INFLUENCE OF THE VACUUM CHAMBER LIMITATION ON DYNAMIC APERTURE CALCULATIONS

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Abstract

In a storage ring the evaluation of the dynamic aperture taking into account the vacuum chamber limitation is more accurate and may display nonlinearities that could not be seen in the conventional absolute dynamic aperture calculations. This has been demonstrated in SESAME case where taking into account the vacuum chamber uncovered the seriousness of a 5th order resonance mainly when high order multipoles were introduced to the lattice. The destructive effect of the 5th order resonance has been avoided by changing the fractional part of the tunes. The results are crosschecked using two tracking codes and verified using the Frequency Map Analysis technique.

INTRODUCTION

The dynamic aperture is the transverse area in the *x-z* plane (*x* for horizontal and *z* for vertical plane) in which the particle betatron motion is stable. It is defined by the maximum initial phase space amplitudes ($x(0)$, $p_x(0)$, $z(0)$, $p_z(0)$) with which the tracked particle doesn't get lost for enough number of turns with respect to the interesting time scale as the damping time for the electrons [1]. It is a local lattice parameter where its horizontal and vertical dimensions at some longitudinal position *s* depend on the optical functions there. In the linear approximation it is transformed using the relation $A_{x,z}(s) = \sqrt{\beta_{x,z}(s)/\beta_{0x,z}}\, A_{0x,z}$, with $\beta_{x,z}(s)$ the *s*-dependent beta function, $\beta_{0\,x,z}$ and $A_{0\,x,z}$ are beta function and dimensions of the dynamic aperture at the calculation point $s = 0$.

In BETA [2] and TRACY-II [3] tracking codes, used in this study, the particle is tracked in different ways. In BETA code the particle coordinates are defined by a column matrix with the components (x, x', z, z', l, δ, 1) where x, z the horizontal and vertical transverse positions, $x' = dx/ds$ and $z' = dz/ds$ are the angles, l and δ are the variations in path length and the relative momentum deviation of the test particle from the synchronous one, whereas the 7th component 1 is used to represent the effect of a kick on the trajectory. When dealing with the 2nd order formalism, the column vector is extended by adding the 2nd order components [4]. The particle tracking is done using the 1st order and 2nd order transfer matrices [4]. In TRACY-II code the particle motion is described by the canonical coordinates (x, p_x, z, p_z , l, δ) with x, z the horizontal and vertical transverse positions and p_x, p_z are their horizontal and vertical conjugate momenta. The tracking is done using the 2nd order or 4th order symplectic integrators where the particle motion is symplectic [5].

Conventionally, the nonlinear beam dynamics is represented by the absolute dynamic aperture calculations where a wide excursion space is offered for the particle [6]. The oscillating particle is considered unstable when it exceeds that space. For more realistic estimation for the dynamic aperture, the vacuum chamber should be included in the calculations since it defines the realistic physical limits to the particle excursion amplitude. In this case, a particle passing close to the chamber borders with nonlinear motion may get lost at the chamber limitation and considered as unstable particle, while it can be described as a stable one if it had larger space to oscillate in as in the absolute dynamic aperture case. In this sense the vacuum chamber may participate in defining the "chamber-limited" dynamic aperture [7]. The importance of including the vacuum chamber can be seen more clearly if the particle nonlinear motion is excited by the effect of high order multipoles [8] for example.

THE DYNAMIC APERTURE WITH VACUUM CHAMBER

In the nonlinear optimization of SESAME storage ring lattice, the vacuum chamber with dimensions $x = \pm 35$ mm for horizontal half-aperture and $z = \pm 15$ mm for the vertical one was included in the dynamic aperture calculations. The vacuum chamber was introduced in TRACY-II as a transverse physical limitation at the entrance and exit of each element in the lattice. The elements of the lattice are divided into many slides, for each the vacuum chamber limitations are introduced. The bending magnets and quadrupoles are the most interesting ones in this consideration. In BETA code, the vacuum chamber was represented by horizontal and vertical scrapers placed at the highest values for beta functions β_x and β_z.

