# COMPENSATION OF STEERER CROSSTALK BETWEEN FLASH1 AND FLASH2

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### Abstract

The free-electron laser in Hamburg (FLASH) [1] is a user facility delivering soft X-ray radiation in the range from 4.2 nm to 50 nm in up to 8000 pulses per second. Ten bunch trains per second with up to 800 electron bunches separated by 1  $\mu$ s are accelerated to energies from about 380 MeV to almost 1250 MeV. Starting from 2014, a second beam line for FEL (free electron laser) operation, FLASH2, has been commissioned. The first beam line, now called FLASH1, and FLASH2 are both driven by the same injector and linac. Downstream of the last accelerating module, the sub-train for FLASH2 is vertically kicked and ejected into the deflecting channel of a horizontal Lambertson septum with a deflection angle of 6.5° [2].

Naturally, in the region of the beam switch the horizontal separation of the two beam lines is rather small. In fact it has been observed that the first steerer magnets (correction dipoles) in each beam line perturb the orbit in the other beam line. This crosstalk can significantly degrade the FEL performance.

We have developed a method for locally compensating the orbit crosstalk using combinations of orbit bumps [3]. The perturbation due to a steerer in one beam line is corrected using additional steerers in the other beam line. This concept has already been tested at FLASH in 2015 and thereby has proven to compensate the crosstalk sufficiently well to ensure unperturbed FEL operation.

### VIRTUAL STEERERS

In order to model the steerer crosstalk quantitatively we introduce the notion of a *virtual steerer*. Given a *real* steerer acting as a dipole magnet on, say, beam line I with kick strength  $\kappa^{I}$  at position  $s^{I}$  (along the reference trajectory in beam line I), its transverse stray field will kick the beam orbit in the other beam line, say II, like there was an additional, *virtual* steerer located at  $s^{II}$  in beam line II, the point closest to the real steerer in beam line I, with horizontal and vertical kick strengths  $\kappa_{x}^{II} \propto \kappa_{y}^{II}$ . The magnetic field outside the iron yoke is not designed to be purely horizontal or vertical, therefore virtual steerers will in general deflect the beam horizontally and vertically at the same time and with different (proportional) kick strength.

To quantify the coupling of the virtual to the real steerer, the crosstalk ratio T is introduced

$$T_{x,y} = \frac{\kappa_{x,y}^{\text{virtual}}}{\kappa^{\text{real}}},\tag{1}$$

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where  $\kappa^{\text{real}}$  is the kick strength of the real steerer in its deflecting plane and  $\kappa^{\text{virtual}}_{x,y}$  are the kick strengths of the corresponding virtual steerer in *x* or *y*.

### Measurement of the Crosstalk Ratio

A convenient way to measure the strength of a virtual steerer and its crosstalk ratio uses orbit response matrices (ORMs). Given *N* steerers  $\{K_i\}_{1 \le i \le N}$  with kicks  $\{\kappa_i\}_{1 \le i \le N}$ , *M* BPMs  $\{B_j\}_{1 \le j \le M}$  measuring beam positions  $\{x_j\}_{1 \le j \le M}$ , and the linear transfer map  $\mathbf{M}_{j \leftarrow i}$ , the evolution of trajectories between  $K_i$  and  $B_j$  reads

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{j} = \mathbf{M}_{j \leftarrow i} \begin{pmatrix} x \\ x' \end{pmatrix}_{i} .$$
 (2)

The difference orbit  $\Delta x_j$  at BPM B<sub>j</sub> after changing the kick strength of steerer K<sub>i</sub> by  $\Delta \kappa_i$  is given by the the ORM **O** 

$$\Delta x_j = (\mathbf{O})_{j,i} \ \Delta \kappa_i := \left(\mathbf{M}_{j\leftarrow i}\right)_{1,2} \ \Delta \kappa_i \,. \tag{3}$$

Therefore it is possible to calculate the kick strength (and thus the crosstalk ratio T) of a virtual steerer, if the transfer matrix element between the virtual steerer and the BPM is computed using the magnetic lattice, the beam rigidity, the magnet excitation curves and the magnet currents.

