

BETA FUNCTION MEASUREMENT AND RESONANCES INDUCED BY SPACE CHARGE FORCE AND LATTICE MAGNETS

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Abstract

J-PARC MR has been operated at tune $(\nu_x, \nu_y) = (22.40, 20.75)$. A new operating point around $(21.4, 21.4)$ has been proposed by simulation studies on space charge effect since 2013. Machine experiments at the operating point has been performed since 2014 and many encouraging results are being obtained. In either operating point, non-structure 3rd order resonances seem an important key for the beam loss. We study space charge and lattice induced resonances in this paper.

INTRODUCTION

Particles move with experience of electro-magnetic field of lattice elements and space charge. We study slow emittance growth arising in a high intensity circular proton ring. Typical mechanism of emittance growth is resonance and chaotic diffusion due to nonlinear force of magnets and space charge.

RESONANCES AND TUNE SPREAD DUE TO SPACE CHARGE FORCE

Emittance growth/hallo formation is characterized by resonance structure. Resonance island appears in phase space. Its modulation due to synchrotron oscillation induces emittance growth. The resonance width [1] is measure of the growth rate.

$$\Delta J_x = 4m_x \sqrt{\frac{U_m}{\Lambda}}. \quad (1)$$

where Λ is determined by nonlinear force/potential,

$$\Lambda \equiv m_x^2 \frac{\partial^2 U_{00}}{\partial J_x^2} + m_x m_y \frac{\partial^2 U_{00}}{\partial J_x \partial J_y} + m_y^2 \frac{\partial^2 U_{00}}{\partial J_y^2}. \quad (2)$$

U_{00} is d.c. component of betatron phase, ϕ_x, ϕ_y .

We first discuss the space charge potential [2]. Beam distribution is assumed to be Gaussian in transverse determined by emittance and Twiss parameters. U contains linear component, which gives a tune shift and Twiss parameter distortion.

Resonance terms, which are Fourier components for betatron phases, are expressed by

$$U_{m_x, m_y}(J_x, J_y) = -\frac{\lambda_p r_p}{\beta^2 \gamma^3} \oint ds \int_0^\infty \frac{du}{\sqrt{2\sigma_x^2 + u} \sqrt{2\sigma_y^2 + u}} \left[\delta_{m_x 0} \delta_{m_y 0} - \exp(-w_{x\eta} - w_y) \sum_{l=-\infty}^{\infty} (-1)^{(m_x + l + m_y)/2} I_{(m_x - l)/2}(w_x) I_l(\nu_x) I_{m_y/2}(w_y) e^{-im_x \phi_x - im_y \phi_y} \right]. \quad (3)$$

where $m_x + l$ and m_y are even numbers due to symmetric feature of space charge potential. Here variables are defined as

$$w_x = \frac{\beta_x J_x}{2\sigma_x^2 + u}, \quad w_{x\eta} = \frac{\beta_x J_x + \eta^2 \delta^2}{2\sigma_x^2 + u}. \quad (4)$$

$$\nu_x = \frac{2\sqrt{2\beta_x J_x \eta \delta}}{2\sigma_x^2 + u}, \quad w_y = \frac{\beta_y J_y}{2\sigma_y^2 + u}. \quad (5)$$

The tune slope $\partial^2 U_{00} / \partial J_x^2$ induced by space charge potential is evaluated by $U_{00}(J_x, J_y)$ in Eq.(3).

$$U_{00}(J_x, J_y) = -\frac{\lambda_p r_p}{\beta^2 \gamma^3} \oint ds \int_0^\infty \frac{dt}{\sqrt{2+t} \sqrt{2r_{yx}+t}} \left[1 - e^{-w_{x\eta} - w_y} \sum_{l=-\infty}^{\infty} (-1)^l I_{l/2}(w_x) I_l(\nu_x) I_0(w_y) \right]. \quad (6)$$

where $t = u/\sigma_x^2$ and $r_{yx} = \sigma_y^2/\sigma_x^2$ and

The tune shift for on energy particle ($\delta = 0$) is given by derivative of U_{00} for J_x as follows,

$$2\pi \Delta \nu_x = \frac{\partial U_{00}}{\partial J_x} = -\frac{\lambda_p r_p}{\beta^2 \gamma^3} \oint ds \frac{\beta_x}{\sigma_x^2} \int_0^\infty \frac{e^{-w_x - w_y} dt}{(2+t)^{3/2} (2r_{yx}+t)^{1/2}} \left[(I_0(w_x) - I_1(w_x)) I_0(w_y) \right], \quad (7)$$