The presented nonlinear calculations are done on an Optics with working point ($Q_x = 7.23$, $Q_z = 5.19$) [9] and are evaluated by tracking the particle for 1000 turns starting from the middle of the Long straight section where $\beta_x = 12.31$ m and $\beta_z = 3.13$ m. The maximum values for β_x and β_z in this optics are $\beta_{xmax} = 12.807$ m in the middle of the focusing quadrupoles at the ends of the long straight sections and $\beta_{zmax} = 21.35$ m in the middle of the bending magnets as shown by Fig. 1. The vacuum chamber vertical half-aperture, $z = 15$ mm, in the bending magnet yields a vertical physical aperture $\Delta z = 5.74$ mm (at $x = 0$) at the calculation point. The presented calculations are done for chromaticities corrected to zero in both planes.

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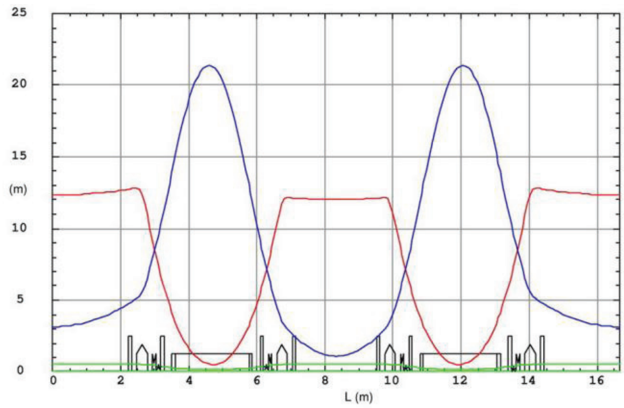

Figure 1: SESAME storage ring optics showing β_x (red), β_z (blue) and dispersion η_x (green).

By introducing the vacuum chamber, signature of the nonlinearities became different from the case of absolute dynamic aperture as shown by Fig. 2 where the results of BETA, which highly agree with TRACY-II results, are presented.

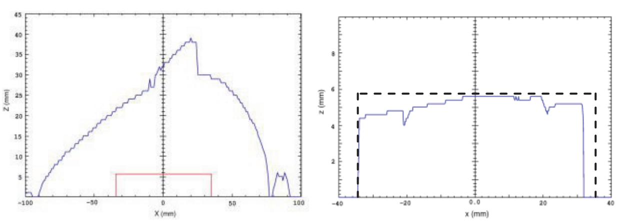

Figure 2: (left) The absolute dynamic aperture (blue) compared to the vacuum chamber size (red). (right) The chamber-limited dynamic aperture. The dashed line shows how the chamber-limited dynamic apertures should be in case of linear particle motion.

Figure 2(right) show that the chamber-limited dynamic aperture is degrading at large x-amplitudes mainly in the left side and, moreover, it has two clear vertical cuts at $x \approx \pm 21$ mm. This indicates a presence of resonances (created by the sextupoles) there which make the particle motion nonlinear and increase the amplitude of particle vertical oscillation to higher than what is allowed by the vacuum chamber. Consequently this particle gets lost on the upper chamber limitation. In case of no chamber limitation, that particle is considered stable by the tracking code due to the large oscillation space offered to it as can be seen in Fig. 2(left).

Applying High Order Multipoles

When the dipole high order multipoles, listed in Table1, were included into the calculations the size of the absolute and chamber-limited dynamic apertures became as shown in Fig. 3.

Table 1: Dipole High Order Multipoles

High Order Component	(ΔB_z/ B)
Sextupole	2.42 x 10^{-4}
Octupole	4.70 x 10^{-5}
Decapole	-3.09 x 10^{-5}
Dodecapole	-1.36 x 10^{-5}
14-pole	-1.17 x 10^{-4}

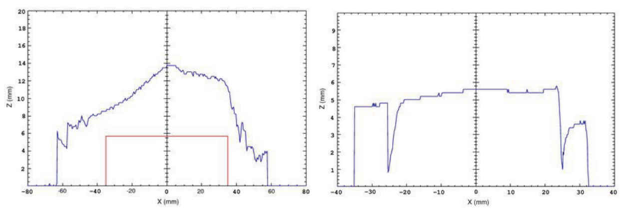

Figure 3: Dynamic apertures under high order multipole effect. (left) The absolute dynamic aperture (blue) compared to the vacuum chamber size (red), (right) the chamber-limited dynamic aperture.

