IMPROVED TEAPOT METHOD AND TRACKING WITH THICK QUADRUPOLES FOR THE LHC AND ITS UPGRADE

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Abstract

The comparison between tracking with thick and thin lens models for the LHC has been studied. A widely-used method to generate thin models is based on the TEAPOT slicing, which, in the original implementation, was limited to a maximum of four slices. In this paper, an improved method is presented, which overcomes the limitation in the number of slices of the original TEAPOT. The performance is analysed and the impact on numerical simulation of the dynamic aperture is evaluated, both for the LHC and its upgrade, HL-LHC.

INTRODUCTION

In computer codes like MAD-X, the accelerator is described as a sequence of elements. A detailed sequence will contain a description of elements with their geometrical dimension and their effect on the beam. For particle tracking the magnet lattice description has to be symplectic. Symplectic lattice descriptions can be obtained by translation to thin lattice, in which the elements are represented by one or several multipole slices (see, e.g., Ref. [1]). The use of thin elements is the easiest example of symplectic integration of the equations of motion. The simplest approach consists of using equally spaced kicks, interspersed with drifts. This is what is done in the case of MAD-X, where the splitting is obtained by translation of the thick sequence to a thin sequence using the `MAKETHIN` module from MAD-X [2]. The field of symplectic integration provides very sophisticated tools to deal with the problem of symplectic tracking (see, e.g., Ref. [3] for a review and Refs. [4, 5, 6] for accounts on special techniques). It is also worth noting that for dipoles and quadrupoles it is possible to find symplectic transformations such that these elements can, in principle, be kept as thick elements for the tracking. Recently, a study has been performed to assess the accuracy of the tracking performed with thin or thick quadrupoles [7].

In this paper the emphasis is put on an extension of the very elegant slicing algorithm for quadrupoles [8] that has been originally implemented in the TEAPOT code [9]. The original implementation was limited to 4 slices, while in this paper an extension of the algorithm to an arbitrary number of slices is presented. This has a positive impact on the number of slices required to achieve a given accuracy for the representation of the machine optics, with also a beneficial effect on the tracking speed.

THICK AND THIN QUADRUPOLES

The transfer matrix for a thick quadrupole in 2 dimensions can be written as

$$\mathbf{M}_{\mathrm{q}}(K,L) = \begin{pmatrix} \cos\sqrt{K}L & \frac{\sin\sqrt{K}L}{\sqrt{K}} \\ -K\sin\sqrt{K}L & \cos\sqrt{K}L \end{pmatrix}, \quad (1)$$

where K is the quadrupole strength and L the full length of the thick quadrupole.

For the transfer matrix of a thin quadrupole we have

$$\mathbf{M}_{\mathrm{th}}(KL) = \begin{pmatrix} 1 & 0 \\ -KL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}, \quad (2)$$

which can be obtained from Eq. (1), by replacing the matrix elements with first order Taylor series terms ($\cos x \to 1$, $\sin x \to x$).

The thin lens transfer matrix has only one non-trivial matrix element $\mathbf{M}_{\mathrm{th}\,2,1} = -KL = -1/f$, which corresponds to a kick of $x' = -x/f$ for a particle travelling at distance x from the magnet's axis and f is the focal length of the lens. A particle travelling parallel to the axis (initial $x' = 0$) is deflected towards the axis which is crossed after a distance of f from the quadrupole.

SIMPLE AND TEAPOT SLICING

In MAD-X, the `MAKETHIN` module provides two types of slicing algorithm, namely `SIMPLE` and `TEAPOT`. We first consider `SIMPLE` slicing using n equidistant slices, $\mathbf{M}_{\mathrm{d}}$ being the matrix of a drift, $L_n = L/(2n)$, and $K_n = 2KL_n$, then the global transfer matrix is given by

$$\mathbf{M}_{\mathtt{SIMPLE}}(n) = \left[\mathbf{M}_{\mathrm{d}}\left(L_n\right)\mathbf{M}_{\mathrm{th}}\left(K_n\right)\mathbf{M}_{\mathrm{d}}\left(L_n\right)\right]^n, \quad (3)$$

where each slice has a thin lens quadruple of strength K_n sandwiched between drift spaces of length L_n.