The tune slope for $\delta = 0$ is given by second derivative of U_{00} as follows,

$$\frac{\partial^2 U_{00}}{\partial J_x^2} = 2\pi \frac{\partial \nu_x}{\partial J_x} = -\frac{\lambda_p r_p}{\beta^2 \gamma^3} \oint ds \frac{\beta_x^2}{\sigma_x^4} \int_0^\infty \frac{e^{-w_x - w_y} dt}{(2+t)^{5/2} (2r_{yx}+t)^{1/2}} \left[\left\{ \frac{3}{2} I_0(w_x) - 2I_1(w_x) + \frac{1}{2} I_2(w_x) \right\} I_0(w_y) \right], \quad (8)$$

Similar formulae for $2\pi \Delta \nu_y$, $\partial^2 U_{00} / \partial J_x \partial J_y$, $\partial^2 U_{00} / \partial J_y^2$ are obtained [2].

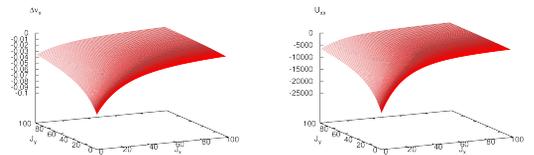


Figure 1: Tune spread $(\Delta \nu_{x,y}(J_x, J_y))$ due to space charge force.

J-PARC MR is designed with super-periodicity 3. Resonances except for structure ones, $m_x v_x + m_y v_y = 3n$, are suppressed. Transfer map of each $1/3$ is expressed by

$$\mathcal{M}^{(i)} = \exp \left(-H_{00}^{(i)} - U_{00}^{(i)} - H_m^{(i)} - U_m^{(i)} \right) \quad i = 1, 2, 3 \quad (9)$$

where i denotes i -th super-period. H_{00} and U_{00} , which are expressed only by J , characterize tune and its spread. H and U are for lattice and space charge, respectively. H_m and U_m are resonance driving term. U_m in each super-period. When the super-periodicity is perfect, three transfer map $\mathcal{M}^{(i)}$ are identical, and non-structure resonances are cancelled, because each resonance term (complex) is taken summation with multiplying terms with shifted phase factor $\phi_{xy} \rightarrow \phi_{xy} + v_{xy}/3$. Figure 2 shows space charge induced resonance driving term, U_{301} and U_{40} given by Eq.(3). The driving terms perfectly agree each other.

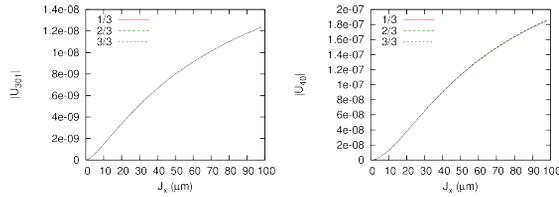


Figure 2: U_{301} and U_{40} for space charge force in the design lattice.

EVALUATION OF RESONANCES USING MEASURED TWISS PARAMETERS

The resonances appear from errors of lattice. Nonlinearity is integrated with beta function and phase φ as shown in Eq.(3). The beta function and phase are measurable quantities. The resonance driving terms are evaluated using measured beta and phase. Figure 3 shows an example of measured beta function and phase. Beam position, in which betatron oscillation is excited by a kicker, is measured by turn-by-turn monitor. Twiss parameters are determined by reconstruction of phase space motion. The phase jump, seen in Fig.3 are mainly from x-y coupled motion, where tune is (21.38, 21.43).

