IOTA – Integrable Optics Test Accelerator at Fermilab

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Collaborative effort

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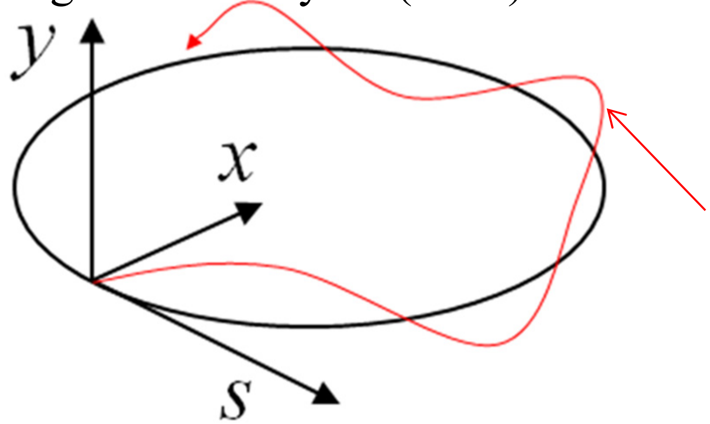
- SNS: V. Danilov
- Budker INP: D. Shatilov
- BNL: H. Witte
- JAI (Oxford)
- Tech X
- Special thanks to E. Forest

- List of posters at IPAC2012:
  - TUPPC090 "Beam Physics of Integrable Optics Test Accelerator at Fermilab"
  - TUEPPB003 "Motion Stability in 2D Nonlinear Accelerator Lattice Integrable in Polar Coordinates"
  - WEPPR012 "Simulating High-Intensity Proton Beams in Nonlinear Lattices with PyORBIT"

Strong Focusing – our standard method since 1952

Christofilos (1949); Courant, Livingston and Snyder (1952)

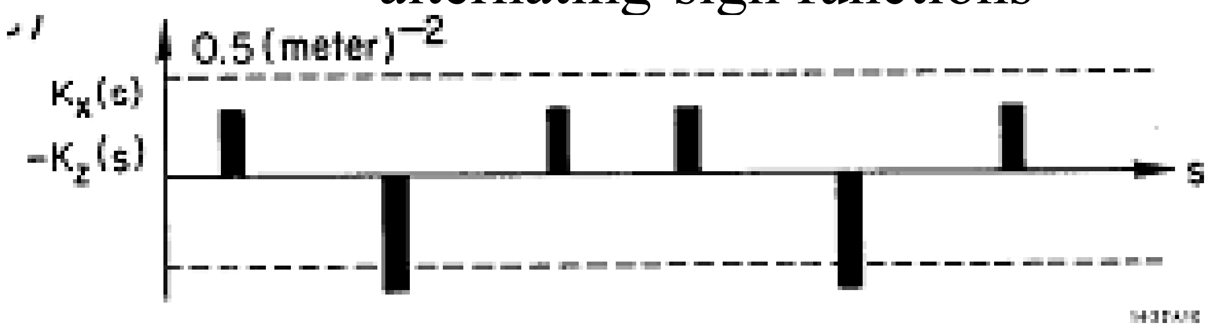


Particle undergoes betatron oscillations

$$\begin{cases} x'' + K_x(s)x = 0 \\ y'' + K_y(s)y = 0 \end{cases}$$

$$K_{x,y}(s + C) = K_{x,y}(s)$$ -- piecewise constant alternating-sign functions

s is "time"



--Magnet lattice and focussing functions in the normal cells of a particular guide field.

Strong focusing

Specifics of accelerator focusing:

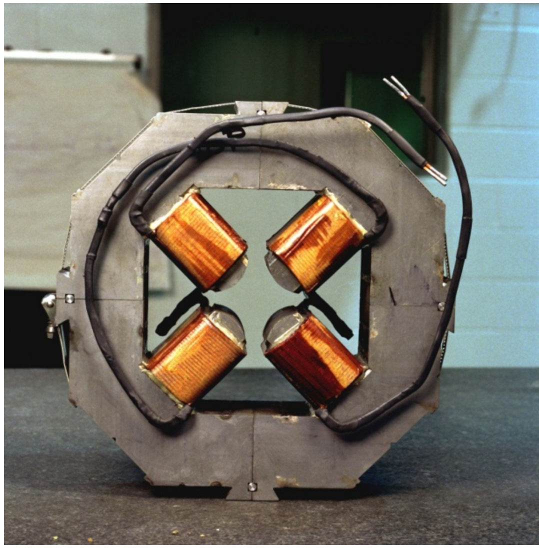
- Focusing fields must satisfy Maxwell equations in vacuum

$$\Delta\varphi(x, y, z) = 0$$

- For stationary fields: focusing in one plane while defocusing in another
  - quadrupole:

$$\varphi(x, y) \propto x^2 - y^2$$

  - However, alternating quadrupoles results in effective focusing in both planes



Courant-Snyder invariant

Equation of motion for betatron oscillations

$$z'' + K(s)z = 0,$$

$$z = x \text{ or } y$$

$$I = \frac{1}{2\beta(s)}\left(z^2 + \left(\frac{\beta'(s)}{2}z - \beta(s)z'\right)^2\right)$$

Invariant (integral) of motion, a conserved qty.

where $\left(\sqrt{\beta}\right)'' + K(s)\sqrt{\beta} = \frac{1}{\sqrt{\beta^3}}$

Linear oscillations

- Normalized variables

$$z_N = \frac{z}{\sqrt{\beta(s)}},$$

$$\psi' = \frac{1}{\beta(s)} \quad \text{-- new time variable}$$

$$p_N = p\sqrt{\beta(s)} - \frac{\beta'(s)z}{2\sqrt{\beta(s)}},$$

- In this variables the motion is a linear oscillator

$$\frac{d^2 z_n}{d\psi^2} + \omega^2 z_n = 0 \qquad I = \frac{1}{2\pi}\oint p_n dz_n$$

- Thus, betatron oscillations are linear; all particles oscillate with the same frequency!

$$H = \omega_x I_x + \omega_y I_y$$

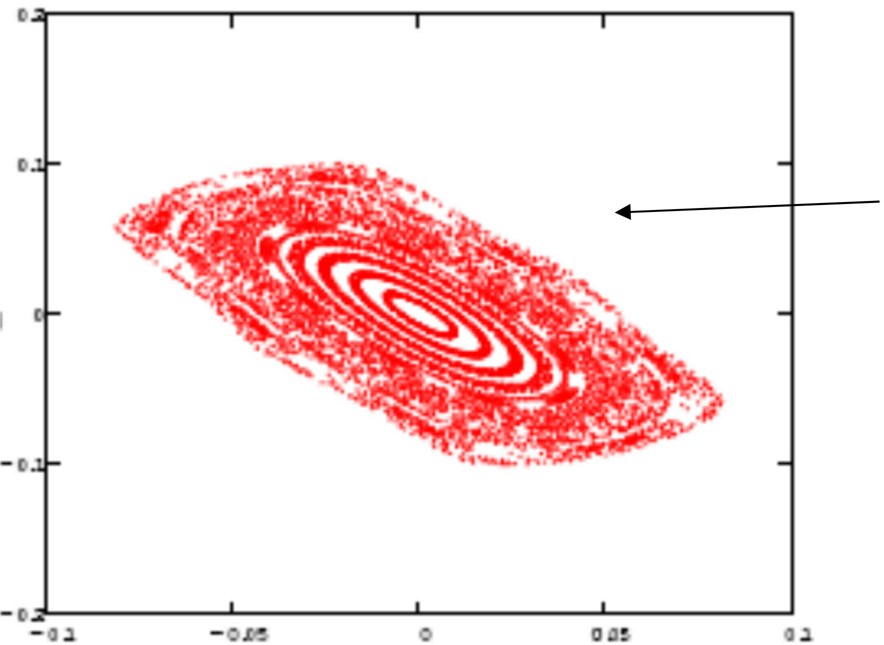
Our goal

- Our goal is to create a practical nonlinear focusing system with a large frequency spread and stable particle motion.

- Benefits:
  - Increased Landau damping
  - Improved stability to perturbations
  - Resonance detuning

Octupoles and tune spread

In all machines there is a trade-off between Landau damping and dynamic aperture (resonances).