We now compare the first terms of the Taylor expansion of this product with that of the $(2,1)$ matrix element of the thick quadrupole

$$\mathbf{M}_{\mathrm{q}}(K,L)_{2,1} = -KL\left(1 - \frac{KL^2}{6} + \cdots\right), \quad (4)$$

$$\mathbf{M}_{\mathtt{SIMPLE}}(n)_{2,1} = -KL \times \left[1 - \frac{KL^2}{6}\left(1 - \frac{1}{n^2}\right) + \cdots\right] \quad (5)$$

and note that the two expressions differ in the second term. We will now show that it it possible to reproduce exactly the second term of Eq. (4) by modifying the slice positions to what will be referred to as `TEAPOT` slicing [9], which is illustrated in Fig. 1 and also defines the symbols used in this

ISBN 978-3-95450-122-9

paper. Indeed, this algorithm is an extension of the original TEAPOT slicing. Positions and distances are expressed in units of the length L of the thick quadrupole. Δ is the distance between slices and δ the distance of the first and last slice to the edge of the magnet.

For a given number of slices n, it is sufficient to specify the distance δ at the edge. The equal distances between the central slices can be obtained from the sum which must be 1 in units of L according to

$$2\delta + (n-1)\Delta = 1\,. \tag{6}$$

Fig. 1 also shows as dashed lines the case of simple equidistant positions, where $\Delta = 2\delta$. The global transfer matrix

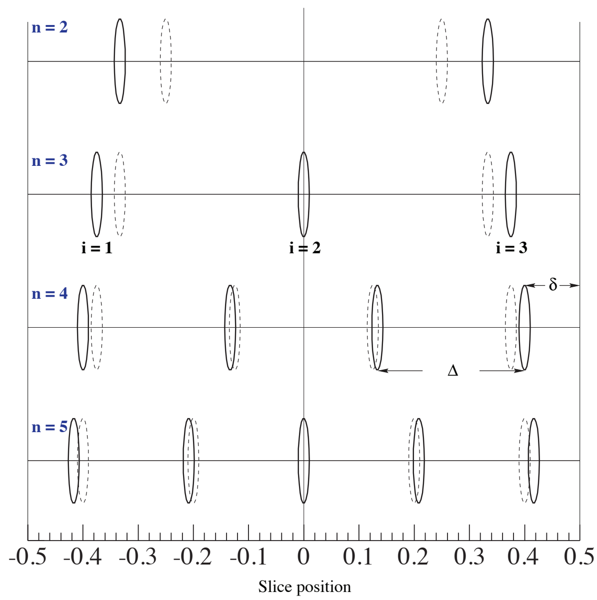

Figure 1: Illustration of TEAPOT-style (solid lenses) and SIMPLE-style (dashed) slice positions for $2 \le n \le 5$ slices. The origin is at the centre of the thick magnet, and ± 0.5 corresponds to the left and right edge of the thick magnet. Δ is the distance between the central slices and δ the distance of the first and last slice to the edge of the magnet.

with TEAPOT slicing can be written as

$$\begin{aligned} \mathbf{M}_{\texttt{TEAPOT}}(K,L,n,\delta) =& \mathbf{M}_{\rm d}(L\,\delta)\times \\ [\mathbf{M}_{\rm th}\,(K_n)\;\mathbf{M}_{\rm d}\,(L\,\Delta)]^{(n-1)}&\mathbf{M}_{\rm th}\,(K_n)\,\mathbf{M}_{\rm d}\,(L\,\delta)\,. \end{aligned} \tag{7}$$

