# BEAM POSITION MONITOR DESIGN FOR DIELECTRIC WAKEFIELD ACCELERATOR IN THz RANGE 

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## Abstract

Dielectric based collinear wakefield accelerator have been broadly selected for the THz accelerator due to its simplicity. In order to move the THz accelerators from the current exploratory research into the practical phase, certain common accelerator components are indispensable. Beam Position Monitor (BPM) is one of them. However, most of conventional BPM techniques are hardly scaled down to THz regime. Here we propose a BPM design which uses the dominant dipole mode excited in the dielectric wakefield accelerators to extract information of the beam position.

## INTRODUCTION

Beam-driven wakefield accelerators are promising candidates for TeV class high energy colliders and for other applications like providing beam for compact FELs [1-2]. A very high accelerating gradient (GV/m level) can be achieved in wakefield accelerators [3]. To take advantage of the high gradient, wakefield accelerators are generally required to operate in THz regime $(>0.1 \mathrm{THz})$. Dielectric-based THz collinear wakefield accelerators (DWA) are preferred because of its simplicity. In order to realize the wakefield acceleration in a practical matter, one inevitable issue is the need to control Beam Breakup ( BBU ) which ensures the drive beam to be able to propagate until it has exhausted most of its energy. It has been shown in $[2,4]$ that single bunch BBU limits the distance over which the drive bunch can propagate in the dielectric lined waveguide before it begins losing electrons into the walls of the structure. A practical design of a DWA has been proposed (extended over 20 m and capable to $\sim 80 \%$ energy extraction efficiency) in which BBU is controlled by employing BNS damping [2]. However, in order to achieve this goal of BBU control, a very tight tolerance on a straightness of the beam trajectory is applied. The beam position monitor (BPM) is a must to track the beam trajectory in particle accelerators. The conventional BPM techniques like the stripline and button BPMs are not suitable for the THz accelerators which has the beam pipe in the order of a few millimetres. The cavity BPM can be used in principle by choosing a higher order dipole mode, however, it will take the extra space of beaimline which has already been tight resulting from the BBU control lattice. In addition, the fabrication precision of a cavity BPM in THz may be prohibitive. Instead of using an extra BPM device, in this paper we present a new method to detect beam position for collinear wakefield accelerators using the TE11 like mode in the beampipe which is naturally converted from

HE11 mode, the dominant dipole mode in a DWA structure.

## PRINCIPLE

In a DWA structure (Note, we only discuss the cylindrical DWA in this paper. The drive bunch traverses along the structure geometrical centre, it will only excite the TM01 mode which is the fundamental-monopole mode to accelerate the trailing bunch. The dipole mode HEM11 will be excited when the particle has an offset to the centre. In the Fig.1, we assume that the particle coordinate in the vacuum region is ( $\mathrm{r} 0, \theta 0$ ), which denotes the distance and azimuthal angle in the cylindrical coordinate.


Figure 1. Cross section view of the dielectric accelerating structure (left). BPM coupler model in CST Microwave Studio (right).

Based on the wakefield theory [5], particle will exchange the energy with the structure and radiate electromagnetic field. The radiation field can be decomposed as guided wave modes in the frequency domain. The peak gradient of each mode can be expressed as

$$
\begin{equation*}
E_{a}=\frac{q \omega}{2}\left(\frac{R}{Q}\right) \frac{1}{1-\beta_{g}} \tag{1}
\end{equation*}
$$

Where q is the charge, $\omega$ is the eigen-mode angular frequency, R over Q is the normalized shunt impedance along the particle trajectory for this mode, $\beta_{g}$ is the group velocity of the eigen-mode. For TM01 mode, this value is consistent and irrelevant to particle position. But for the dipole mode, this value is proportional to the particle offset to the center. The power extracted out at the end of dielectric-lined waveguide is [6]:

$$
\begin{equation*}
P_{s}=\frac{E_{a}^{2} v_{g}}{4 k_{L}}=\frac{1}{4} q^{2} \omega\left(\frac{R}{Q}\right) v_{g}\left(\frac{1}{1-\beta_{g}}\right)^{2},\left(0 \leq t \leq \tau_{s}\right) \tag{2}
\end{equation*}
$$

Where the $\tau_{s}$ is the power pulse length, which is determined by the waveguide length and group velocity.

