# MODEL DRIVEN MACHINE IMPROVEMENT OF COSY BASED ON ORM DATA 

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#### Abstract

In order to improve the existing model of the COoler SYnchrotron (COSY) Jülich a technique called Linear Optics from Closed Orbit (LOCO) [1], originally used at light sources, is favored. For the use at COSY, a hadron storage ring, the LOCO algorithm was implemented in a newly developed $\mathrm{C}++$ program, which uses MAD-X ${ }^{1}$ for optics calculation and ROOT ${ }^{2}$ for data analysis and illustration. First results of the benchmarking of the program with simulated data are discussed.


## INTRODUCTION

COSY's diverse capabilities of phase space cooling and the flexibility of its lattice with respect to ion-optical settings makes it an ideal test facility for accelerator technology developments [2-4]. High demands on beam control and beam based measurements have to be fulfilled for future experiments such as the proposed precursor experiment for a first measurement of the electric dipole moment of the deuteron (see [5] and references within).
One major task toward reaching this goal is the improvement of the model of COSY, which currently reaches a precision of $\Delta \beta / \beta \approx 30-50 \%$ [6]. A technique called LOCO, that was successfully applied for the calibration and correction of linear optics at light sources [1,7,8], will be used. For the application on a proton storage ring that operates at nonrelativistic energies a new program, utilizing the LOCO algorithm, was written.

## LOCO PROGRAM

## Algorithm

LOCO is based on the analysis of a measured orbit response matrix (ORM), which contains thousands of data points reflecting the focusing structure of the ring [1]. The closed orbit ORM is defined by:

$$
\begin{equation*}
\binom{\vec{x}}{\vec{y}}=M \cdot\binom{\vec{\Theta}_{x}}{\vec{\Theta}_{y}}, \tag{1}
\end{equation*}
$$

where $(\vec{x}, \vec{y})$ are the measured horizontal and vertical shifts of the closed orbit at all beam position monitors (BPMs) for a change in the deflection strength of the steering magnet by $\left(\vec{\Theta}_{x}, \vec{\Theta}_{y}\right)$. A typical ORM measurement at COSY utilizes approximately 40 steering magnets and about 60 BPMs (30 horizontal, 30 vertical) along the ring, resulting in about 2400 ORM entries.
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${ }^{1}$ The MAD-X Program, http://madx.web.cern.ch
${ }^{2}$ The ROOT data analysis framework, https://root.cern.ch
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The model response matrix, for comparison with the measurement, is calculated by the MAD-X accelerator optics program. One example ORM, calculated with the COSY model is shown in Fig. 1.
The goal of the LOCO algorithm is to improve the COSY model by comparing the model ORM and the measured ORM. A $\chi^{2}$-minimization is performed by adjusting the model parameters until the ORMs are equal. The $\chi^{2}$ is defined as the squared sum of the differences between the model and the measured ORM entries ( $M_{\text {mod }}, M_{\text {meas }}$ ), weighted with the inverse of their measurement errors squared $\left(\sigma_{M_{\text {meas }, i j}}\right)$ :

$$
\begin{equation*}
\chi^{2}=\sum_{i, j} \frac{\left(M_{\mathrm{mod}, i j}-M_{\mathrm{meas}, i j}\right)^{2}}{\sigma_{M_{\mathrm{meas}, i j}}^{2}}=\sum_{k=i, j} E_{k}^{2} \tag{2}
\end{equation*}
$$

The indices $i$ and $j$ denote the BPM and the steerer magnet, respectively. The model parameters $K_{l}$ under investigation so far are listed in Tab. 1. By varying these, the error vector $d E_{k} / d K_{l}$ can be determined for every parameter:

$$
\begin{equation*}
-E_{k}=\frac{d E_{k}}{d K_{l}} \cdot \Delta K_{l} \tag{3}
\end{equation*}
$$

Combining $d E_{k} / d K_{l}$ for all parameters results in the Jacobian matrix $d E_{k} / d K$. Applying a singular value decomposition (SVD) to this non-square matrix allows for the determination of its pseudoinverse and the direct recalculation of the parameter error $\Delta K$ :

$$
\begin{equation*}
\frac{d E_{k}}{d K}=U S V^{\mathrm{T}}=\sum \vec{u}_{l} s_{l} \vec{v}_{l}^{\mathrm{T}} \text { and } \tag{4}
\end{equation*}
$$



Figure 1: Calculated orbit response matrix of COSY.
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Table 1: Parameters under investigation.

| Parameter name | Number |
| :--- | ---: |
| BPM calibration | 60 |
| BPM roll $(\psi)$, shift $(s)$ | $2 \cdot 60$ |
| Steerer calibration | 40 |
| Steerer roll $(\psi)$, position $(s)$ | $2 \cdot 40$ |
| Deflection angle (offset) | 40 |
| Gradient of quadrupole families | 14 |
| Gradient of individual quadrupoles | 56 |
| Quadrupole rotations $(\phi, \theta, \psi)$ |  |
| and misalignments $(x, y, s)$ | $6 \cdot 56$ |
| Quadrupole coefficient of dipoles (K1) | 24 |
| Sextupole coefficient of dipoles (K2) | 24 |
| Sextupole coefficient of quadrupoles $(\mathrm{K} 2)$ | 56 |

$$
\begin{equation*}
\Delta K=-\sum \vec{v}_{l} \frac{1}{s_{l}} \vec{u}_{l}^{\mathrm{T}} \cdot E_{k} \tag{5}
\end{equation*}
$$

Since the response matrix is not linear to most of the parameters LOCO must be iterated until it converges to the best set of parameters.