It starts with a drift of length δ, followed by $n-1$ thin quads spaced by Δ, and ends with a last thin slice at distance δ from the end. The Taylor expansion of the $(2,1)$ matrix element is

$$\begin{aligned} \mathbf{M}_{\texttt{TEAPOT}}(K,L,n,\delta)_{2,1} = -KL\times \\ \left[1-\frac{KL^2}{6}\left(1+\frac{1}{n}\right)(1-2\delta)+\cdots\right]. \end{aligned} \tag{8}$$

Choosing

$$\delta = \frac{1}{2}\frac{1}{1+n}\,, \tag{9}$$

allows reproducing the thick quadrupole expression given in Eq. (4). A comparison of SIMPLE and TEAPOT slicing with numerical values for δ, Δ is given in Table 1.

Table 1: Comparison of distances used in TEAPOT and SIMPLE slicing

n	δ	Δ TEAPOT	Δ SIMPLE
2	1/6	$n/3 = 0.\bar{6}$	$1/n = 0.5$
3	1/8	$n/8 = 0.375$	$1/n = 0.\bar{3}$
4	1/10	$n/15 = 0.2\bar{6}$	$1/n = 0.25$
m	$1/[2(1+m)]$	$m/(m^2-1)$	$1/m$

The aim of the TEAPOT algorithm is to improve the convergence of the $(2,1)$ matrix element towards the true solution given by the thick quadrupole. It is also possible to show that such an algorithm has a beneficial impact on the convergence of the other two independent elements, namely $(1,1)$ and $(1,2)$. In both cases the converge rate is quadratic in n and a plot is given in Fig. 2.

It is worth mentioning that even if the computations reported here are based on the use of focusing quadrupoles the conclusions holds true in general.

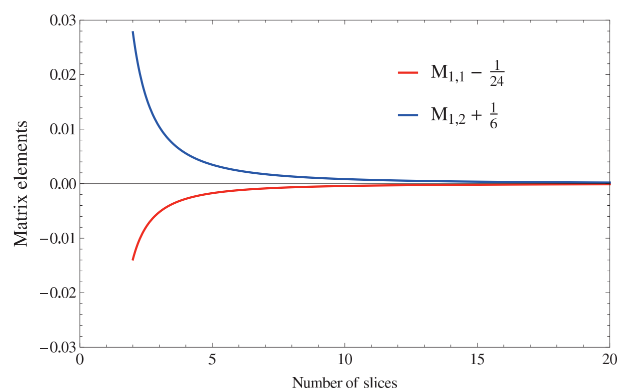

Figure 2: Convergence of the matrix elements $\mathbf{M}_{\texttt{TEAPOT}}(K,L,n,\delta)_{1,1}$, $\mathbf{M}_{\texttt{TEAPOT}}(K,L,n,\delta)_{1,2}$ towards the thick quadrupole values.

RESULTS FOR THE LHC

The first test of the proposed algorithm has been performed by using the standard LHC lattice, which represents a realistic and complex benchmark. As already shown in Ref. [7] the approach consists of slicing the main quadrupoles in the arcs and evaluating the resulting β-beating. The plot is shown in Fig. 3, where the dependence of the β-beating for both slicing methods is shown as a function of the number of slices. The much better performance of the TEAPOT method is clearly seen, featuring a β-beating two orders of magnitude smaller for the same number of slices than that of SIMPLE, even if the slope of the two curves is essentially the same.

It is worth noting that for HL-LHC squeezed optics [10] the improvement is over one order of magnitude when 16 TEAPOT slices are used, reducing the β-beating when the thin optics is not re-matched to 4% (6%) for the case of $\beta^* = 15$ cm (10 cm).

The ultimate test is, however, the computation of the dynamic aperture (DA) for the various configurations. Here, the original study presented in Ref. [7], where the DA comparison between different types of slicing was performed, has been pursued. The good agreement found for injection optics has been verified for more stringent conditions, such as those provided by the collision optics with two insertions sqeezed to $\beta^* = 55$ cm.