If we take the attenuation and bunch charge density into consideration, we can get the power level as

$$
\begin{equation*}
P_{s}=\frac{1}{4} q^{2} \omega\left(\frac{R}{Q}\right) v_{g}\left(\frac{1}{1-\beta_{g}}\right)^{2}\left(\exp \left(-\frac{\omega^{2} \sigma_{z}^{2}}{2 c^{2}}\right)\right)^{2} \exp \left(-2 \alpha_{0} v_{g} t\right),\left(0 \leq t \leq \tau_{s}\right) \tag{3}
\end{equation*}
$$

Then we need to calculate the R over Q . The strongest dipole mode excited by the particle is HEM11, its field in the vacuum region can be written as (paraxial approximation)

$$
\left\{\begin{array}{c}
E_{r}=\left[-\frac{j \omega \mu_{0}}{k_{1}^{2} r} A_{1} \frac{j}{2}\left|k_{1}\right| r-\frac{j \beta}{k_{1}} B_{1} \frac{j}{2}\right] \cos \left(\phi-\theta_{0}\right) e^{j(\omega t-\beta z)} \\
E_{\phi}=\left[\frac{j \omega \mu_{0}}{k_{1}} A_{1} \frac{j}{2}+\frac{j \beta}{k_{1}^{2} r} B_{1} \frac{j}{2}\left|k_{1}\right| r\right] \sin \left(\phi-\theta_{0}\right) e^{j(\omega t-\beta z)} \\
E_{z}=A_{1} \frac{j}{2}\left|k_{1}\right| r \cos \left(\phi-\theta_{0}\right) e^{j(\omega t-\beta z)}  \tag{4}\\
H_{r}=\left[-\frac{j \omega \varepsilon_{0}}{k_{1}^{2} r} A_{1} \frac{j}{2}\left|k_{1}\right| r-\frac{j \beta}{k_{1}} B_{1} \frac{j}{2}\right] \sin \left(\phi-\theta_{0}\right) e^{j(\omega t-\beta z)} \\
H_{\phi}=\left[-\frac{j \omega \varepsilon_{0}}{k_{1}} A_{1} \frac{j}{2} J_{1}^{\prime}\left(k_{1} r\right)-\frac{j \beta}{k_{1}^{2} r} B_{1} \frac{j}{2}\left|k_{1}\right| r\right] \cos \left(\phi-\theta_{0}\right) e^{j(\omega t-\beta z)} \\
H_{z}=B_{1} \frac{j}{2}\left|k_{1}\right| r J_{1}\left(k_{1} r\right) \sin \left(\phi-\theta_{0}\right) e^{j(\omega t-\beta z)}
\end{array},\right.
$$

This mode has polarization angle which is determined by the particle azimuthal angle. Assuming the particle trajectory is parallel to the structure $\mathbf{z}$-axis which means its coordinate $(\mathrm{r} 0, \theta 0)$ keeps same at each z position, we can get the R over Q value:

$$
\begin{equation*}
\frac{R}{Q}=\frac{V^{2}}{\omega U}=\left.\frac{\int_{0}^{L_{c}} E_{z} d z}{\omega U}\right|_{\substack{r=r_{0} \\ \theta=\theta_{0}}}=\frac{A_{1}^{2}\left|k_{1}^{2}\right| r_{0}^{2} L_{c}^{2}}{4 \omega U} \tag{5}
\end{equation*}
$$