In the FLASH switchyard, the steerer that is the closest to the other beam line, which has an iron length of 10 cm, a gap height of 4 cm and a distance from the outer edge of its iron yoke to the center of the neighboring beam line of only 5 cm, has measured crosstalk ratios of  $T_x = 0.2751 \pm 0.0043$  and  $T_y = 0.166 \pm 0.030$ .

# BUMP-BASED COMPENSATION OF THE STEERER CROSSTALK

Local orbit bumps are the designated method to shift the transverse position of a beam locally without affecting the orbit in the rest of the machine [4]. The basic building block of local orbit manipulations that is general enough to be applied to arbitrary lattices consists of three steerers in a so-called *three-bump*.

### Local Orbit Three-Bumps

A local orbit three-bump consists of three steerers. The 1<sup>st</sup> steerer produces a betatron oscillation with given amplitude and initial phase that is finally compensated at the position of the  $3^{rd}$  steerer by linear combination of the betatron oscillations due to the  $2^{nd}$  and  $3^{rd}$  steerers. The  $2^{nd}$  and  $3^{rd}$  steerer can compensate an incoming betatron oscillation with any phase given their phases difference is not an integer multiple of  $180^{\circ}$ . Since the betatron oscillations are always linearly superposed, the following three statements are always true:

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With steerers K<sub>1</sub>, K<sub>3</sub>, K<sub>3</sub> at betatron phases  $\varphi_1 < \varphi_2 < \varphi_3$ and with kicks strengths  $\kappa_1$ ,  $\kappa_2$ ,  $\kappa_3$ ,

- 1. if  $\varphi_3 \varphi_2 \neq k\pi$ ,  $k \in \mathbb{N}$ , then real constants  $C_{2,123}$ and  $C_{3,123}$  exist so that  $\kappa_2 = C_{2,123}\kappa_1 \& \kappa_3 = C_{3,123}\kappa_1$ define a closed bump  $\forall \kappa_1$ ;
- 2. if  $\varphi_3 \varphi_1 \neq k\pi$ ,  $k \in \mathbb{N}$ , then real constants  $C_{1,123}$ and  $C_{3,123}$  exist so that  $\kappa_1 = C_{1,123}\kappa_2$  &  $\kappa_3 = C_{3,123}\kappa_2$ define a closed bump  $\forall \kappa_2$ ;
- 3. if  $\varphi_2 \varphi_1 \neq k\pi$ ,  $k \in \mathbb{N}$ , then real constants  $C_{1,123}$ and  $C_{2,123}$  exist so that  $\kappa_1 = C_{1,123}\kappa_3 \& \kappa_2 = C_{2,123}\kappa_3$ define a closed bump  $\forall \kappa_3$ .

Here and in the following we use the following convention for the bump coefficients: The bump coefficient is denoted by C. The first lower index indicates the dependent steerer it controls. The second lower index-triple denotes the steerers involved in the bump. Herein the independent steerer which controls the other two is denoted in bold face. For an explicit derivation of the bump coefficients see e.g. [4]. Whenever (later) bump coefficients are defined for a special beam line and in a special plane (x or y), it is indicated by upper indices.

## Long Three-Bump



Figure 1: The method of the long three-bump. The black box indicates the absence of (noticeable) crosstalk due to either shielding or sufficient separation. Only one phase plane is shown. The other is trivially included by adding two non-crosstalking real steerers for the other plane in beam line II.

The long three-bump is the simplest scheme to correct the steerer crosstalk. The orbit perturbation produced by the first virtual steering coil is corrected using two real steerers far enough downstream, so that their influence on the other beam line is negligible. This is visualized in Fig. 1 and the closed bump condition for local compensation in one plane in terms of  $\kappa_1^I$  reads

$$\begin{aligned}
\kappa_{2}^{\text{II}} &= C_{2,123}^{\text{II}} T_{1}^{\text{II}} \kappa_{1}^{\text{I}} \\
\kappa_{3}^{\text{II}} &= C_{3,123}^{\text{II}} T_{1}^{\text{II}} \kappa_{1}^{\text{I}}.
\end{aligned}$$
(4)

We note that the long three-bump compensation scheme only contains closed bump conditions parametrized in terms of the first steerer.

