A LATTICE CORRECTION APPROACH THROUGH BETATRON PHASE ADVANCE *

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Abstract

Most lattice correction algorithms, such as LOCO [1], rely on the amplitude of the BPM signals. However, these signals are a mixture of the BPM gain and beta-beat. Even though BPM gain can be fitted by analyzing the statistics of all the BPMs in a ring accelerator, we found the uncertainty is on the order of a few percent. On the other hand, the betatron phase advance, which is obtained from the correlation of two adjacent BPMs, is independent of the BPM gain and tilt error. It was found at NSLS-II that the measurement precision of the phase advance is typically 1mr, which corresponds to about 0.2% of beta beat. The phase error can be corrected similarly using a reversed response matrix, and at NSLS-II the phase error can be corrected to <10mr (p-p) in less than half an hour. The same technique can be applied to the nonlinear lattice. By comparing the phase advance differences between the on- and off- orbit lattices, the sextupole strength error can be identified. Simulation and experimental results are presented in the paper.

INTRODUCTION

The equivalence of betatron phase or amplitude was probably realized earlier but the first example of application was only found recently in a 1993 PAC paper by Castro of CERN [2]. Therein the phase was calculated from a Fourier Transform of the BPM turn-by-turn signal, then the phase deviation was converted to beta function for correction. The measurement uncertainty was determined to be 4-5mr, which corresponded to 5% of beta beat. The approach was pursued at many facilities afterwards, for example, Cornell [3], SSRL [4], LHC [5], RHIC [6], Diamond [7], and PSI [8]. In [3] it was shown that the phase beat can be decomposed into two sinusoidal wave, very similar to the beta beat. And it was shown analytically that linear coupling does not affect the betatron phase to the first order. In [4] and [5] it was pointed out that the phase beat is equivalent to the beta beat, and can be corrected directly just like the beta beat correction. The consensus in almost all the references is that the phase measurement and correction is fast and immune to BPM calibration error. Another advantage that was not mentioned so frequently is that turn-by-turn signal does not require stored beam; therefore the method is useful in situations like commissioning. At NSLS-II the beta-beat was corrected down to several percent at commissioning using turn-by-turn data [9, 10]. So far, this technique is limited by a 5mr phase error and the residual beta beat is usually 5-6% [8], which is similar to the results when the method was first proposed.

This paper will start from the improvement of the phase calculation algorithm, and show an approach to 1mr resolution. The linear and nonlinear lattice correction at NSLS-II is presented as an example.

REFINED FOURIER TRANSFORM

The BPM turn-by-turn signal is pseudo-sinusoidal due to damping and oscillation from the other planes. The first step of the analysis is to identify the main frequency. Many methods have been tried and we found NAFF [11] is the most accurate for this purpose. The idea is to find the frequency of the maximum amplitude that fits the oscillation data. A Hanning window significantly improves the precision. If the frequency ν is known, the standard way to calculate the phase of a discrete signal x_n, n=1,2,3$\cdots$,N is

$$a = \sum_i^N x_i \sin(2\pi\nu i) \quad b = \sum_i^N x_i \cos(2\pi\nu i) \tag{1}$$

and the phase ϕ

$$\phi = \tan^{-1}(-a/b). \tag{2}$$

However the error from Eq.(2) is $1/N + \delta\phi_e + \pi N\delta\nu$, where $\delta\nu$ is the error of the frequency. The $1/N$ term is due to damping or $N\nu$ being a non-integer. The $\delta\phi_e$ term is caused by noises in the signal. The third term could be large and completely invalidate the calculation; therefore the frequency must have a precision much less than $1/N$.

The $1/N$ term can be overcome by a refined phase search similar to the frequency search in NAFF, or analytically. Assume the signal can be approximated by

$$x_i = Ae^{-\alpha\theta_i}\cos(\nu\theta_i + \phi + \epsilon), \tag{3}$$

where $\theta_i = 2\pi i$, A and α are the amplitude and the damping coefficient determined from other methods. ϵ is the correction to the phase in Eq.(2). ϵ is given by

$$\epsilon = \frac{e_1 - e_2}{e_3 - e_4} \tag{4}$$

where

$$\begin{aligned} e_1 &= \sum_i \frac{A}{2}\exp(2\alpha\theta_i)\sin 2(\nu\theta_i + \phi) \\ e_2 &= \sum_i x_i \exp(\alpha\theta_i)\sin(\nu\theta_i + \phi) \\ e_3 &= \sum_i x_i \exp(\alpha\theta_i)\cos(\nu\theta_i + \phi) \\ e_4 &= \sum_i A\exp(2\alpha\theta_i)\cos 2(\nu\theta_i + \phi). \end{aligned}$$

