DESIGN AND OPTIMISATION STRATEGIES OF NONLINEAR DYNAMICS FOR DIFFRACTION LIMITED SYNCHROTRON LIGHT SOURCES

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Abstract

This paper introduces the most recent achievements in the control of nonlinear dynamics in electron synchrotron light sources, with special attention to diffraction limited storage rings. Guidelines for the design and optimization of the magnetic lattice are reviewed and discussed.

INTRODUCTION

The last five years have witnessed a renaissance of storage ring based light sources, driven by the requirement of an ever decreasing electron beam emittance. Despite the concept of diffraction limited lattice based on Multi Bend Achromats (MBA) was already known since the seminal work of D. Einfeld in 1993 [1, 2], it was not until the funding on MAX IV in 2009 [3] that the light source community has significantly re-engaged with the concept of MBA and a more decisive reduction of the emittance.

An important cause underpinning this renaissance has been the growing confidence in the design and optimisation of very aggressive ultra-low emittance ring lattices. In this paper we present the main strategies for the design of such lattices, the theory and the tools used for their optimisation. We then provide a number of examples taken from the existing project highlighting the main trends in the field.

ULTRA-LOW EMITTANCE LATTICES

The equilibrium emittance in a storage ring is given by

$$\varepsilon_x = C_q \frac{\gamma^2}{J_x} \frac{\oint H(s)/\rho^3(s)ds}{\oint 1/\rho^2(s)ds} \sim C_q F \frac{\gamma^2\theta^3}{J_x} \propto \frac{\gamma^2}{N_b^3} \qquad (1)$$

where $C_q = 3.84\cdot10^{-13}$ m, γ is the relativistic factor of the electrons, J_x is the horizontal damping partition number' H(s) the dispersion invariant, $\rho(s)$ the bending radius of the dipole magnet. The second formula is valid in the limit of small bending angles θ and N_b is the number of bendings in the ring. For a synchrotron light source to be diffraction limited, the electron beam emittance ε_x should be lower than the photon beam emittance $\lambda/4\pi$. Therefore diffraction limited light sources in the hard X-rays (12 keV) require emittances in the 10 pm range. Formula (1) provides the basic guidance for the design of low emittance lattices showing clearly the benefits of using a large number N_b of bending magnets. It also provides theoretical minimum emittance conditions for the contribution of a single dipole in a cell or for a single dipole in a cell matched to achromatic conditions [4]. These two conditions form the building blocks for the construction of a large variety of lattices proposed for (quasi-)diffraction limited ring and upgrade of existing machines as reported in Table 1.

Table 1: (Quasi-)Diffraction Limited Rings and Upgrade Proposals, Main Parameters

	En. (GeV)	C (m)	ε_x (pm)	Natural ξ_h, ξ_v
MAX IV	3	528	330	-50;-50
ESRF-EBS	6	844	133	-97;-84
SIRIUS	3	518	280	-113;-80
APS-U	6	1104	65	-138;-108
ALS-II	2	197	50/50	-65;-68
Diamond II	3	562	120	-129;-94
Elettra U	2	260	280	-79;-47
SPRring-II	6	1320	150	-155;-142
SLS-II	2.4	288	132	-69;-34

ANALYTICAL TOOLS

The characterisation of the nonlinear beam dynamics in storage rings is historically based on the analysis of the Hamiltonian motion [5] or nonlinear one turn map formalism [6, 7]. The limitation on the dynamic aperture (DA) and momentum aperture (MA) comes from the presence of nonlinearities in the ring that excite nonlinear resonance driving terms.

Starting from the Hamiltonian of the betatron motion the effect of the nonlinear magnetic field is usually analysed by deriving analytic expressions for the resonance driving terms. The general form for the driving terms can be written as [8]

$$h_{jklmp}(s) = \oint V_{mn}(s')\beta_x^{(j+k)/2}(s')\beta_y^{(l+m)/2}(s')\delta^p e^{i[(j-k)\phi_x(s,s')+(l-m)\phi_y(s,s')]}ds'$$

where j, k, l, m, p are non-negative integers, $V_{mn}(s)$ describes the azimuthal distribution of the gradient of the multipole generating a potential with power $x^m y^n$, e.g. for a normal sextupole b_3 we have the terms $b_3(s)(x^3 - 3xy^2)$. The resonance driving term $h_{jklmp}(s)$ excites the resonance (m, n) where m = j – k, and n = l – m. The order of the resonance is N = |m|+|n| = j+k+l+m. Notice that h_{11001} and h_{00111} drive the horizontal and vertical chromaticities respectively. The driving terms are s-dependent complex coefficients which depend on powers of the optics functions and the phase advance around the ring. Each