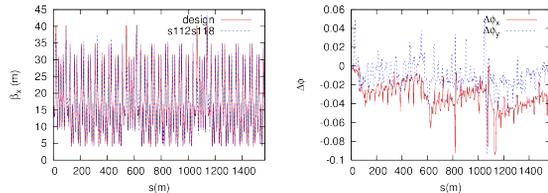


Figure 3: Measured beta function and phase for shot 112 (x) and 118 (y).

Space Charge Induced Resonances

Space charge induced resonance driving terms are evaluated by the measured beta and phase in Eq.(3). Figure 4

shows U_{301} and U_{40} . 3 lines are slightly different each other in U_{40} . $2/3$ is somewhat larger than others, $1/3$ and $3/3$, for U_{301} . Tune shift and slope are given in Fig.1. The resonance width is $\Delta J_x \sim \sqrt{10^{-9}/2} \times 10^4 \approx 0.22 \mu\text{m}$ for U_{301} . The width is small compared with the injected beam emittance $\varepsilon \approx 20 \mu\text{m}$. For the measured optics errors in beta and phase, space charge induced resonances are very weak.

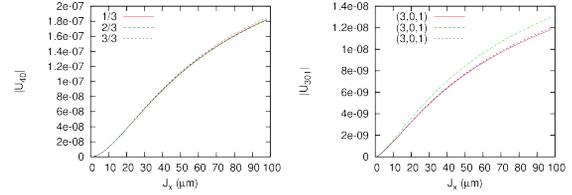


Figure 4: U_{301} and U_{40} for space charge force given by measured beta function and phase.

Lattice Magnet Induced Resonances

Main nonlinear magnet in J-PARC MR is sextupole. The sextupole magnets are located with keeping the super-periodicity, therefore resonance driving term H_m should be cancelled except for structure resonance. Driving term of 3rd order resonance is evaluated by

$$H_{30} = \frac{J_x^{3/2}}{6} \int \beta_x^{3/2} K_2(s) e^{-3i\phi_x} ds \quad (10)$$

Figure 5 shows real and imaginary parts of H_{30} in the design lattice. The resonance term is identical in every super-periodicity, therefore the driving term is cancelled by betatron phase rotation, $\phi_{xy} \rightarrow \phi_{xy} + v_{xy}/3$. J-PARC lattice is also designed that the resonance term is cancelled in each super-periodicity.

Figure 6 shows resonance term using measured beta and phase. Tune shift and slope are mainly caused by space charge force, U_{00} , in Fig.1. The resonance width for the 3rd order lattice resonance under the space charge tune spread is $\Delta J_x \sim \sqrt{50 \times (10^{-5})^{3/2} / (6 \times 2 \times 10^4)} \approx 3.6 \mu\text{m}$. This width is 1/10 of the emittance. The resonance can be source of emittance growth.

The measured betatron phase fluctuates around 0.02. The integral for space charge is performed continuously, thus detailed fluctuation of the phase are smeared out. While sextupole is localized, thus the fluctuation may be emphasized on the resonance terms.

Elaboration of beta and phase measurement is necessary. Anyway, we can say that space charge induced resonance is weak compared with lattice magnet induced ones.

DIRECT SIMULATION USING MEASURED TWISS PARAMETERS

A code SCTR supports space charge simulation using measured beta and phase [3]. SCTR also supports frozen space charge potential to study Hamiltonian motion under the space charge force. Figure 7 (a) and (b) shows phase

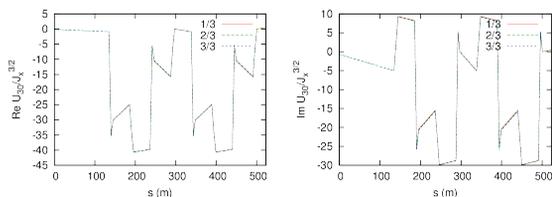


Figure 5: Real and imaginary parts of H_{30} due to sextupole magnets given by design beta function and phase.