Assuming the absence of phase degeneracy we only need one three-bump per plane in the second beam line to correct

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the virtual kicks due to all crosstalking steerers from the other beam line. Unfortunately, since the bump is closed potentially far downstream of the initial distortion, the compensation is possibly not "local" enough.

### Interleaved Short Three-Bumps



Figure 2: The method of interleaved short three-bumps.

Each virtual perturbation is closed as local as possible at the expense of back-reacting on the other beam line and causing the need for crosstalk compensation in that beam line. This method is based on a chain of successively correcting perturbations in one beam line while introducing new, weaker perturbations downstream in the other. The chain terminates when the new perturbations become negligible due to shielding, separation and/or the multiplying up the crosstalk coefficients to higher order. Only one such chain (starting with the most upstream crosstalking steerer) is needed per plane, since any perturbation from the crosstalk of a steerer downstream the first is is corrected by the subchain starting from there. Figure 2 shows this case.

To illuminate the decaying magnitude of the successive corrections we introduce the smallness parameter  $0 < \epsilon \ll 1$  via

$$T_i = \tilde{T}_i \cdot \epsilon \text{ with } \tilde{T}_i = O(1) .$$
(5)

Using the example from Fig. 2, assuming the perturbation starts with  $\kappa_1^{I}$ , and propagating the perturbations down the chain we find for the closed bump conditions:

$$\kappa_{2}^{\text{II}} = T_{1} C_{2,124}^{\text{II}} \kappa_{1}^{\text{I}} = O(\epsilon)$$

$$\kappa_{4}^{\text{II}} = T_{1} C_{3,124}^{\text{II}} \kappa_{1}^{\text{I}} + T_{3} C_{2,346}^{\text{II}} \kappa_{3}^{\text{I}} = O(\epsilon)$$

$$\kappa_{3}^{\text{I}} = T_{2} C_{2,235}^{\text{I}} \kappa_{2}^{\text{II}} = O(\epsilon^{2})$$

$$\kappa_{5}^{\text{I}} = T_{2} C_{3,235}^{\text{I}} \kappa_{2}^{\text{II}} + T_{4} C_{2,457}^{\text{II}} \kappa_{4}^{\text{II}} = O(\epsilon^{2})$$

$$\kappa_{7}^{\text{I}} = T_{4} C_{3,457}^{\text{II}} \kappa_{4}^{\text{II}} = O(\epsilon^{2})$$

$$\kappa_{6}^{\text{II}} = T_{3} C_{3,346}^{\text{II}} \kappa_{3}^{\text{I}} = O(\epsilon^{3}).$$
(6)

Using the interleaved short three-bumps the initial perturbation is corrected locally at the expense of introducing weaker (higher order  $\epsilon$ ) perturbations downstream, which have to be corrected successively. The orbit distortion produced by the downstream steerers is always less, since they are in general further away from the other beam line ( $\tilde{T}_j \leq \tilde{T}_i$ for i < j) and since they are of higher order in  $\epsilon$ . Therefore the orbit perturbation is always reduced until it becomes negligible. We note that the interleaved short three-bump compensation scheme only contains closed bump conditions parametrized in terms of the first steerer.

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### Recursively Coupled Short Three-Bumps



Figure 3: Visualization of the recursively coupled short three-bumps.

The recursively coupled short three-bumps are the most elegant solution to correct the steerer crosstalk. Only the crosstalking steerers, but at least two real steerers per beam line and plane are needed and the correction takes place as local as possible. A schematic can be seen in Fig. 3. The initial perturbation from the one of the crosstalking steerers of, say, beam line I is corrected using two real steerers in beam line II that can be downstream or upstream of the virtual steerer introduced by the crosstalk. Since these two steerers will in general produce crosstalk in beam line I, the perturbation is compensated using two real steerers in beam line I, closest to the virtual steerers crosstalking from beam line II. Each of the two real steerers in beam line I can be up- or downstream of any of the two virtual steerers. In particular one of them (and that is the case in Fig. 3) can be the steerer that initially introduced the perturbation on beam line II. The steerers selected in the last step in beam line I will again crosstalk to beam line II but, as we have seen before in the previous subsection, the back-reacting crosstalk due to compensating crosstalk is one order higher in  $\epsilon$ . We might therefore hope to find a convergent recursion for the steerer kicks. The recursion for the situation given in Fig. 3 with in total 4 real and 4 virtual steerers can be written in matrix form:

