ANALYSIS OF TRANSVERSE FIELD INDUCED BY CURVED BERYLLIUM WINDOW IN MUON IONIZATION COOLING CAVITY*

T. Luo†, D. Li, LBNL, Berkeley, CA 94720, USA

Abstract

The beryllium windows are used in the vacuum RF cavity for muon ionization cooling to increase the cavity shunt impedance. The windows are curved for predictable thermal deformation and RF frequency detuning. This curvature also introduces transverse field, which can affect the transverse beam emittance. In this paper, we will analyze this transverse field and evaluate its effect on the transverse emittance.

INTRODUCTION

Recently, interest has increased in the possibility of using muon in high-energy physics as the colliding particles in μ^+-μ^- colliders [1]. The liability of muons is that they are created in a diffuse phase-space. As a result, the volume of the 6-Dimensional (6D) phase-space must be rapidly reduced via ionization cooling [2] by several orders of magnitude in order to be able to further accelerate it.

To reduce the transverse emittance, the beam is strongly focused with high magnetic fields and subsequently sent through an absorber material to reduce the overall momentum. The beam regains longitudinal momentum in RF cavities, resulting in an overall loss in transverse emittance. Longitudinal emittance reduction is achieved by shaping the absorbers into wedges and providing a bending magnetic field, generating a dispersion such that particles with higher energy are sent through more material.

Thin curved Be windows have been proposed in the cooling channel to increase shunt impedance of the closed-cell rf cavities. In this paper, we will analyze the transverse field in the RF cavity due to the Be window curvature and evaluate its effect on the transverse emittance.

CURVED BE WINDOW

In the current muon collider design, the muons are generated by the pion decay from proton spallation. Due to the short life time of muon and large initial emittance, how to reduce the beam emittance within its lifetime to meet the luminosity requirement is one of the challenges in the muon collider design. So far the only practical way is the ionization cooling scheme. In this scheme, the muon beam loses energy by ionization interaction in the absorber and regains the longitudinal momentum in the RF cavity. The faster the RF cavity can recover the longitudinal energy lose, the quicker the cooling rate, which is critical due to muon's short life time.To increase the cavity acceleration efficiency, two beryllium windows which are almost transparent to the muon beam are placed on the cavity side walls. This can increase the cavity shunt impedance by a factor of two.

To control the muon scattering and emittance dilution in the Be window, the window itself must be thin, within a few millimeters at most. With the RF thermal heating, the thin Be window will easily undergo thermal deformation and detune the cavity frequency. If the window is flat and relaxed, it can bulge in either direction, thus the frequency detuning is unpredictable and hard to control. One proposed method is to pre-stretch the Be window to prevent thermal deformation. However this method turns out to be quite technically challenging. Another method, which has been applied to the RF cavity in the Muon Ionization Cooling Experiment (MICE) [3–5], is to curve the Be window to make the direction of the thermal expansion predictable. The MICE Be window is shown in Figure 1. By curving two windows in the same direction, they will bulge in the same direction under thermal deformation, thus partially cancels the frequency detuning induced by each window.

Figure 1: Beryllium window for MICE cavity, with 0.38 mm thickness, 42 cm diameter and about 18 mm depth of the bow.

The thermal stress, deformation and frequency detuning by the curved Be window have been investigated in 201 MHz MICE RF cavity [6] and 805 MHz rectilinear cooling channel RF cavity [7]. Besides the mechanical effects, the curved window also alters the RF field in the cavity and introduce the transverse kicking field, which is the focus of this paper.

TRANSVERSE FIELD IN MICE RF CAVITY

The MICE RF cavity with Be windows, as shown in Figure 2, is the only kind of cavity with such windows that has been manufactured and commissioned. Two such cavities are to be placed in the MICE cooling channel, where the muon beam emittance will be studied thoroughly. The electric and magnetic field in the cavity are solved by Omega3P

* Work supported by the Office of Science, U.S. Department of Energy under DOE contract number DE-AC02-05CH11231
† tluo@lbl.gov

ISBN 978-3-95450-147-2

[8] for the model of a quarter of cavity, as shown in Figure 3 and 4.

Figure 2: Installed Be window on MICE cavity (left) and Be window installation (right).

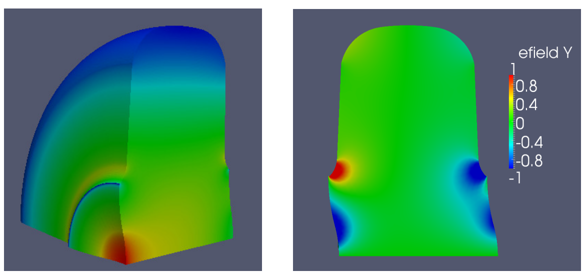

Figure 3: Electric field solved by Omega3P: left the E field magnitude, right the E field transverse component.

.

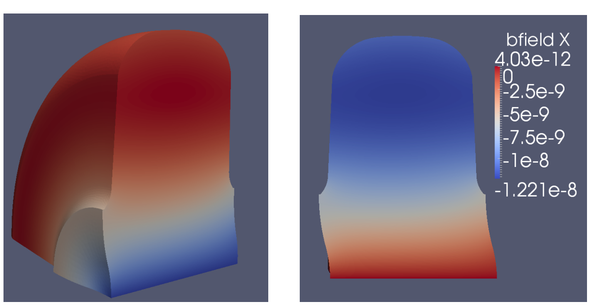

Figure 4: Magnetic field solved by Omega3P: left the B field magnitude, right the B field azimuthal component.

.