ISBN 978-3-95450-147-2

magnet type will contribute to specific resonance driving terms and the formula for the $h_{jklmp}(s)$ allow us to get an insight as to what elements are building up large resonant driving terms and to devise strategies to compensate them. Usually the compensation works in first order of perturbation theory and should be checked with numerical tracking. A closer analysis of the expression for the driving terms reveals that sextupoles excite resonance of order 3, 1 in first order with the sextupole gradient, octupoles excite resonance of order 4, 2, and 0 (i.e. amplitude dependent tuneshift) in first order with the octupole gradient and so on. The formula is sufficient to show that in a cell with thin lens sextupoles, located at position with same β functions, the contribution of two sextupoles to all driving terms cancel if the phase advance between the two sextupoles is

$$\Delta\mu_x = (2n_x + 1)\pi;\ \Delta\mu_y = n_y\pi \quad (2)$$

If we want to cancel simultaneously also the contribution of skew sextupoles the phase advance should be

$$\Delta\mu_x = (2n_x + 1)\pi;\ \Delta\mu_y = (2n_y + 1)\pi \quad (3)$$

Higher order driving terms generated by sextupoles are linear combinations of products of first order driving terms, hence a perfect cancellation of first order driving term will entail a cancellation to all orders. However, it is often the case that the geometric constraints on the available layout prevent achieving the correct phase advance for cancellation. In such cases one can use interleaved sextupole families [9], but it is clear that the phase advance between a pair of sextupoles is generally disturbed by the interleaved sextupoles which introduce an amplitude dependent phase advance and spoil the cancellation. Furthermore, it should be noticed that this cancellation is based on a thin lens model for the multipolar errors and it breaks down for thick elements. The cancellation also breaks down when we consider the off-momentum dynamics, as the momentum dependent phase advance may be shifted and fail to satisfy the conditions for cancellation.

Another strategy often used to compensate specific resonance driving terms is to seek their cancellation with N repeated identical cells whose total phase advance, after N cells, is set to a multiple integer of 2π. In this way the compensation of the sextupoles is not obtained within the cell as done with (2) or (3) but by summing up N contributions to the driving terms, one per cell, so that the total sum is eventually zero. The driving terms are therefore cancelled at the end of the N cells.

The most general approach to correct the driving terms is to build a Jacobian matrix relating the driving terms we want to minimise to the sextupole families available and invert it with the Singular Value Decomposition (SVD) approach [8]. If the Jacobian matrix is square and well behaved the inversion is possible and the correction is trivial. If however the matrix is degenerate or non-square, the matrix inversion is difficult because of the presence of small singular values in the SVD decomposition and the correction might require very strong sextupoles. This degeneracy is usually an indication that the sextupole families do not sample the phase advance in a sufficiently dense way. The extension of this concept to octupoles and decapoles and the corresponding resonance driving terms is reported in [10]. In all cases the robustness of the compensation scheme has to be validated by considering the effects of magnet misalignment and field errors.

NUMERICAL TOOLS

A complementary approach to the optimisation of the linear and nonlinear beam dynamics is given by the use of numerical tools for the direct optimisation of the DA and the MA of the ring, responsible for the injection efficiency and the Touschek lifetime. The field has benefitted enormously from the increased computation capabilities provided by relatively large computer clusters. On the one hand, parallelised algorithms or distributed applications allow the efficient exploration of vast portions of multi-dimensional parameter spaces. On the other hand, the direct optimisation of the DA and MA, if not directly the injection efficiency and the Touschek lifetime, can now be used with reasonable machine time runs. These calculations are often done with full 6D tracking using realistic machine model including magnetic systematic and random errors, fringe fields, element misalignments, operating chromaticities, engineering apertures and insertion devices (IDs). This combination of high throughput and accurate computation of the beam dynamics has been a major step forward in the optimisation of the ultralow emittance rings.

