

# COMPARING THE PERFORMANCE OF MOGA AND MOPSO IN OPTIMIZATION OF THE HEPS PERFORMANCE\*

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## Abstract

The High Energy Photon Source (HEPS), a kilometre-scale diffraction-limited storage ring light source, with a beam energy of 5 to 6 GeV and transverse emittances of a few tens of pm.rad, is to be built in Beijing. A preliminary design with a hybrid 7BA lattice, a natural emittance of 60 pm.rad and a circumference of about 1.3 kilometers, has been made. Based on this design, we optimized the linear and nonlinear performance of the ring with the MOGA and MOPSO algorithms. From comparison of the performance of these two algorithms, it was found that MOPSO promises higher diversity than MOGA, while MOGA can reach better convergence than MOPSO. To reach a true Pareto front, a successive and iterative implementation of the PSO and MOGA, rather than using either of these two algorithms, is suggested.

## INTRODUCTION

Along with continuous advances in accelerator technology and unceasing pursuit of higher quality photon flux, effort is being made worldwide to design or construct the diffraction-limited storage rings (DLSRs) [1] with the aim to push brightness and coherence beyond the limits of existing third generation light sources (3GLSs), by reducing the emittance to approach the diffraction limit for the range of X-ray wavelengths of interest to the scientific community. Different from 3GLSs that generally consist of DBAs or TBAs, in a DLSR design multi-bend achromat (MBA) lattice with compact layout as well as strong focusing is usually adopted to achieve an ultralow emittance within a reasonable circumference.

A kilometre-scale storage ring light source with a beam energy of 5 to 6 GeV, named the High Energy Photon Source (HEPS), has been proposed to be built in Beijing. Recently a preliminary design [2] based on the concept of ‘hybrid MBA’ [3] was developed for the HEPS. The ring consists of 48 identical hybrid seven-bend achromats (7BAs), with a natural emittance of 60 pm.rad at 6 GeV and a circumference of about 1.3 kilometers. The layout and optical functions of a single 7BA are shown in Fig. 1.

Each 7BA uses high-gradient (up to 80 T/m) quadrupoles and defocusing dipoles in the middle two unit cells, to create a compact layout with ultralow emittance. Also, it uses four outer dipoles with longitudinal gradients to generate two dispersion bumps, for a more efficient chromatic correction than available in a standard MBA. Moreover, a  $-I$  transportation between each pair of sextupoles, with phase advance at or close to  $(2n+1)\pi$  (where  $n$  is an integer) in both  $x$  and  $y$  planes, is designed to cancel

most of the nonlinearities induced by the sextupoles. The long straight section for insertion device (ID) is of 6 m. To reserve as much space as possible for diagnostics and other equipment, only four families of multipoles are placed in the dispersion bumps (see Fig. 1) for nonlinear optimization. Due to the small number of free knobs, we were able to do a grid scan of the nonlinear performance with respect to the multipole strengths in a reasonable time. However, it was found very difficult to simultaneously optimize the DA and MA for this design, even with the adoption of the dispersion bumps and the local cancellation scheme. The compromise solution predicts an ‘effective’ DA of 2.5 (or 2.2) mm in the  $x$  (or  $y$ ) plane and an ‘effective’ MA of 2.4%. Since magnetic and misalignment errors were not considered in the particle tracking, the effective DA or MA, which counts only the stable motions with small tune variations (i.e., with fractional part of the tunes in the range of  $[0, 0.5]$ ), rather than the DA or MA defined in a conventional sense (i.e., the area with all surviving particles after tracking over typically a few thousand turns), is used here to give a more accurate estimation of the actual ring acceptance.

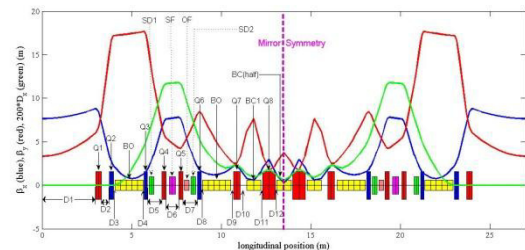


Figure 1: Optical functions and layout of a single hybrid 7BA of the HEPS preliminary design.

Due to the small DA, we proposed to use on-axis longitudinal injection for beam accumulation in HEPS by using RF cavities of double frequencies [4]. This method can greatly reduce the requirement of the MA to the level of 3%.

With the same layout as the original design, by including both the quadrupole and multipole strengths in global scan and minimizing the required chromatic sextupole strengths followed with tune space survey, we were able to attain a better design [5] with a similar emittance, 59.4 pm.rad, but with larger MA ( $\sim 3\%$ ) and DA ( $\sim 2.5$  mm in  $x$  and 3.5 mm in  $y$  plane).