From the EM field solution, we can calculate the transverse kick at different radial positions. Since we are most interested in the beam with the maximum acceleration, we first determine the corresponding RF phase ϕ_0, which gives the maximum V_z:

$$V_z = \int E_z(z)dz = \int E_z(\beta ct)\cos(\omega t + \phi)\beta c dt,$$

when $\phi = \phi_0$.

Then we can calculate the transverse momentum kick ΔP with ϕ_0 for these maximum-boosted muon:

$$\Delta P = \int (F_E + F_B)dt,$$

where

$$F_E = e \cdot E_r = e \cdot E_r(\beta ct)\cos(\omega t + \phi_0),$$

and

$$F_B = e \cdot \beta c \cdot B_\theta = e \cdot \beta c \cdot B_\theta(\beta ct)\cos(\omega t + \phi_0 - \pi/2),$$

, with β the relativity factor and $-\pi/2$ from the phase lag between E and B field. The calculation results for MICE cavity are shown in Figure 5.

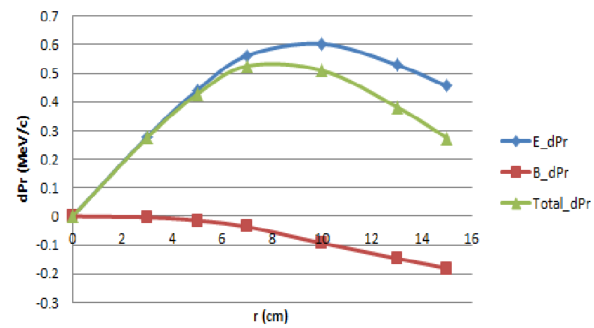

Figure 5: The transverse kick on a 200 MeV/c muon beam due to E field, B field and the total kick, normalized to 10 MV longitudinal acceleration.

From these results we can see that:

1. The E and B field kick the beam in the opposite directions. For an ideal pillbox cavity with the RF phase for maximum acceleration, E and B kicks have the same magnitude but opposite directions, thus cancel each other. Due to the curved windows facing the same direction, the symmetry is broken and these two fields don't cancel each other anymore, resulting a net transverse kick.

2. For 10 MV longitudinal acceleration, the corresponding transverse momentum can be as high as 0.5 MeV/c, which is significant compared with the magnitude of transverse emittance cooling of MICE [9]. Thus one need to include the Be window curvature in the RF field calculation and apply the accurate field map for the muon particle tracking.

3. Within $r < 5$ cm, the transverse kicker $\Delta P \propto r$, which provides a linear focusing/defocusing effect like a quadrupole, but uniform in all the transverse radian directions.

4. There are two cavities in the MICE cooling channel, and their windows can face the same or opposite directions, thus enhancing or canceling each other's focusing/defocusing effect.

5. For the beam with RF phase other than ϕ_0, the focusing/defocusing effect varies. For example, for the beam with the RF phase $\phi_0 + \pi$, the focusing/defocusing is completely opposite to the case of ϕ_0.

6. Notice that in this calculation, we have assumed the beam doesn't move transversely when passing through the cavity. This may not be the case for certain cooling channels, especially where the strong focusing magnetic field is not parallel to the beam axis. For more accurate calculation one should track the beam in the RF cavity with magnetic field background.

7. For future such cavities, if one prefers to reduce the transverse kick, the two Be windows shall face the opposite directions to preserve the mirror symmetery in the beam direction. The frequency detuning will be the sum from each cavity but still be predictable.

The results are double checked with Panofsky-Wenzel (PW) theorem:

$$P_{\perp} = -\frac{ie}{\omega}\frac{d}{dr}\int E_z dz,$$

which only requires the E field map. The i factor indicates the $\pi/2$ phase difference between the E_z integration and the transverse kick $P_{\perp}$. The calculation by PW theorem is consistent with the direct fields integration.

CONCLUSION

In this paper, we studied the transverse field induced by the curvature of the Be window in the vacuum RF cavity for muon ionization cooling. For MICE cavity, the transverse field may considerably affect the muon transverse momentum and contribute to the emittance cooling. The window curvature should be taken into the RF field calculation and muon particle tracking. Within small r range, the window curvature can introduce a linear focusing or defocusing effect, depending on the curvature direction and the beam RF phase.

ACKNOWLEDGMENT

This work is supported by the Office of Science, U.S. Department of Energy under DOE contract number DE-AC02-05CH11231.

REFERENCES

[1] C. M. Ankenbrandt et al., Phys. Rev. ST Accel. Beams 2, 081001 (1999).

[2] D. Neuffer, Part. Accel. 14, 75 (1983).

[3] M. Bogomilov et al., (MICE Coll.), JINST 7, P05009 (2012).

[4] The MICE collaboration, "Characterisation of the muon beams for MICE", Euro. Phys. J. C:, 73, pp1-14, (2013).

[5] The MICE collaboration, "Pion contamination in the MICE muon beam", J. Inst., 11 P03001 (2016).

[6] T. Luo et al., "Progress on the MICE 201 MHz cavities at LBNL", IPAC2015, WEPTY046.

[7] T. Luo et al., "Thermal and Lorentz force analysis of beryllium windows for a rectilinear muon cooling channel", IPAC2015, WEPTY047.

[8] K. Ko, et al., "Advances in Parallel Electromagnetic Codes for Accelerator Science AND Development", LINAC2010, Tsukuba, Japan, 2010.

[9] V. Blackmore et al., " MICE Ionization Cooling Demonstration", MICE tech notes 452.

ISBN 978-3-95450-147-2