NUMERICAL CALCULATION OF ELECTROMAGNETIC FIELDS IN ACCELERATION CAVITIES UNDER PRECISE CONSIDIRATION OF COUPLER STRUCTURE*

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Abstract

The acceleration with superconducting radio frequency cavities requires dedicated couplers to transfer energy from the radio frequency source to the beam. Simultaneously, higher order mode couplers are installed to effectively suppress parasitic modes. Therefore, the numerical eigenmode analysis based on real-valued variables is no longer suitable to describe the dissipative acceleration structure. At the Computational Electromagnetics Laboratory (TEMF) a robust parallel eigenmode solver to calculate the eigenmodes in the lossy acceleration structure is available. This eigenmode solver is based on complex-valued finite element analysis and utilizes basis functions up to the second order on curved tetrahedral elements to enable the high precision elliptical cavity simulations. The eigenmode solver has been applied to the TESLA 1.3 GHz accelerating cavity to determine the resonance frequency, the quality factor and the corresponding field distribution for all 192 eigenmodes up to the 5^{th} dipole passband (3.12 GHz).

INTRODUCTION

During the design phase of superconducting radio frequency (RF) accelerating cavities a challenging and difficult task is to determine the electromagnetic field distribution inside the structure with the help of proper computer simulations. So far the most efficient commercially available eigenmode solvers are based on real-valued analysis, which is sufficient to describe the entire electromagnetic field in the lossless acceleration structure. In reality, because of the dissipative acceleration structure a complex-valued eigenmode solver can be used to determine the field distribution efficiently [1]. A robust parallel complex-valued eigenmode solver has been developed at the Computational Electromagnetics Laboratory (TEMF) and is applied to the TESLA 1.3 GHz accelerating cavity to determine the resonance frequency, the quality factor and the corresponding field distribution for 192 eigenmodes up to the 5^{th} dipole passband (3.12 GHz) [2].

THEORETICAL BACKGROUND

Generally, Maxwell's equations are the mathematical foundation of the eigenmode analysis for resonating structures. To describe the electromagnetic field distribution inside the elliptical RF accelerating cavities with high precision, the continuous Maxwell's formulation has been transformed to a suitable matrix equation with the help of the finite element method (FEM) [3]. For the FEM discretization the tetrahedral grids and higher order curvilinear elements (Fig.1) have been be applied to satisfy the demand for high-precision modeling of the elliptical cavity [4].

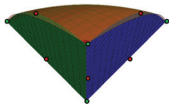

Figure 1: Curvilinear tetrahedral element [4].

Because energy transfer occurs in the dissipative superconducting cavities, lossy boundary conditions have to be considered [3]. For simulation of the accelerating cavities with input coupler and higher order mode (HOM) couplers, port boundary conditions can be applied. To model a true port interface accurately a two-dimensional (2-D) modal expansion of the resulting electromagnetic field in the specified boundary plane has been performed [3]. Once the port frequency is fixed, the modal field pattern in the boundary plane can be determined with the help of a 2-D eigenvalue formulation [3]. Then, the resonance frequency of the eigenmodes in cavities can be obtained by iterative evaluation steps [3]. Due to the dissipative acceleration structure the obtained angular resonance frequency is complex,

$$\underline{\omega} = \omega + j\alpha \tag{1}$$

where the real part $\omega = 2\pi f$ represents the angular resonance frequency of the eigenmode and the imaginary part describes the damping behavior of the oscillation. The quality factor is specified by

$$Q = \frac{\omega}{2\alpha}. \tag{2}$$

Implementation

The geometric modeling of the accelerating structure with the tetrahedral meshing is performed with CST MICROWAVE STUDIO [5]. The eigenmode solver is generally computationally demanding due to the precision modeling of elliptical cavities with curved tetrahedral meshes as well as the complex-valued calculation process. To achieve a good performance on simulation time, a distributed memory architecture using MPI parallelization strategy has been utilized for the implementation [3].

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Application

Firstly, the eigenmode solver has been applied to the TESLA 1.3 GHz accelerating cavity (Fig.2) to determine the characteristic values (resonance frequency and quality factor) for all modes in the 1st monopole passband [2]. The TESLA 1.3 GHz cavity is composed of a 9-cell cavity, the input coupler as well as the up- and downstream HOM couplers. Port boundary conditions are used to define the boundary conditions for three couplers and both beam tubes.