Nevertheless, it is worthy to check whether there exist designs with even better nonlinear performance. To this end, we systematic scanned all the possible tuneable element parameters, including the lengths and positions of all magnets, bending angles of dipoles and gradients of quadrupoles and multipoles, within specific ranges that are determined by practical or optical constraints. The

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lattice performance was optimized using two stochastic optimization algorithms, e.g., the multi-objective genetic algorithm (MOGA) and multi-objective particle swarm optimization (MOPSO) algorithm. The MOGA methods mimic the process of natural selection and evolution of species, and have been widely used in many accelerator optimization problems [e.g., 6-8]. While MOPSO emulates the self-organizing behavior of social animals living in group, and recently was applied to optimization of the linac operation and ring nonlinear dynamics [9, 10]. It has been demonstrated both methods are powerful and effective in solving the problems where the objectives are piecewise continuous and highly nonlinear, and have many local optima. Nevertheless, study [10] indicated that MOPSO converges faster, and is not as dependent on the distribution of initial population as MOGA. In this study, we compared the performance of these two algorithms in an optimization problem with already-known answer, from which we showed that optimization with a rational combination of MOPSO and MOGA will be more effective than using any of them.

### MOGA OPTIMIZATION FOR FIXED ID SECTION LENGTH

Pervious HEPS lattice design experience indicated that minimization of the chromatic sextupole strengths followed with tune space survey can effectively improve the nonlinear performance. Thus, we first chose to look at the optimal trade-offs between the emittance and the sextupole strengths required for chromaticity correction, for the case with fixed ID section length,  $L_{ID} \equiv 6$  m. One of the MOGA methods, NSGA-II [11], was used.

In the optimization we used 26 optimizing variables and two objectives (weighted emittance and sextupole strengths). To ensure enough diversity in the initial population, we first randomly generated lots of possible combinations of optimizing variables with large fluctuations around those of the original design, from which we selected 6000 solutions with stable optics and used them as the initial population.

For the sake of the comparison of sextupole strengths between different solutions, the sextupoles in the lattice were grouped in just two families (SD, SF) and their lengths are set to 0.2 m, such that there is a unique solution of the sextupole strengths ( $K_{sd}$ ,  $K_{sf}$ ) for specific corrected chromaticities ([0.5, 0.5] in this study). And the calculated sextupole strengths were then represented with a nominal strength,

$$K_s = \sqrt{(K_{sf}^2 + K_{sd}^2) / 2}. \quad (1)$$

For a specific combination of optimizing variables, before evaluating the horizontal natural emittance  $\epsilon_{x0}$  and  $K_s$ , the optics was matched by tuning several quadrupoles to ensure (if feasible) the achromatic condition (with  $K_{Q3}$  and  $K_{Q4}$ ) and the  $-I$  transportation between each pair of sextupoles (with  $K_{Q5}$ ,  $K_{Q6}$  and  $K_{Q7}$ ). Moreover, in lattice

evaluation some constraints were considered, and the degree of the violation of the constraints were measured with a series of weight factors. These factors were then multiplied by  $\epsilon_{x0}$  and  $K_s$  to get the values of the two objectives. The constraints include a reasonable maximum value of beta function  $\beta_{x,y}$  [ $\max(\beta_{x,y}) \leq 30$  m], reasonably low beta functions in ID section for high brightness ( $1.5 \text{ m} < \beta_y < 4 \text{ m}$  and  $1.5 \text{ m} < \beta_x < 15 \text{ m}$ ), stability of the optics [ $\text{Tr}(M_{x,y}) < 2$ , with  $M_{x,y}$  being the transfer matrix of ring in  $x$  or  $y$  plane], fractional tunes in  $(0, 0.5)$  that is favorable against the resistive wall instability, etc. If a specific constraint is satisfied, the corresponding weight factor will be one; otherwise the factor will be assigned a value of above 1, and more violated the constraint is, the larger the factor will be. In this way, among the solutions with the same or similar  $\epsilon_{x0}$  and  $K_s$ , the desirable ones (meet all constraints) will have smaller objective values than those that violate certain constraints, and will be assigned a higher rank with the non-dominated sorting and have higher priorities for survival and reproduction in the evolution chain.

Note that only the gradients of quadrupoles were used as optimizing variables. To optimize the quadrupole lengths, we performed an optimization, where the quadrupole gradients were varied in larger ranges than available. From the covering range of gradients of the final population, we adjusted the quadrupole lengths in such a way that all the gradients are below but close to their upper limits. Then, another optimization was implemented with the modified quadrupole lengths. The population evolved over 1000 generations (see Fig. 2 for the final population). It was noticed that the population was already very close to the final population at generation 600.

### MOGA AND PSO OPTIMIZATIONS FOR VARIABLE ID SECTION LENGTH

In the above optimization, all possible element parameters are tuned except the length of ID section  $L_{ID}$ . One can consider that if with a shorter  $L_{ID}$ , there will be larger room for variation of the position of magnets and the length of dipoles, and thus it would be feasible to achieve designs with better performance.