Where the $U$ denotes the stored energy in dielectric waveguide, and the $L_{c}$ is the length of the waveguide. Substituting Eq. 5 into Eq.3, we can get the power at the end of dielectric structure. The HEM11 mode will propagate behind the beam till the end of DWA structure. Since HEM11 cannot be supported in the empty circular waveguide it will convert to TE11 like mode in the metal cylindrical beam pipe. According to the mode matching theory, the power amplitude of the HEM11 mode has a linear attenuation in this conversion, here we can assume the factor to be $\kappa$. As shown in the Fig.1, three couplers separated by 120 -degree along the circumference are used to extract the TE11 mode out. The broadside of each coupler waveguide is along the beam direction (z-axis), which means that the $E_{\varphi}$ of the field in cylindrical tube is coupled with the electric field in the coupler waveguide. The TE11 $E_{\varphi}$ field can be described as

$$
\begin{equation*}
E_{\varphi}=j \omega \mu T V_{0} J_{1}^{\prime}(T r) \cos \left(\varphi-\theta_{0}\right) e^{-j \beta z} \tag{6}
\end{equation*}
$$

Where the T denotes the transverse angular wave number and $\mathrm{V}_{0}$ denotes the amplitude coefficient, which is proportional to the accelerating voltage. Thus the azimuthal distribution of electric field can be described with the cosine function. And the attenuation factors from the end of dielectric-lined waveguide to each of the coupler rectangular waveguide are

$$
\left\{\begin{array}{c}
\beta_{3}=\Gamma^{2} \cos ^{2}\left(0-\theta_{0}\right)  \tag{7}\\
\beta_{4}=\Gamma^{2} \cos ^{2}\left(2 \pi / 3-\theta_{0}\right) \\
\beta_{5}=\Gamma^{2} \cos ^{2}\left(4 \pi / 3-\theta_{0}\right)
\end{array}\right.
$$

Where $\Gamma$ is the s-parameter from the position 1 to the position 3 in the Fig.1, since it is a symmetric structure the $\Gamma$ is the same value from position 1 to 4 and 1 to 5 . Finally, we can get the power detected at the 3 coupler ports:

$$
\left\{\begin{array}{l}
P_{s 3}=\frac{\kappa}{16} q^{2} \frac{A_{1}^{2}\left|k_{1}^{2}\right| r_{0}^{2} L_{c}^{2}}{\omega U} v_{g}\left(\frac{1}{1-\beta_{g}}\right)^{2}\left(\exp \left(-\frac{\omega^{2} \sigma_{z}^{2}}{2 c^{2}}\right)\right)^{2} \exp \left(-2 \alpha_{0} v_{g} t\right) \Gamma^{2} \cos ^{2}\left(\theta_{0}\right) \\
P_{s 4}=\frac{\kappa}{16} q^{2} \frac{A_{1}^{2}| |_{1}^{2} \mid r_{0}^{2} L_{c}^{2}}{\omega U} v_{g}\left(\frac{1}{1-\beta_{g}}\right)^{2}\left(\exp \left(-\frac{\omega^{2} \sigma_{z}^{2}}{2 c^{2}}\right)\right)^{2} \exp \left(-2 \alpha_{0} v_{g} t\right) \Gamma^{2} \cos ^{2}\left(2 \pi / 3-\theta_{0}\right) \\
P_{s 5}=\frac{\kappa}{16} q^{2} \frac{A_{1}^{2}\left|k_{1}^{2}\right| r_{0}^{2} L_{c}^{2}}{\omega U} v_{g}\left(\frac{1}{1-\beta_{g}}\right)^{2}\left(\exp \left(-\frac{\omega^{2} \sigma_{z}^{2}}{2 c^{2}}\right)\right)^{2} \exp \left(-2 \alpha_{0} v_{g} t\right) \Gamma^{2} \cos ^{2}\left(4 \pi / 3-\theta_{0}\right) \tag{8}
\end{array}\right.
$$

Where the $\alpha_{0}$ is the attenuation factor to indicate the power loss in the dielectric layer when the microwave propagates along the dielectric wall.
Since the voltage signal is proportional to the square root of the power, we can find the relationship between BPM voltage signal and particle position as

$$
\begin{equation*}
V \propto r_{0} \cos \left(\theta_{0}\right) \tag{9}
\end{equation*}
$$

The Eq. 9 indicates that the signal extracted out from the 120-degree separation waveguides contains information of the beam coordinate, thus can be used as BPM. It is worth to point out that the beam position obtained from this BPM device reflects the history of beam trajectory in the upstream neighbouring DWA structure. In addition, the TE11 mode coupler, due to its orientation, is naturally isolated from the TM01 mode, which eliminates the common mode interferences often occurring in the cavity BPM.