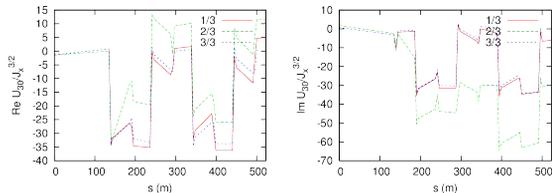


Figure 6: Real and imaginary parts of H_{30} due to sextupole magnets given by measured beta function and phase.

space trajectory in a frozen potential for design and measured lattice, respectively. Beam is parabolic distribution with maximum emittance 40 mm.mrad, and the population is $N_p = 2.5 \times 10^{13}$. Beam particles with the amplitude $2J_x > 40 - 50$ mm.mrad are lost for the measured lattice. Beam loss simulation with self-consistent beam distribution (no frozen) is performed. Figure 8 shows beam loss for the design lattice and that with measured beta/phase. Beam loss is very large for measured lattice as is guessed in Fig.7

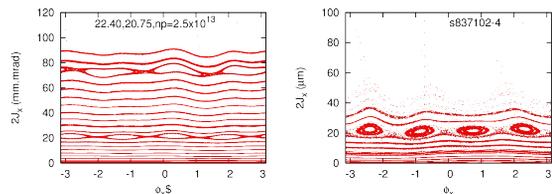


Figure 7: Phase space trajectory in a frozen potential with parabolic distribution with $N_p = 2.5 \times 10^{13}$. Left and right plots are drawn for the design lattice and that with measured beta/phase, respectively.

SUMMARY

Resonances induced by space charge are evaluated by integrals of Fourier component of space charge force with considering beta function and phase. Resonances induced by lattice magnets are also integrals of nonlinear magnets with considering beta function and phase.

The beta and phase is observable quantity. Measured beta and phase are used for the integral. We evaluated fourth and their order resonances.

J-PARC MR has 3 super-periodicity. One turn map is not sufficient to analyze the resonance behavior. We calculate resonance terms in each super-period. Breaking of the super-periodicity: that is, difference of resonance terms in each

super-period is index of the resonance strength. The reso-

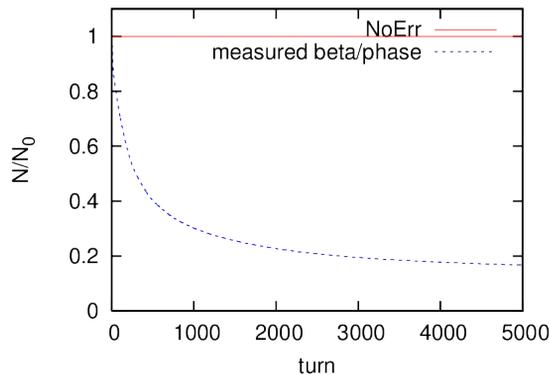


Figure 8: Beam loss for the design lattice and that with measured beta/phase.

nance width is evaluated by the resonance strength combined with tune slope induced by space charge force.

Space charge induced resonances are good symmetry. Nonstructural resonances due to space charge force, fourth and third order resonances are cancelled almost. Residual effects induces resonances with the width of 0.22 μm , which is negligible small compare with the emittance 20 - 40 μm of injected beam. Space charge force is integrated along the whole ring smoothly; therefore the resonances are similar strengths in each 1/3 rings.

Lattice induced resonances are integrated in each sextupole position. The resonance term is not cancelled well now. Especially, 3-rd order resonance term at $s = 522 + 140$ m breaks symmetry. Residual resonance terms give the width of 3.6 μm which is 10% of the emittance 20-40 μm . The resonance should give strong effect with couple to the synchrotron motion.

Space charge simulation based on the measured beta/phase. Strong third order resonance was seen in phase space plot using measured beta/phase. Beam loss simulation also showed strong effect: beamloss.

The betatron phase seems wrong values, when x-y coupling in data is strong. We need elaboration for the jump seen in $s = 522 + 140$ m. Sophisticated date analysis should be done [4].

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