$$\begin{pmatrix} \Delta \kappa_1^{\mathrm{II}} \\ \Delta \kappa_4^{\mathrm{II}} \\ \Delta \kappa_1^{\mathrm{I}} \\ \Delta \kappa_3^{\mathrm{I}} \end{pmatrix}_i = \begin{pmatrix} 0 & 0 & T_1 \cdot C_{l,124}^{\mathrm{II}} & T_3 \cdot C_{l,234}^{\mathrm{II}} \\ 0 & 0 & T_1 \cdot C_{3,124}^{\mathrm{II}} & T_3 \cdot C_{3,234}^{\mathrm{II}} \\ T_2 \cdot C_{1,123}^{\mathrm{I}} & T_4 \cdot C_{1,134}^{\mathrm{I}} & 0 & 0 \\ T_2 \cdot C_{3,123}^{\mathrm{I}} & T_4 \cdot C_{2,134}^{\mathrm{I}} & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta \kappa_2^{\mathrm{II}} \\ \Delta \kappa_4^{\mathrm{II}} \\ \Delta \kappa_1^{\mathrm{I}} \\ \Delta \kappa_3^{\mathrm{I}} \end{pmatrix}_{i-1}$$
(7)

or more compactly

$$\Delta \vec{\kappa}_i = \mathbf{K} \, \Delta \vec{\kappa}_{i-1}, \tag{8}$$

with  $(\Delta \kappa)_i = (\kappa)_i - (\kappa)_{i-1}$  and  $\kappa$  = any of the  $\kappa$ 's involved. In passing, we note that the recursively coupled short threebump scheme involves bump coefficients of all three types. Let the starting perturbation be an arbitrary vector  $\vec{\kappa}_0$  and  $\Delta \vec{\kappa}_0 := \vec{\kappa}_0$ , then  $\vec{\kappa}_n$  after *n* iterations is given by

$$\vec{\kappa}_n = \sum_{i=0}^n \Delta \vec{\kappa}_i = \sum_{i=0}^n \mathbf{K}^i \, \vec{\kappa}_0 \,. \tag{9}$$

In the limit  $n \to \infty$  rediscover the geometric series for matrices. If all eigenvalues  $\lambda_i$  of **K**, fulfill  $|\lambda_i| < 1$ , then [5]

$$\sum_{i=0}^{\infty} \mathbf{K}^i = (\mathbb{1} - \mathbf{K})^{-1}, \tag{10}$$

# We note that **K** is $O(\epsilon)$ and that we do not expect the bump coefficients to blow up unless the phase advances between steerers are close to degenerate. In fact for the switchyard in FLASH all $\lambda_i$ are real and their moduli are well below 1, in fact of the order of 0.2 rather.

### **MEASUREMENTS AT FLASH**

To demonstrate the practical feasibility of this concept, we have measured beam orbits at FLASH with and without correction for all three schemes and various initial perturbations. Fig. 4 shows as example the horizontal difference orbits in FLASH1 from the switchyard to the SASE undulator after changing a crosstalking steerer in FLASH2 by 0.6 mrad with (blue) and without (red) compensation using interleaved short three-bumps. Obviously the uncorrected difference orbit is up to 0.5 mm, while the corrected difference orbit outside the correction section is within  $2\sigma$  of zero. Note there were also small drifts due to external sources during the measurement. Furthermore, we demonstrated that



Figure 4: Measurement of the horizontal difference orbits FLASH1 downstream the switchyard after changing the kick of a crosstalking FLASH2 steerer with (blue) and without (red) bump compensation.

the SASE pulsenergy can be preserved in FLASH1 while steering the beam in the crosstalking region of FLASH2 with bump compensation active.

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