## SIMULATION AND CALIBRATION



Figure 2. Simulation model in CST (left). The voltage achieved at port $3 \sim 5$ (right).
Take an example of 300 GHz 0.2 m long DWA module used in [2]. The outer and inner diameter of the dielectriclined waveguide are 2.1228 mm and 2 mm , and the material is quartz whose epsilon is 3.75 . These parameters set the working frequency at 300 GHz . We first simulated the 3 120-degree separation waveguides structure in frequency domain to verify that the voltage at each
waveguide port is cosine function of the field polarization angle.
Different polarization TE11 mode is imported from port 1. Three voltage monitors set at port $3 \sim 5$ to detect the voltage. Fig. 2 (Right) shows that the voltage of each port varies with the phase, i.e. the polarization angle of the TE11 mode following the cosine function.

However, limited by the signal to noise ratio it is challenging to get the accurate beam azimuthal position from the signal of one waveguide port. Specific calibration method using three ports is needed to calculate the accurate beam position. Assuming the signal processing systems for each of three coupler waveguide are the same, we can derive the particle azimuthal angle using Eq. 8

$$
\left\{\begin{array}{l}
\frac{V_{3}}{V_{4}}=\frac{\cos \left(\theta_{0}\right)}{\cos \left(120-\theta_{0}\right)}  \tag{10}\\
\frac{V_{3}}{V_{5}}=\frac{\cos \left(\theta_{0}\right)}{\cos \left(240-\theta_{0}\right)}
\end{array}\right.
$$

It means if we measure the voltages at three coupler waveguide ports at the same time, then we can solve Eq. 10 to get the particle azimuthal angle $\theta 0$ in the DWA structure.
To achieve the particle radial position information, we need to do the calibration for this BPM. Based on the Eq. 4 and Eq.9, we know that the HEM11 dipole mode strength is proportional to the particle radial position, so the signal detected at the coupler waveguide port is proportional to the particle radial position r0 too.


Figure 3. The field map in the metal tube behind dielectric-lined waveguide (left). The model of 200 mm dielectric-lined waveguide with BPM (right).
Assuming the particle has an offset r 0 in the positive x direction, the cross section of the wakefield in the metal tube after the DWA structure is shown in Fig.3. For this particle coordinate ( $\mathrm{r} 0, \theta 0$ ), where $\theta 0=0$, the voltage signal of coupler waveguide port at $\mathrm{pi} / 2$ position is only determined by r 0 . Therefore, in the calibration mode, we can scan the emulated particle radial position (e.g. an electric wire) from 0 to rmax and record the signal information at $\mathrm{pi} / 2$ waveguide port as the calibration result.

Figure 4 shows the simulation results. The length of DWA structure is 200 mm in order to save the simulation time. The BPM part consists of a metal cylindrical tube and 3 rectangular waveguides. These waveguides are standard WR3 which can extract TE11 mode out for detecting the power or voltage signal.


Figure 4: The voltage 3 FFT value versus particle radial position r0 (top) and voltage 3 FFT value versus particle azimuthal angle $\theta 0$ (bottom).
The results shown in Fig. 4 indicate that the voltage at BPM waveguide port is proportional to the particle radial offset r 0 and also proportional to the cosine function of particle azimuthal angle. As to the resolution, the simulation results show that the maximum voltage detected at the port 3 is about $4.4 \mathrm{~V} / \mathrm{nC} / \mathrm{um}$, which means that the resolution can reach 1 nanometre for 1 nC beam if the noise level of electrical facility is 4.4 mV . This is benefit from the large R over Q of this long structure.

## CONCLUSION

A BPM technique based on the dipole mode of THz DWA structure is proposed. It may help pave the path toward the future wakefield accelerator facilities.

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