Figure 3(left) indicates that the absolute dynamic aperture is still enough larger than the physical one, consequently the high order multipoles seem tolerable by the optics. But when the vacuum chamber is included in the calculations in Fig. 3(right), we can see that these high order multipoles amplify the two cuts to a level that cannot be accepted resulting in a dynamic aperture much smaller than the physical one. Hence these high order multipoles are not tolerable by the optics, contradicting the indication given by Fig. 3(left). It can be noticed that the two cuts also have been shifted outward from the center. This is due to the distorted tune shifts with x and z-amplitudes. Propagation of the two vertical cuts down through the dynamic aperture indicates that the driving resonance is excited causing higher amplitudes for the particle nonlinear vertical oscillations so that the particle gets lost on the upper chamber limitation at lower vertical heights.

This explanation is more clarified by Fig. 4 which shows behavior of the vertical oscillation amplitude of the particle versus x-position at z = 4.8 mm without and with high order multipoles. The blue lines represent the vertical physical aperture Δz = ±5.74 mm at the calculation point.

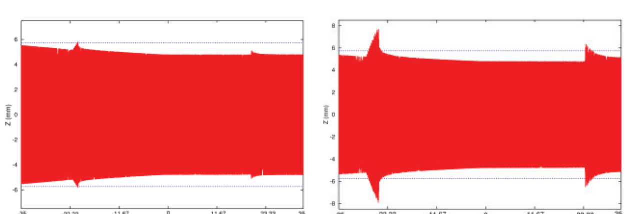

Figure 4: The vertical oscillation amplitudes versus x for a particle tracked at the vertical height z = 4.8 mm, without (left) and with (right) high order multipoles.

The gradually increasing vertical amplitude with x in the left sides of Fig. 4 explains the dynamic aperture degradation in the left hand side of Figs. 2(right) and 3(right). The two drastic increments in vertical amplitude at $x \approx \pm 21$ mm which are amplified by high order multipole effect stand behind the two seen cuts since they cause the particle to exceed the vertical acceptance of the vacuum chamber.

The Vacuum Chamber is a Simple Tool to Show Inner Dynamic Nonlinearity

The above investigation shows that introducing the vacuum chamber limitations into the calculations was a simple tool to uncover the inner nonlinearity in SESAME dynamic aperture which couldn't be seen in case of absolute dynamic aperture calculations.

TREATING THE PROBLEM

In order to understand the problem deeply and to define the corresponding destructive resonance, the dynamics was more investigated using the known Frequency Map Analysis technique [10-14]. The investigation showed that the systematic 5^{th} order resonance $3Q_x + 2Q_z = 32$ is the driving force of the drastic increment in the vertical oscillation amplitude of the particle which consequently gets lost at the vacuum chamber wall. Effect of this resonance becomes stronger when it encounters the particle at larger oscillation amplitude [15]. When the high order multipoles of Table 1 are applied, this resonance is more strengthened by the decapole component.

It was possible to avoid the impact of the 5^{th} order resonance by changing the working point ($Q_x = 7.23$, $Q_z = 5.19$) to ($Q_x = 7.28$, $Q_z = 5.19$), hence moving the working point farther from this resonance so that the particle crosses it at higher x-amplitude that is already outside the vacuum chamber dimensions. Figure 5 shows this result through the less destructed chamber-limited dynamic aperture and the smooth vertical oscillations with x-amplitude even with high order multipoles.

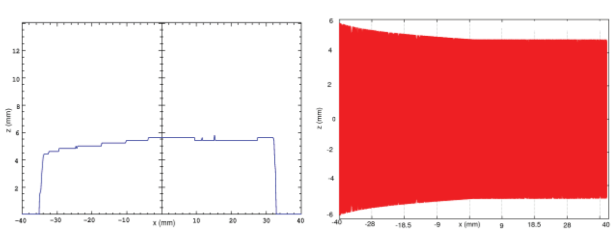

Figure 5: Chamber-limited dynamic aperture (left) and vertical oscillation amplitude with x (right), under effect of high order multipoles, for the new working point (($Q_x = 7.28$, $Q_z = 5.19$).

CONCLUSION

This study showed that including the vacuum chamber limitation in the dynamic aperture calculations could be a simple tool, other than the complicated FMA technique, to uncover the nonlinearity in the inner structure of the dynamic aperture. In SESAME case it revealed a seriousness of existing 5^{th} order resonance, mainly when high order multipoles are included, something which couldn't be noticed in case of absolute dynamic aperture calculations.

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