Several methods have been proposed to explore the parameters space and to define the objectives of the optimisation. Systematic scan of quadrupoles and sextupoles (e.g. GLASS [11]) are feasible in rings with a limited number of magnet families. These become quickly impractical when the parameters space has five of more dimension. Genetic algorithms have instead proved to be very effective in sampling the parameter space and providing good solution for the nonlinear dynamics. The use of Multi-Objective Genetic Algorithms (MOGA) to the optimisation of storage ring light sources was pioneered by M. Borland, successfully improving the lifetime of APS using sextupoles [12, 13]. MOGA searches can be parallelised, they are numerically robust, not involving derivatives, and capable of finding global minima, giving the best trade-off between conflicting objectives in the so-called Pareto front. They are now commonly used to optimise both the linear and the nonlinear optics of ultra-low emittance rings.

The application of MOGA has taken different flavours: the objectives chosen were initially proxy for DA and MA, like tuneshift with amplitude, driving terms which could be computed much quicker than the full DA and MA. However, as is well known, quantities computed perturbatively do not necessarily guarantee a clear correlation with the DA and MA (see e.g. [14]), therefore they are nowadays replaced by the direct calculations of the DA and MA. Diffusion rates in frequency maps (FM) have been proposed as more accurate evaluators of the beam dynamics as they carry information on the regularity of motion, beside the extent of the DA [15]. Tracking should be performed in 6D for at least one synchrotron cycle to capture losses at injection due to a mismatch in the incoming beam to the RF bucket. Eventually the direct simulation of the injection process can be used [16].

The lattices are further evaluated with numerical tools such as FM [17], or spectral line analysis [18]. It is important to observe that these algorithms and the accurate computation of the beam dynamics on which they rely upon, have proven to be in very good agreement with the experimental data taken at several existing machines [19-21].

OPTIMISATION STRATEGIES

A good choice for the linear optics is a mandatory starting point for a successful optimisation of the nonlinear dynamic and it is generally accepted that the optimisation of the linear optics and nonlinear optics are interleaved processes.

The simplest task in the optimisation of the linear optics is the choice of the betatron working point in order to avoid destructive resonances. Tune scans often involving change of integer part of the tune are made to explore the best working point. Other relevant quantities are the minimum beta in the straight section and the overall maximum beta in the cell as they both conspire to build the natural chromaticity. The value of maximum dispersion in the cell defines the strength of the sextupole necessary for chromaticity correction. The linear optics can be further tuned to allow the cancellation of driving terms over one or more cells or via symmetry as discussed in the previous paragraph. More advanced strategies foresee the determination of key quantities of the linear optics that have specific effects on the nonlinear dynamics once the chromaticity is corrected (see HMBA optimisation below).

Once a good solution of the combined analysis of linear-nonlinear optics is found, we proceed with the further optimisation of the parameters strictly related to the nonlinear optics (i.e. sextupole gradients, lengths, correlated drifts, etc. ditto for octupoles and higher order multipoles if required) to control tuneshifts and resonance driving terms. The resonance cancellation schemes proposed in the previous section are rigorously valid only for thin element multipoles, while the sextupoles are thick and interleaved. Therefore, in practice there is room for further optimisation of the nonlinear dynamics, either by departing from the exact requirements for phase cancellation (2) and (3), or by adding more sextupoles families within the cell but also by assuming different families across two or more cells. Additional octupoles have proven to be useful in many cases.

The final step is the used of numerical algorithms. MOGA optimisations have been run on virtually all new projects, using sextupole families and targeting objectives related to DA and MA. Following the MOGA optimisation a Pareto-front describing the best trade-off between the objectives is generated and the solutions that best balance large DA and large MA can be chosen. It is however possible to conceive a simultaneous optimisation of the linear and nonlinear optics directly with MOGA by using quadrupole and sextupole families as parameters.

The robustness of the optimisation must be checked with respect to the presence of IDs and errors resulting from the mis-powering of elements, intrinsic systematic or random errors, misalignments. As stated, MOGA can be run directly on models containing IDs and errors therefore taking fully into account their effect as shown in the APS studies [22]. It is worth noticing that the latest lattice designs take fully into account the possibility of correcting the effect of these errors in the optics. This has important consequences on the tolerances defined for such errors. The suite of corrections strategies used in the optimisation goes well beyond the traditionally orbit and tunes correction, but includes first turn threading, dispersion free steering, coupling free steering [23], beam based alignment, LOCO [24].

