SPACE CHARGE INDUCED COLLECTIVE MODES AND BEAM HALO IN PERIODIC CHANNELS

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Abstract

The collective mode instabilities of periodically focused high intensity beams based on the Vlasov-Poisson equation are investigated both analytically and numerically. It is found that the broadened collective stop bands resulting from space charge induced structure resonance in long periodic channels predict well the areas where the rms emittance growth accompanied with n-fold phase space structure (beam halo) would take place. We believe that the formed beam halo, which is depicted in action-angle frame, could be understood as a side-effect of the collective beam mode.

COLLECTIVE BEAM INSTABILITY

As the only known self-consistent beam distribution profile in a certain periodic focusing channel, which decides the zero beam current phase advance, the ideal KV [1] beam is used to find the parameter region, where the space charge effect is taken into account, that the beam cannot tolerate any perturbation even from noise. From the analytical point of view [2, 3], the perturbed space charge potential is assumed as

$$V_n = \sum_{m=0}^{n} A_m(s)x^{n-m}y^m + \sum_{m=0}^{n-2} A_m(s)x^{n-m}y^m + ... \quad (1.1)$$

With appropriate boundary conditions, the integrals of the discontinuity of the surface electric field $I_{j;k,l}(s)$, caused by the perturbed space charge potential with strength $A_j(s)$ from period to period, could be constructed. The constructed $I_{j;k,l}(s)$ has to meet the condition,

$$\frac{1}{C_{k,l}(s)}\frac{d}{ds}\left[\frac{1}{C_{k,l}(s)}\frac{dI_{j;k,l}(s)}{ds}\right] + I_{j;k,l}(s) = -\frac{1}{C_{k,l}(s)}A_j(s)$$

$$(1.2)$$

where $C_{k,l}(s)$ is a function of the matched beam betatron oscillations without perturbation which are assumed stable. With higher order modes were included, more motions of $I_{j;k,l}(s)$ would be constructed. Denoting $\xi = (I, I')$, the Eqs. (1.3) could be equivalently transformed to $\xi'(s) = M(s)\xi(s)$ as the form of Matthews equation. The stability of the system is decided by the eigenvalues of map $M(L)$, where L is the period length [4]. Whenever the eigenvalues leave the unit circle, it represents perturbation could obtain sustain kick and the

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original beam profile will turn into a certain pattern. One application of this method is the study of the envelope instability [5~9]. The term "structure resonance" is also used as the same process in several other papers. In the following we do not distinguish the terminology "structure resonance" and "unstable collective mode".
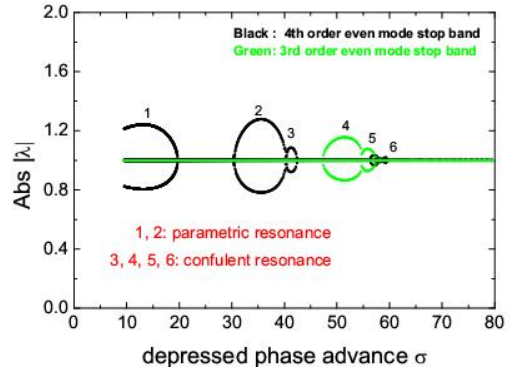

Figure 1: The stop 3rd order and 4th order stop bands in FD channel when the zero beam current phase advance is 80 degree.

As known, the 2nd order collective mode (envelope instability) will be depressed when zero beam phase advance σ_0 is chosen less than 90 degree. Figure 1 shows the 3rd and 4th order stop bands when $\sigma_0 = 80$ degree.

BEAM HALO FORMATION

The issue of beam halo formation has been studied for quite a long time since more attentions have been paid on the physics of high intensity beam and various experimental results obtained from different applications [10, 11]. Till now, the beam halo is still mostly defined qualitatively from the geometry characteristics of the beam profiles, where the terms "core" and "tail" are used [12]; or the inflection point of density function [13]. Numerical definition raised in Ref. [14] by using the information of 4th order moment, actually is still not widely accepted. From the theoretical point of view, for a long time, beam halo mechanism has been mostly regarded as the simple and non-self-consistent particle core resonance, accompanied with chaotic motion if beam is pushed into the space charge limit. Whereas, this single particle core resonance is limited, cannot explain the phenomenon, as seperatrix crossing, found in self-consistent simulation and experiment. However, it clearly shows interaction and physical map between the oscillation core and single particle. Through, the current beam halo definition is controversial; the mechanism of halo formation is not quite clear, different approaches are still

raised to decrease the deterioration from beam halo at variety applications theoretically and experimentally [15-17].

In this following, we mainly focus on the beam halo study in the simulation side. In the long focusing channel discussed, neglecting the motion in longitudinal, we show how the 4D actions J, which are supposed to be invariant, are modified by the collective mode; the information got from the 4D action-angle phase space includes the physics obtained from the normal projected 2D x-px phase space profile.

NUMERICAL SIMULATION

A number of 20K macroparticles with nearly rms matched initial KV beam is tracked in periodic channels with the TOPO code, which deals with the space charge self-consistently with classic Particle-In-Cell (PIC) method, to show the beam behaviour under the influence of collective mode. One aim is to verify the theory prediction from the ideal KV beam; another is to extend the availability and validity to the rms equivalent non-KV beams. The preliminary result shows that the stop bands agree well with the region where emittance growth takes place [3].
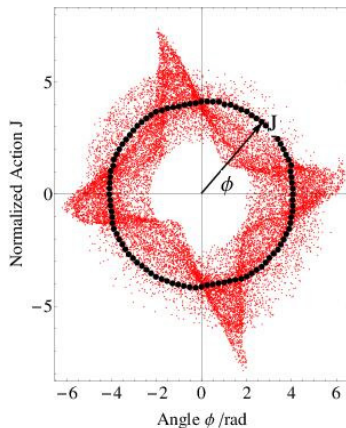

Figure 2: The action and angle in FD channel at period 18 with rms KV initial beam under the condition $\sigma_0 = 80$, $\sigma = 35$.

Figure 2 shows the beam profile in the action angle phase space at the 18th period for an ideal rms matched KV initial beam evolving in the FD channel, in which the initial beam is supposed to be located at the unit circle $J = 4$. The 4-fold phase space structure represents the influence of the 4th order collective mode. The projected 2D phase space as x-px shows similar pattern. No doubt, these particles manipulating the 4-fold phase space will turn into beam halo ultimately. In the case of non-uniform WB beam case, the result is similar to the ideal KV beam. However, because of the damping from the nonlinear density profile, beam will suffer less beam halo and emittance growth.

CONCLUSION AND DISCUSSION

When beam is evolving in a periodic linear focusing channel, the collective mode will play an important role to affect the beam behaviour. It will lead to the n-fold phase space structure which will turn into beam halo ultimately. The usual 2D phase space actually is just the projection of the 4D phase space in action-angle frame. Discussion on the transits phase space evolution, the saturations of the space charge nonlinearity and emittance growth, damping from the non-uniform density distribution could be found in further discussion [3].

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