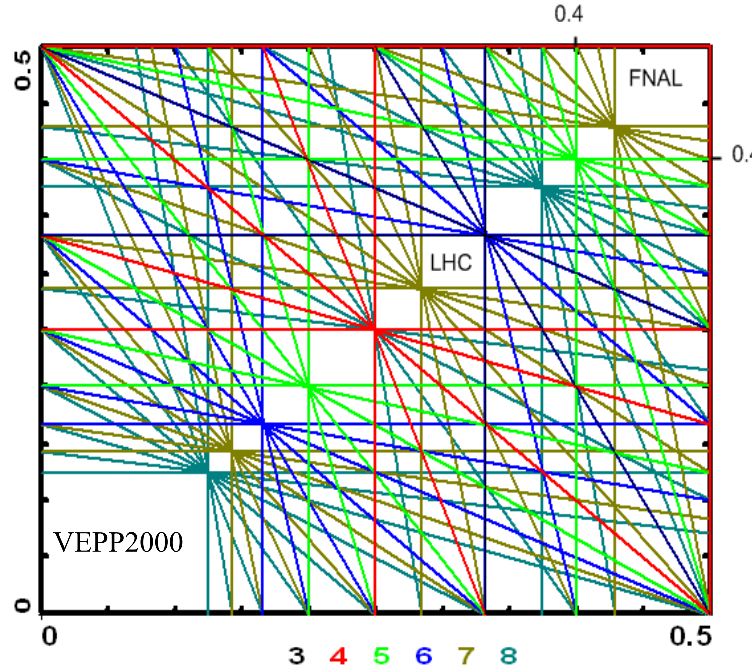


Typical phase space portrait:

1. Regular orbits at small amplitudes
2. Resonant islands + chaos at larger amplitudes;

Linear vs nonlinear

- Accelerators are linear systems by design (freq. is independent of amplitude).
- In accelerators, nonlinearities are unavoidable (SC, beam-beam) and some are useful (Landau damping).
- All nonlinearities (in present rings) lead to resonances and dynamic aperture limits.
- Are there "magic" nonlinearities that create large spread and zero resonance strength?
- The answer is – yes
  (we call them "integrable")



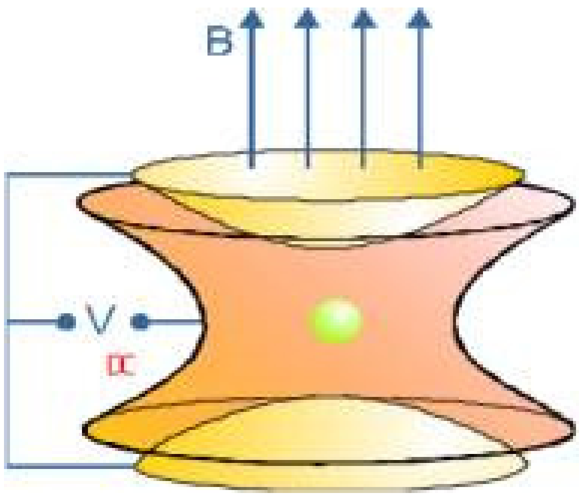
$$k\nu_x + l\nu_y = m$$

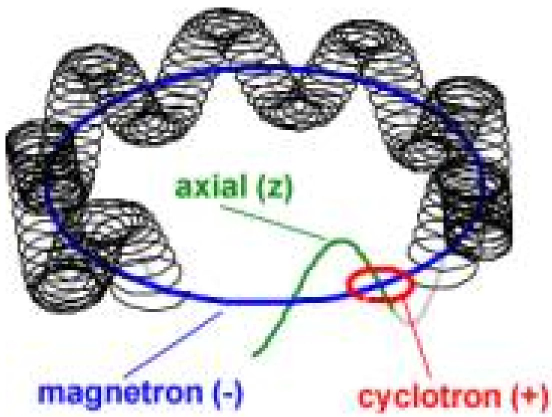
3D: $$H = F(J_1, J_2, J_3)$$

Ideal Penning trap

- The ideal Penning trap is a LINEAR integrable and system
  - It is a linear 3-d oscillator

$$H = \omega_1 J_1 + \omega_2 J_2 + \omega_3 J_3$$





Kepler problem – nonlinear integrable system

- Kepler problem: $$V = -\frac{k}{r}$$
- In spherical coordinates: $$H = -\frac{mk^2}{2\left(J_r + J_\theta + J_\phi\right)^2}$$
- Example of this system: the Solar system

Nonlinear systems can be more stable!