EXAMPLES

In what follows we describe the application of the optimisation strategies just discussed to the lattice design proposed for new rings and upgrade of existing rings.

Multi-Bend-Achromat (MBA)

Conventional MBA cells [2, 4] are based on the use of the Theoretical Minimum Emittance (TME) cell, flanked by two end-bending cells that suppress the dispersion in the straight sections, also designed to reach the theoretical minimum contribution to the emittance. Normally the theoretical minimum values are not reached and we speak of detuned TME cells or TME-like cells.

Such lattices generate strong focussing and a small dispersion. The natural chromaticity builds up in the high gradient quadrupoles with large optics function and is notoriously difficult to correct since the small dispersion throughout the cell calls for very large sextupole gradients in interleaved families. The chromaticity correction is usually achieved by placing the chromatic sextupoles at the dispersion peaks. However large dispersion is in direct contrast to lowering the emittance and the resulting values for the dynamic aperture and the local momentum aperture are small. For these reasons lattice designers have generally stayed reasonable far from achieving the TME conditions in order achieve optics with reduced natural chromaticity that allow a satisfactory corrections on the nonlinear dynamics.

This is the baseline design strategy of the MAX IV cell (Fig. 1). The control of the nonlinear dynamics is based on a combination of analytical and numerical tools to correct the driving terms of nonlinear resonances and validation with numerical tracking supported by Frequency Map analysis (FMA). The use of small octupoles to control the detuning with amplitude proved to be beneficial [25].

ISBN 978-3-95450-147-2

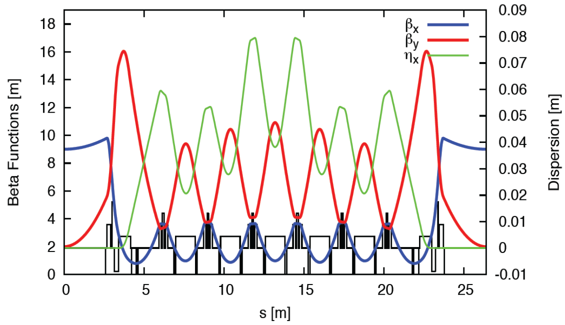

Figure 1: MAX IV 7BA cell [25].

A similar cell was devised for the design of the SIRIUS ring [26] by using a 5BA with a more aggressive lattice tuning that delivers an emittance of 280 pm, smaller than MAX IV, despite a similar circumference. Again the chromaticity sextupoles are evenly distributed along the cell. Similarly the correction of the nonlinearities was left to a suite of tracking based tools and MOGA. An example taken from the SIRIUS lattice is given in Fig. 2 where DA and Touschek lifetime where optimised using a set of 14 sextupole families [27]. Plots like the one shown in Fig. 2 have become common among lattice designers.

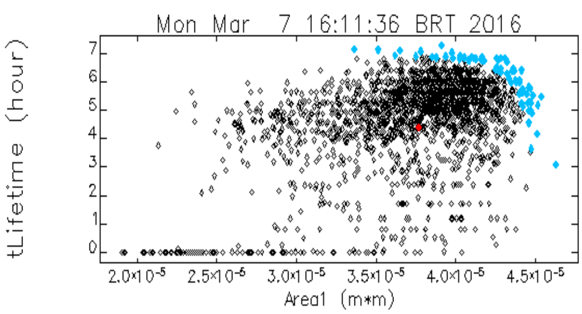

Figure 2: Pareto front (blue dots) for DA and Touschek Lifetime for the SIRIUS 5BA lattice, the red dot is the start of the optimisation.

A noticeable example of lattice design based on TME-like cells is given by Pep-X lattice [28] delivering an emittance of 29 pm reduced to 10 pm with damping wiggler, i.e. a truly diffraction limited ring at 1Å wavelength. The correction of the nonlinear resonances is based on the tailoring of the phase advance between the cells so that a large number of driving terms are compensated after one arc (made of 8 cells). The dynamic aperture at injection is also enlarged by designing a special optics with large β_x in the injection straight. Although the lattice symmetry is broken the compensation of the driving term is still effective in generating sufficient dynamic aperture and momentum aperture. The scheme has successfully corrected geometric resonance up to fourth order resonance. The cancellation of the driving terms after 8 cells is shown in Fig. 3 [28].