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The results from Eq.(4) is very close to numerical search if the signal is regular. The amplitude of ϵ is on the order of a few mili-radian; therefore the correction is critical for our purpose.
A simple simulation was carried out to test the algorithm. In this case a Gaussian noise with the amplitude $A \cdot R$ is added to Eq.(3). The amplitude, frequency and phase were calculated by the script, and compared with the input values. Figure 1 shows the difference versus the noise ratio R.
At NSLS-II the BPM turn-by-turn noise for a 10mA 100-

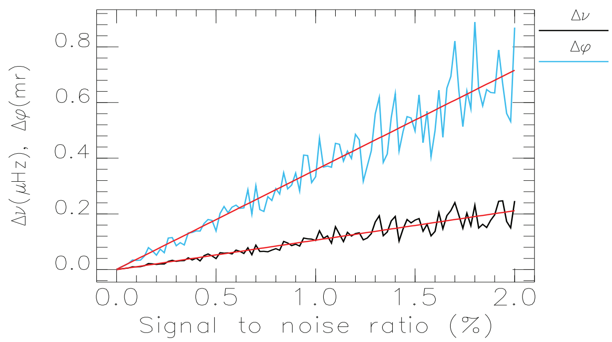

Figure 1: The frequency (ν, μHz) and phase (ϕ, mr) error grow linearly with signal-to-noise ratio (R, %). ν and ϕ are obtained from Naff fitting of 1000 points. 20 seeds were averaged for each R value. The fitted lines are given by $\delta\nu = 1.06 \times 10^{-5}$ Hz/% R, and $\delta\phi = 3.58 \times 10^{-2}$ Radian/% R; therefore $\delta\phi \approx \pi N \delta\nu$.

bunch train is about 10 μm [12], or 1% if the oscillation amplitude is 1mm. This corresponds to $\delta\nu \sim 10^{-7}$ in Fig.1. The order of magnitude agrees with the frequency deviation found from the statistics of 180 BPMs in a single event. The phase uncertainty of 0.4 mr (rms) will be the ultimate limit in this case.

RESOLUTION AND LINEAR CORRECTION

For M BPMs in a ring accelerator, the phase vector is defined as [5]

$$\Delta\phi_m = \phi_{m+1}^{meas} - \phi_m^{meas} - (\phi_{m+1}^{mod} - \phi_m^{mod}), \quad (5)$$

where subscript indicates the BPM index and the superscript stands for "measurement" or "model". ϕ_{M+1} refers to the first BPM but starts from the next turn.
In order to find out the actual phase measurement precision at NSLS-II, we varied the strength of a quadrupole (QH1G2C02A), and measured the phase vector before and after. Figure 2 shows the comparison of the measured and calculated phase vector for a current change of 0.5A. The measured phase was fitted to the calculated phase and a scale factor $f_x = 1.015$ was applied to the measured value to account for beta function deviation, hysteresis, transfer-function uncertainty, and inaccuracy of the hard-edge modeling. The error bar was determined from the standard deviation of 10 repetitive measurement, of which the averaged deviation is 1.7mr, however, the difference to the model is calculated to be 0.7 mr (rms of 180 BPMs). The greater measurement fluctuation is probably caused by orbit drifting, power supply ripple, and rf jitter; but averaging clearly reduces the effects of these errors.

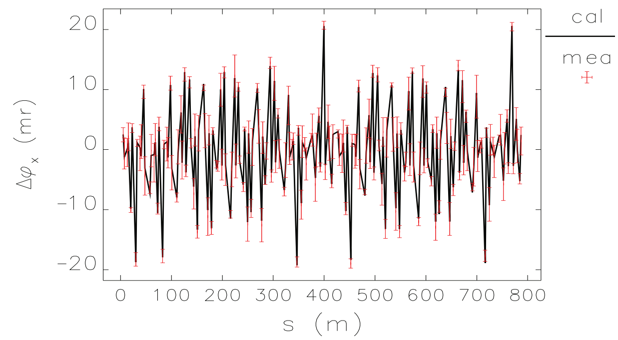

Figure 2: Comparison of the horizontal phase change obtained from measurement and calculation when the current of QH1G2C02A is changed by 0.5A, or, $\Delta K_1 = 8.4 \times 10^{-4}$ $1/m^2$.

The same measurement was repeated at ΔI=0.05,0.1,0.25,0.5,2A, and the deviation to the model is shown in Fig.3 for both transverse planes. The amplitude is about 0.5-1mr, and slightly smaller in the vertical plane.

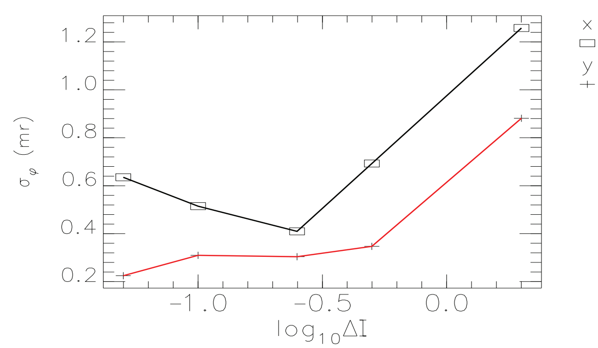

Figure 3: The standard deviation of the phase discrepancy when the quadrupole current is changed by ΔI=0.05,0.1,0.25,0.5,2A.