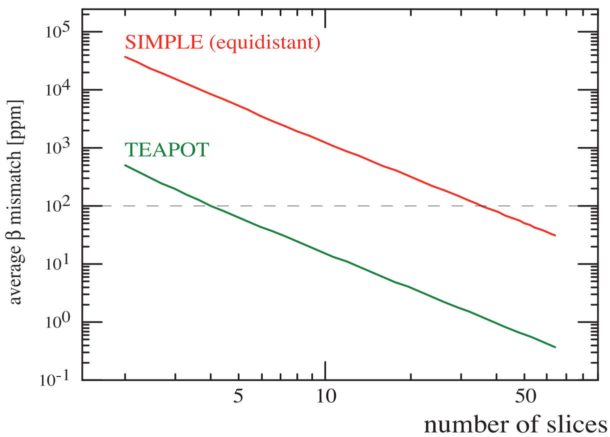

Figure 3: β-beating in the LHC ring as a result of `TEAPOT` or `SIMPLE` slicing of the arc main quadrupoles.

The initial conditions for tracking are distributed uniformly over phase space angles, with 30 pairs over 2σ amplitude range. The maximum number of turns is 10^5 and the momentum off-set is 0.27×10^{-3}, corresponding to 3/4 of the bucket height. For each of the three lattice models considered, i.e., nominal model with thin quadrupoles (11 phase space angles), model with all quadrupoles thick (59 phase space angles), model with thin main quadrupoles and thick insertion quadrupoles (59 phase space angles), the full set of magnetic field imperfections for all magnet classes have been assigned. Sixty realisation of magnetic field errors are considered. The results are plotted in Fig. 4.

Overall, there is a good agreement, at least within the error bars, between the DA values for the three models. The agreement improves for small and large values of the phase space angles, corresponding to mainly horizontal, respectively vertical, motion. On the other hand, around $45°$, when the motion is intrinsically 4D, the agreement gets worse. In this respect the possibility of having better slicing algorithm implies improving the tracking accuracy, without having to move to thick tracking, which is extremely heavy in terms of CPU-time. One should also stress that such an agreement is certainly worse than for the case of the injection optics.

CONCLUSIONS AND OUTLOOK

The extended `TEAPOT` algorithm proved to have an excellent performance, reproducing the machine optics with a relatively small number of slices. It has been implemented in the `MAKETHIN` module of MAD-X [11], thus improving the efficiency of tracking studies. The difference between thick and thin lens tracking might be non-negligible for pushed optics configurations, such as the squeezed optics for a collider. In this respect an efficient slicing algorithm will be a true advantage.

New options will also be implemented, such as the capability of leaving classes of quadrupoles thick after slicing, i.e., represented by a sequence of kicks, used to assign multipolar errors, and thick quadrupoles instead of drifts as currently done. These changes will be coupled to the possibility of performing tracking using thick quadrupoles, thus expanding the spectrum of possible applications.

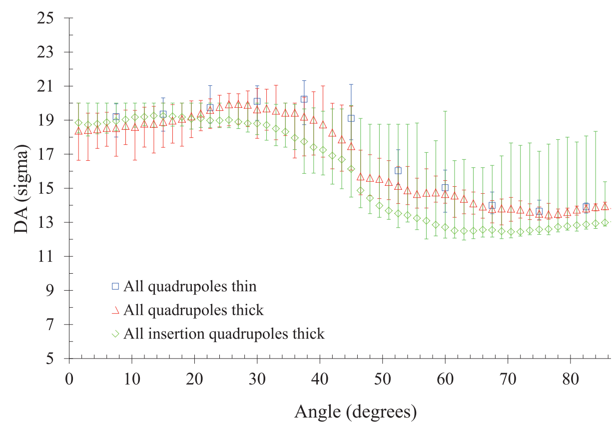

Figure 4: DA as a function of phase space angle for the three lattice models considered in this paper. The error bars refer to the minimum and maximum DA value over the sixty realisations used.

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ISBN 978-3-95450-122-9