Many other projects have been proposed along the same concept [29]. A radical solution to the problem of the small DA consists in giving up completely the off-axis injection for multi-turn accumulation, proposing new injection schemes based on swap-out on-axis injection. In this way the dynamic aperture required is much reduced and the lattice design can be pushed further. On-axis injection requires a DA of few beam sigma, significantly less than what needed in off-axis injection, and the additional complexity of the accumulator and injection system appears to be manageable [30]. Swap-out injection based lattices will still require good lifetime: despite MBA lattices have a small dispersion invariant and hence the excitation amplitude of Touschek scattered particle is reduced, the MA should still be optimised to reach few %. Along these lines, the ALS upgrade [31] is based on a 9BA lattice delivering 100 pm emittance.

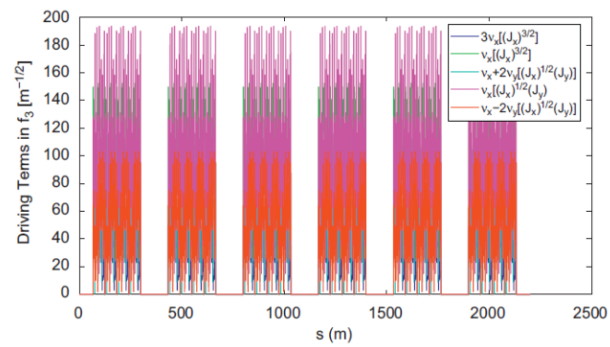

Figure 3: Cancellation of sextupole resonances after 8 cells at Pep-X.

Hybrid MBA

A variant of the MBA concept was developed for the 7BA cell at the ESRF [32] shown in Fig. 4. The so-called Hybrid MBA cell has two separate dispersion bumps used to ease the chromaticity correction. The dispersion is allowed to grow between dipole one and two and symmetrically between dipole six and seven. Chromatic sextupoles are placed in the dispersion bump and the phase advance between them is tailored to 3π in the horizontal plane and π in the vertical plane. As done at MAX IV, additional octupoles are used to further control the detuning with amplitude. By setting the phase advance between the pairs to an odd multiple of π, most of the driving terms generated by the sextupoles are cancelled. The hybrid cell also uses longitudinal bending gradient to further minimise the emittance. The optimisation of the longitudinal profile of the magnetic field shows that the high field region is concentrated in the region where dispersion is small, i.e. at the end of the dipole and is reported in [33]. The hybrid-MBA cell has been preferred to the MBA cell also by the APS upgrade [22, 34]. The large number of cells (40 compared to the 32 cells of ESRF) allows reaching smaller emittance. Furthermore the present lattice has been designed with the assumption that the injection will be based on a swap-out scheme [34] and therefore the requirements on the on-momentum dynamic aperture are somewhat relaxed. The present baseline design has 67 pm and similar route has been followed in the first studies of the Chinese High Energy Photon Source (HEPS) [35].

ISBN 978-3-95450-147-2

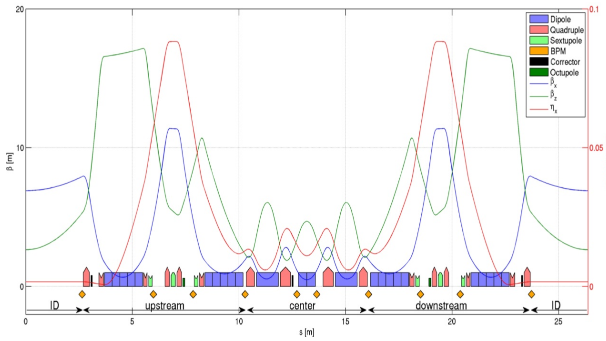

Figure 4: The ESRF cell [32].

In the optimisation of the Hybrid MBA cell, a number of other parameters of the linear optics are found to have a noticeable effect on the nonlinear optics and therefore are included in the combined linear/nonlinear optics optimisation. These are the values of the optics functions (β, α, D_x) at the high dispersion peak, i.e. optics functions at the chromatic sextupoles and the value of β_y at the centre of the cell which helps controlling the overall emittance. Specific matrix elements between the chromatic sextupoles help controlling the detuning with amplitude in the different planes (e.g. M_{34} for dQ_x/dJ_y) and the cross detuning with amplitude (M_{12} for dQ_x/dJ_y). While these interdependencies can be extracted rigorously from the analysis of the corresponding terms in the Hamiltonian, it is often the case that the most critical quantities are selected from a systematic empirical analysis of their contribution to the nonlinear dynamics once the chromaticity correction is fixed. In this sense the ability of the lattice designer is still a crucial element in the success of the nonlinear optimisation without which the computing power and tools offered by MOGA type of optimisation are unlikely to succeed.