## Implementation

The implementation of the LOCO algorithm is visualized in Fig. 2. At the beginning of the program, all relevant data such as input parameters and settings required for the initialization of the LOCO procedure are read from the configuration file. Depending on the choice of the user a reference ORM ( $M_{\text {meas }}$ ) is prepared from real data or generated using random parameter settings. In the latter case, the parameter generation is correlated to a user-defined Gaussian distribution. The ORM is calculated using a specially developed MAD-X script that provides orbit data for different deflection strengths of every single steering dipole. These


Figure 2: Flow chart (top to bottom) of the new LOCO program.
simulated orbit data are analyzed in the same way as the measured orbit data [9].
The LOCO algorithm now starts with the calculation of an undisturbed ORM ( $M_{\text {mod }}$ ) using the existing COSY lattice without parameter errors. The resulting matrix $E_{k}$ derived according to Eq. 2 forms the starting point of the first iteration of the LOCO algorithm.
Subsequently, one parameter is varied in three to five steps and a new ORM is calculated for every step. The change of every ORM entry with respect to the reference entry is plotted in a separate graph as function of the parameter variation. The slope of a linear fit to these data points yields the entries of the error vector $d E_{k} / d K_{l}$. This procedure is repeated for every parameter. All resulting error vectors are combined to the Jacobian matrix $d E_{k} / d K$. The pseudoinverse of this matrix, calculated using Armadillo libraries ${ }^{1}$, is than multiplied to $E_{k}$ (Eq. 5) yielding the new parameter error $\Delta K$ for the next iteration. After every iteration the collected data are stored into a ROOT file containing the optical functions, the ORM data and a ROOT Tree with all parameter settings.

## BENCHMARKING

For benchmarking the newly developed program hundreds of simulations have been performed for various sizes and types of parameter errors as well as for combinations of parameter types. In addition, the sensitivity of the algorithm to the BPM errors, the step sizes of parameter variations and the truncated rank of the matrix in the SVD analysis have been studied.
Exemplary discussed here is the LOCO analysis of simulated gradient errors of all 56 quadrupole magnets along COSY. The example shown in Fig. 1 is the calculated ORM for a Gaussian distributed ( $\sigma=1 \%$ ) change of the gradients. Executing the LOCO algorithm adjusts the parameters of the lattice model in order to minimize the $\chi^{2}$ (Eq. 2). A histogram of the differences $\Delta M_{i, j}$ between the calculated and the reference ORM is displayed in Fig. 3. The rms of the deviation is significantly improved with the number of


Figure 3: Histogram of the difference $\Delta M_{i, j}$ between the calculated and the reference ORM for five iterations. A decreasing rms indicates an improving agreement.


Figure 4: Top panel: $\Delta \mathrm{K}=\mathrm{K}_{\mathrm{mod}}-\mathrm{K}_{\text {meas }}$ for 56 quadrupole gradients is shown for the starting conditions (red curve) and after up to five iterations.
Bottom panel: The $\Delta \beta / \beta_{\text {meas }}$ in the horizontal plane can be improved by about four orders of magnitude assuming almost perfect BPMs.
iterations. Figure 4 displays in the top panel the deviation of the reconstructed parameter settings with respect to the measurement ( $\Delta \mathrm{K}=\mathrm{K}_{\text {mod }}-\mathrm{K}_{\text {meas }}$ ) and in the bottom panel the $\beta$-beat for five iterations. As visible $\Delta K$ is improved from $9.3 \cdot 10^{-3}$ to $1.6 \cdot 10^{-5}$ meaning that the randomly generated variations are very well detected. In the case of gradient errors $K$ is a dimensionless scaling factor. The $\beta$-beat of initially 0.23 was decreased to $6.4 \cdot 10^{-5}$. These numbers were achieved assuming an error of $10^{-9} \mathrm{~m}$ for the position measurement of the BPMs, this corresponds to almost ideal BPMs.
The effect of the error of the position measurement on the reconstruction of different model parameters was carefully studied. In Fig. 5 the resulting rms of $\Delta \beta_{x} / \beta_{x}$ for the variation of quadrupole gradients is plotted as function of the iteration number assuming BPM errors from 1 nm up to 1 mm . Summarizing this graph, the optical functions can still sufficiently be reproduced assuming $\Delta x=10^{-5} \mathrm{~m}$. The reconstruction of the parameter settings is usually a bit worse. This statement is true for most of the parameters named in Tab. 1. Sometimes good optics correction is achieved with incorrect parameter settings (e.g. transverse quadrupole misalignment). This effect is still under investigation.
An example of the sensitivity of the algorithm to the step
size of the parameter variation is shown in the bottom panel of Fig. 5. Here, the reconstruction of the roll $(\psi)$ of all BPMs is plotted for step sizes ranging from 0.001 to 2 rad. Only for very large steps a significant effect was seen. In general, the sensitivity to the step size is different for all parameters. Other ongoing investigations deal with the sensitivity for the different parameter types and to the truncated rank of the Jacobian matrix in the SVD analysis.


Figure 5: Top: The determination of $\Delta \beta_{x} / \beta_{x}$ for adjusting the gradients of all 56 quadrupole magnets clearly depends on the error of the beam position measurement, which is varied from 1 nm up to 1 mm .
Bottom: Reconstruction of the rolls of 60 BPMs for different step sizes of parameter variation.

## SUMMARY AND OUTLOOK

A new LOCO program for the use at COSY was successfully developed and brought into operation. Benchmarking tests show promising results with respect to the use of the program with measured data. Besides finalizing the benchmarking and the implementation of additional features into the program, such as an automatic step size finder or a second minimization algorithm, a first analysis of measured data is planned. An automated ORM measurement system, which was implemented in the past year, allows to collect one full ORM data set within 30 minutes [9]. Several of these measurements with different settings of the quadrupole gradients were accomplished in 2015. The goal of the LOCO analysis of these data is the determination of the applied changes.

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