximation for transfer map coefficients (Table 1).

DISCUSSION AND CONCLUSION

As is known, for a small amount of errors it is possible to determine the exact error magnitudes. If there are many errors in the lattice, then generally they can not be recover exactly, but still can be fitted to obtain coefficients of the transfer map. BPM data can be obtained either from turn-by-turn measurements or by scanning orbit with correctors.



(a) (b)

Figure 4: a) Set (red) and fitted (blue) quadrupole errors; b) set (red) and fitted (blue) sextupole errors.

Table 1: Transfer Map Coefficients up to Second Order for a single cell in case of quadrupole and sextupole errors.

	with set errors	with fitted errors
x	-1.194 x + 9.308 p_x -13.68 x^2 -83.81 $x\ p_x$ 40.96 $p_x{}^2$ + 1.861 y^2 -106.7 $y\ p_y$ -419.6 $p_y{}^2$	-1.194 x + 9.308 p_x -13.69 x^2 -83.84 $x\ p_x$ 41.19 $p_x{}^2$ + 1.85 y^2 -106.9 $y\ p_y$ -420.2 $p_y{}^2$
p_x	-0.2909 x + 1.431 p_x -1.826 x^2 -24.75 $x\ p_x$ 56.25 $p_x{}^2$ -3.929 y^2 -66.4 $y\ p_y$ -209. $p_y{}^2$	-0.2909 x + 1.431 p_x -1.826 x^2 -24.78 $x\ p_x$ 56.4 $p_x{}^2$ -3.938 y^2 -66.52 $y\ p_y$ -209.3 $p_y{}^2$
y	1.661 y + 10.31 p_y 27.78 $x\ y$ + 74.18 $x\ p_y$ -36. $y\ p_x$ -331.9 $p_x\ p_y$	1.661 y + 10.31 p_y 27.8 $x\ y$ +74.24 $x\ p_y$ -36.09 $y\ p_x$ -332.5 $p_x\ p_y$
p_y	-0.2548 y -0.9797 p_y 2.038 $x\ y$ + 17.65 $x\ p_y$ -41.71 $y\ p_x$ -229.3 $p_x\ p_y$	-0.2548 y -0.9797 p_y 2.044 $x\ y$ + 17.69 $x\ p_y$ -41.78 $y\ p_x$ -229.7 $p_x\ p_y$

BPM alignment errors can also be considered as parameters. BPM random errors are usually not important, but special attention should be paid to such systematic errors as monitor calibration and monitor rotation error. We also would like to compare the method to other methods such as Response Matrix Method. In the next stage, more realistic model between BPMs should be considered, since BPM measures the beam centroid position. As we noticed, COSY INFINITY is limited with respect to the number of parameters, computation of transfer maps and generating functions is planned to be performed based on Lie transformation technique.

REFERENCES

[1] M. Berz and K. Makino, "COSY INFINITY 9.0 Beam Physics Manual" (2006)

[2] M. Berz, "Modern Map Methods in Particle Beam Physics", Academic Press, 1999, ISBN 0-12-014750-5