In most cases the optics parameter are used directly as parameters for the optimisation while the objectives are DA and MA or proxies. In this way it is assumed that the optimisation process is capable of matching exactly the linear optics in a way that the optics parameter chosen are effectively achievable. A sufficient number of knobs (e.g. quadrupole gradients, lengths, correlated drifts, etc) must be provided to match the number of optics constraints indicated. This is the case, e.g. at the ESRF where nine quadrupole in the half cell are used to match nine optics constraints [36]. The only caveat is that the chromaticity is corrected each time with the two families of chromatic sextupole to the nominal operating values.

The DDBA/DTBA Concepts

Another design parameter that is often quoted as quality factor for light sources is the ratio between circumference and the length of the straight section available for IDs in the lattice. In operating rings, SOLEIL's DBA lattice has the higher ratio exceeding 40% of the ring available for straight sections.

In the investigation of the Diamond lattice it proved possible to show that a 4BA cell could be modified by adding a straight section in the middle of the cell and hence effectively turning it into a double-double bend achromat (DDBA) [37]. Such lattice still maintains a very low emittance and the optics function can be tailored to minimise beta and dispersion function in the additional straight section.

This concept has been recently merged with the hybrid MBA cell in an attempt to combine the best of both cell designs. By removing the central gradient dipole the 7BA cell has been transformed in two mirror symmetric cells with three dipoles each, renamed Double Triple Bend Achromat (DTBA), as shown in Fig. 5 [38]. The optimisation of the beam dynamics follows the line of the Hybrid MBA cell.

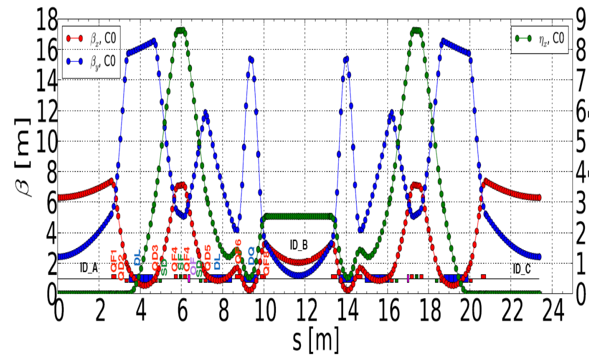

Figure 5: The DTBA cell.

Reverse Bend Lattices

Another interesting concept in low emittance lattice design, revamped in the SLS-II upgrade, is the use of reverse bends. The basic cell is designed to approach the TME condition but reverse bends are introduced flanking the main dipole in the TME cell. Reverse bends provide a greater flexibility in the optics to disentangle the β_x function from the dispersion. In the SLS-II design the reverse bends angle is 10% of the main bends and they are built as gradient dipole providing additional horizontal focussing. The gradient increases the damping partition number J_x in (1) hence minimises further the emittance. The procedure for the optimisation of the nonlinear optics is then similar to the one followed in the conventional MBA [39]. Nine sextupole families are used to suppress sextupole resonances up to second order, octupole resonances up to first order and chromatic terms up to third order. MOGA is then used to optimise simultaneously the DA on-momentum and at ±3% [40].

ACKNOWLEDGEMENTS

The author acknowledges the contribution of R.P. Walker and P. Raimondi to this work. Material and discussions with A. Alekou, M. Apollonio, M. Borland, N. Carmignani, D. Einfeld, W. Guo, R. Hettel, E. Karantzoulis, S. Leemann, L. Liu, S. Liuzzo, I. Martin, Y. Papaphilippou, T. Pulampong, V. Sajaev, C. Steier, A. Streun, C. Sun and M. Venturini are also acknowledged. This work is supported by the EuCARD-2 network cofunded by the partners and the European Commission under Capacities 7th Framework Programme, Grant Agreement No. 312453.

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