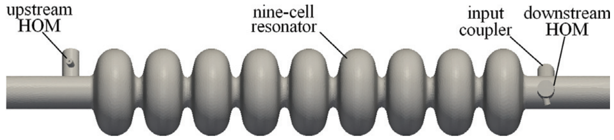

Figure 2: TESLA 9-cell 1.3 GHz superconducting RF cavity with beam tubes as well as the input coupler and two HOM couplers.

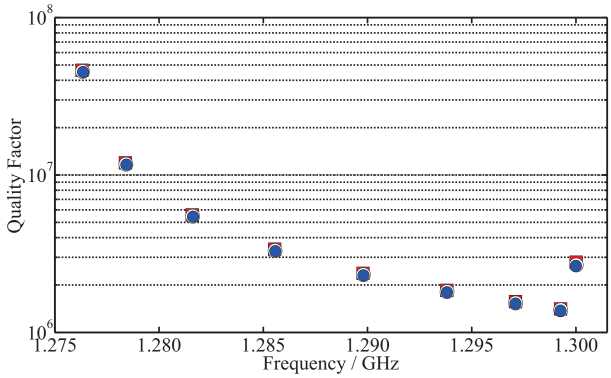

Figure 3: Quality factors versus frequencies for the monopole modes in the 1st monopole passband for different discretizations. The calculations are performed on meshes with 315.885, 1.008.189 and 3.081.614 tetrahedrons indicated by red squared, white circled and blue data points.

The resonance frequency f as well as the quality factor Q of the eigenmodes can be determined simultaneously by the complex formulations (1) and (2) [3]. In Fig.3 the colored points indicate the accurate values of frequency and quality factor for the monopole eigenmodes in the 1st monopole passband. In brief, the simulation results have shown a remarkable competence of the complex-valued eigenmode solver for the difficult numerical calculation of the dissipative elliptical accelerating structure.

COMPUTATION OF EIGENMODES

Since the complex-valued eigenmode solver can perform the calculation of electromagnetic fields in the lossy acceleration structure as required, the resonance frequency, the quality factor (Fig.4) and the corresponding field distribution for 192 modes up to the 5th dipole passband (3.12 GHz) in TESLA 1.3 GHz cavity have been determined [2].

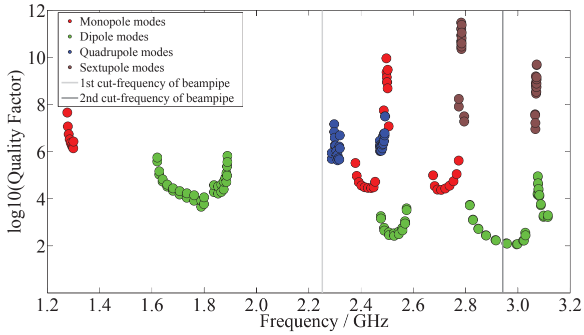

Figure 4: Quality factors versus frequencies for all eigenmodes up to the 5th dipole passband (3.12 GHz).

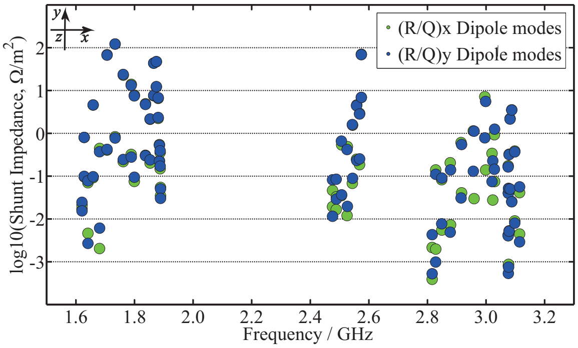

Figure 5: Transverse shunt impedance R/Q versus frequency for dipole modes up to the 5th dipole passband (3.12 GHz).

In addition the transverse shunt impedances of the eigenmodes for all dipole modes up to the 5th dipole passband (3.12 GHz) have been determined (Fig.5). According to Fig.5 the shunt impedances of several dipole modes are about 100 Ω/m^2. In the near future the shunt impedances of all 192 eigenmodes up to the 5th dipole passband (3.12 GHz) will be calculated and published in a TESLA report at DESY.

POST-PROCESSING

After determination of the resonance frequency and the corresponding field distribution of each eigenmode, there are two essential post-processing tasks to be done. The first one is to smooth the electromagnetic field using Kirchhoff's integral theorem, so that the shunt impedance of the eigenmode can be computed precisely. Secondly, it is very hard and inconvenient to manually identify each eigenmode type. For this reason a dedicated algorithm has been developed to automatically identify the eigenmode type in a batch mode operation.