To explore the potential of the lattice with a shorter ID section, the  $L_{ID}$  was included in the optimizing variables and varied in a range of [5, 7] m. The MOGA final population obtained above, with small modifications, was used as the initial population of the new optimization. The modifications include generating random values drawn from a normal distribution with an average of 6 m for the variable  $L_{ID}$ , and accordingly adjusting the values of the variable for the length of the drift D6 to keep the circumference unchanged. The standard deviation of the random seeds for  $L_{ID}$  was set to a small value (0.1 m) to ensure that most of individuals of the initial population have stable optics.

For comparison, the same initial population was evolved over 800 generations with both MOGA and MOPSO. It was found that the solutions with  $L_{ID}$  of larger

than 6 m are gradually phased out in the evolution of both algorithms. In addition, as shown in Fig. 2, most of the solutions at the last generation of MOGA and MOPSO have better performance (i.e., with smaller  $K_s$  at a specific emittance) than those optimized for  $L_{ID} \equiv 6$  m.

On the other hand, the difference in the performance of these two algorithms is also obvious. For MOGA, the  $L_{ID}$  values of the final solutions are all above 5.75 m, which do not exceed the covering range of  $L_{ID}$  values in the initial population. While for MOPSO, a majority of solutions have  $L_{ID}$  values of close to 5 m, and in particular, predict smaller sextupole strengths than with MOGA. Actually this difference has been well explained [10]. In MOPSO, each surviving individual adjusts its moving pace and direction in variable space at every iterative step, according to its own historical experience and relative position within the population. The new solutions are not generated from the existing good solutions, as what is done in MOGA. Thus, MOPSO intrinsically allows more diversity than MOGA, and does not need a diverse seeding in the initial population.

However, one can see from Fig. 2 that the MOPSO solutions distribute rather loosely in the objective space. Also, in the low emittance region (e.g., at  $\epsilon_{x0}$  of about 45 pm.rad), some solutions still have  $L_{ID}$  values of about 6 m, with even larger  $K_s$  than those optimized for  $L_{ID} \equiv 6$  m. As shown in Fig. 3, the situation did not substantially change even after 500 more generations of evolution with MOPSO. By contrast, after evolving the MOPSO population at generation 800 over the same generations with MOGA, the population reached a good convergence to the Pareto optimal front, predicting solutions with  $L_{ID}$  values all close to 5 m and with superior performance over those optimized for  $L_{ID} \equiv 6$  m in the whole emittance range of interest.

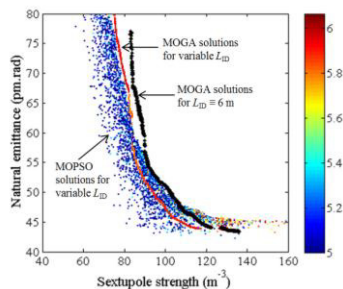


Figure 2: MOGA solutions for  $L_{ID} \equiv 6$  m (black curve) and MOGA and MOPSO solutions for variable  $L_{ID}$ .

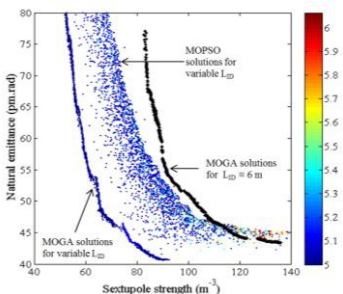


Figure 3: Solutions after 500 more generations of MOGA and MOPSO.

As expected, by optimizing first with MOPSO and then with MOGA, the results show that a shorter  $L_{ID}$  can greatly benefit the lattice design. By shortening the ID section from 6 m to about 5 m, the emittance can be further reduced to about 40 pm.rad, or at the emittance of 60 pm.rad the nominal sextupole strength  $K_s$  can be further decreased by at least 40%. Of course, it is worth mentioning that a shorter  $L_{ID}$  implies a reduced drift space for IDs that are used to create high radiation flux, which, in turn, will affect the machine performance.

## CONCLUSION

From this study, one can learn that MOGA depends significantly on the distribution of initial population. If without enough diversity in the initial population, MOGA will probably converge to a local optimum instead of the true global optima, especially for a very complicated multi-objective problem with many optimizing variables and local optima. Worse still, the MOGA itself cannot give a measure of the diversity of a population. Consequently, if applying MOGA to a typical exploratory multi-objective problem and without another effective algorithm (e.g., MOPSO in this study) for comparison, one will not be able to know for sure whether the final solutions reveal optimal trade-offs between the different objectives. In short, to make an effective MOGA optimization, it is critical, and also challenging, to seed the initial population with high enough diversity. Fortunately, as demonstrated, this difficulty can be overcome with MOPSO, which has the intrinsic ability of breeding more diversity in the process of population evolution. And once the diversity in solutions is ensured, MOGA can reach a better convergence than MOPSO to the true global optima. Therefore, evolving the population with a rational combination of MOPSO and MOGA would be more effective than using either of these two algorithms.

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