The phase error can be corrected with the reversed response matrix from model. At NSLS-II the peak deviation can be corrected to below 10mr after a few iterations [13]. The process takes about 20-30 minutes due mostly to repetitive measurements. A typical residual phase vector is plotted in Figure 4. After correction the lattice was measured by LOCO [1] and the residual beta beat was determined to be about 0.9% (x) and 0.6 % (y) rms.

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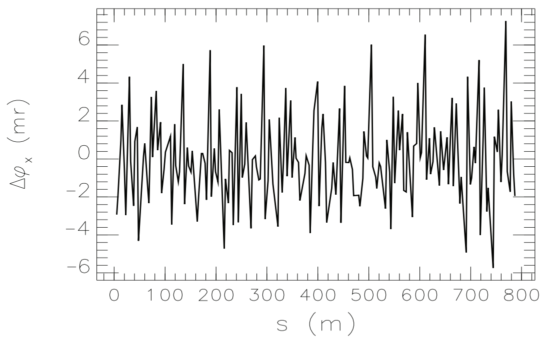

Figure 4: Typical residual phase error.

SEXTUPOLE CORRECTION

This precise measurement technique was applied at NSLS-II to correct the sextupole settings. The idea is to change a horizontal orbit corrector (C) and generate an orbit wave with 2-4mm peak. This lattice differs from the original due to focusing of the off-centered sextupoles. We term this lattice as Corrector-C. The phase vector of Corrector-C has a maximum amplitude of 50-100mr and can be measured with the same technique. Figure 5 shows a typical measurement and comparison with model.

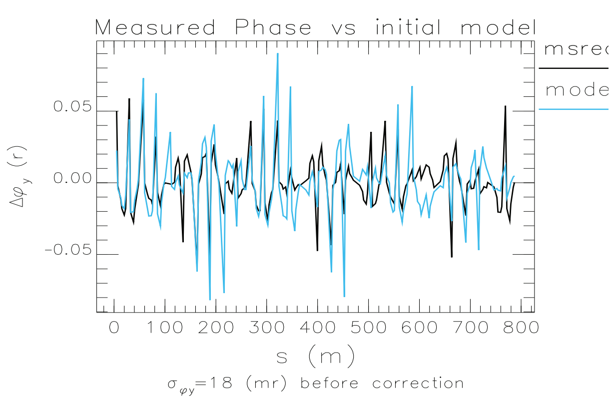

Figure 5: .The measured and the calculated vertical phase vector of lattice CH1XG2C30A. The corrector was changed by 0.18mr.

The same measurement was repeated at 10 correctors. A correction matrix was calculated from the model and adjustment to the sextupole strength was obtained. The model with the updated sextupole settings was compared again to the measurement. A 10% reduction in discrepancy was achieved in the vertical plane but the horizontal plane remained the same.

We measured the dynamic aperture before and after sextupole correction. The criterion for the boundary was 20% of beam loss to save machine study time. The results are plotted against simulation in Figure 6. The simulation is for the ideal lattice, but with physical aperture and magnet errors. All the numbers were normalized to the injection point. We speculated a few reasons for the small improvement: 1) the sextupoles are partially powered in series; 2) the dynamic aperture is already large; 3)other factors, such as orbit leaking into the vertical plane, and linear coupling.

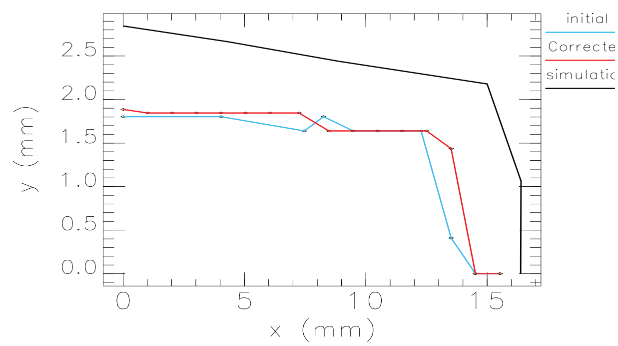

Figure 6: Dynamic aperture before and after sextupole correction, and comparison with simulation.

CONCLUSION

With a refined phase calculation technique the betatron phase measurement precision was improved to 1mr at NSLS-II. Quadrupole and sextupole settings were corrected at NSLS-II with this technique. The linear lattice can be corrected to below 10mr (1% in terms of beta beat) and a small improvement (on top of 15mm) in dynamic aperture was achieved.

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