Field reconstruction

The knowledge of the field components in the vicinity of the particle beam axis is required to determine the shunt impedance R/Q of the eigenmodes. But as is shown in Fig.7 for the TM_{010}, π mode, the specified unsmooth transverse electric field components $\mathrm{E_y}$ off axis ($\mathrm{x} = 5\mathrm{mm}$) for example, which have been directly calculated by complex-valued FEM analysis, are certainly up to four orders of magnitude smaller than the maximum longitudinal field value E_{z0}, but should be continuous and smooth due to the homogeneous vacuum condition inside the cavity. In order to calculate the shunt impedance of the eigenmode with higher precision, the field components can be supplementarily smoothed on a physically motivated basis by using Kirchhoff's integral theorem.

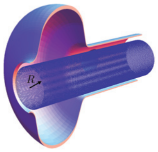

Figure 6: Closed surface ($R = 25\mathrm{mm}$) inside the cavity used for field reconstruction.

According to Kirchhoff's integral theorem the electromagnetic field at an arbitrary position inside a closed surface can be determined once the surface field components are available (Fig.6). Fig.7 indicates a noticeable improvement. The reconstructed field components E_y by using Kirchhoff's integral are much more smooth than the field components from standard FEM analysis, so that the shunt impedance of the eigenmode can be calculated more precisely.

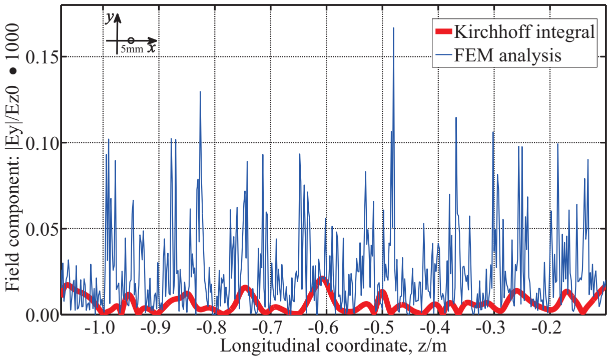

Figure 7: Evaluation of the transverse electric field components E_y off axis ($x = 5\mathrm{mm}$). The field components E_y are normalized to the maximum longitudinal field value E_{z0}. All calculations are performed on symmetrical meshes with 2.997.778 curvilinear tetrahedrons.

Automatic identification of eigenmode type

The first step of the algorithm is to sample the longitudinal components E_z of an eigenmode along a circle in the individual cell (Fig.8). The radius of the circle has to be selected properly to avoid incorrect identification of the eigenmode type because the magnitude of E_z in the vicinity of the particle beam axis for some higher order modes may be too weak. According to the number of zero crossings for the sampled E_z along the circle the mode type can be automatically identified (Fig.9). For example two zero crossings refers to a dipole mode in Fig.9.

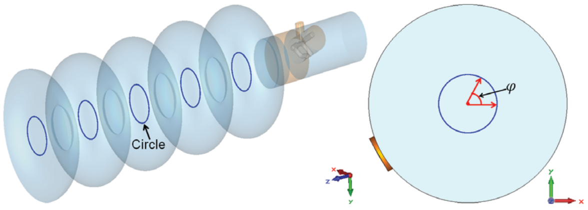

Figure 8: Sample path of the electric field strength $\vec{E}$ along a circle.

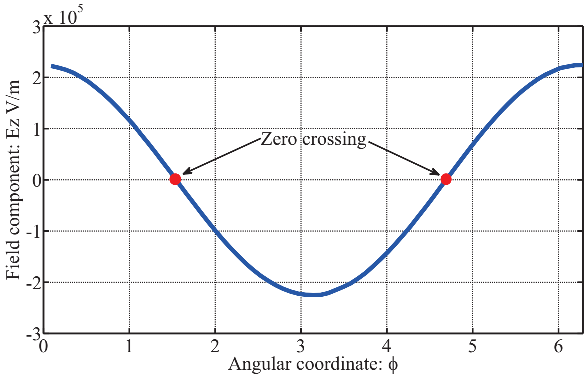

Figure 9: Field components E_z along the specified circular sample path.

CONCLUSION

A robust parallel eigenmode solver on the basis of complex-valued finite element analysis, which utilizes basis function up to the second order on curved tetrahedral elements and port boundary conditions, has been successfully applied to analyze the electromagnetic field inside a lossy TESLA 1.3 GHz accelerating cavity. In order to calculate the shunt impedance of the eigenmodes with higher precision than standard FEM analysis, the electromagnetic field inside the cavity can be smoothed on a physically motivated basis using Kirchhoff's integral theorem. Lastly, some custom-made post-processing routines have been developed to ease the necessary post-processing steps.

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