- 1D systems: non-linear oscillations can remain stable under the influence of periodic external force perturbation. Example:

$$\ddot{z} + \omega_0^2 \sin(z) = a \sin(\omega_0 t)$$

- 2D: The resonant conditions

$$k\omega_1(J_1, J_2) + l\omega_2(J_1, J_2) = m$$

are valid only for certain amplitudes.

Nekhoroshev's condition guaranties detuning from resonance and, thus, stability.

Russian Math. Surveys 32:6 (1977), 1–65
From Uspekhi Mat. Nauk 32:6 (1977), 5–66

N. N. Nekhoroshev

AN EXPONENTIAL ESTIMATE OF THE TIME OF STABILITY OF NEARLY-INTEGRABLE HAMILTONIAN SYSTEMS

Example of a "good" nonlinear system

- Suppose that

$$\frac{d^2 z_n}{d\psi^2} + \omega^2 z_n + \alpha z_n^3 = 0, \quad \text{where } z_n \text{ is } x_n \text{ or } y_n$$

- This would be a nonlinear equivalent to strong focusing

- We do NOT know how realize this particular example in practice!

On the way to integrability

1) Colliding beams:

   a) Round beam – angular momentum conservation- 1D motion in *r* (Novosibirsk, 80's, realized at VEPP2000, tune shift around 0.15 achieved);

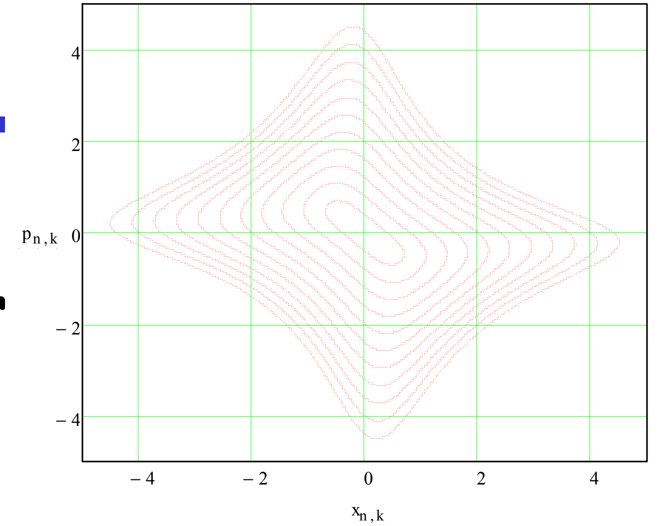
   b) Crab waist - decoupling x and y motion (P. Raimondi (2006), tune shift 0.1 achieved at DAΦNE).

2) Numerical methods to eliminate resonances (e.g. J. Cary and colleagues; D. Robin, W. Wan and colleagues);

3) Exact solutions for realization– our goal. The list is presented in next slides

   Major limiting factor: fields must satisfy Maxwell eqtns.

1-D nonlinear optics



- In 1967 E. McMillan published a paper

SOME THOUGHTS ON STABILITY
IN NONLINEAR PERIODIC FOCUSING SYSTEMS

Edwin M. McMillan

September 5, 1967

- Final report in 1971. This is what later became known as the "McMillan mapping":

$$x_i = p_{i-1}$$
$$p_i = -x_{i-1} + f(x_i)$$

$$f(x) = -\frac{Bx^2 + Dx}{Ax^2 + Bx + C}$$

$$Ax^2p^2 + B\left(x^2p + xp^2\right) + C\left(x^2 + p^2\right) + Dxp = \text{const}$$

If $A = B = 0$ one obtains the Courant-Snyder invariant

- Generalizations (Danilov-Perevedentsev, 1992-1995)

2D case with realistic fields

1. The 1-D McMillan mapping was extended to 2-D round thin lens by R. McLachlan (1993) and by D-P (1995)

Round lenses can be realized only with charge distributions:

a) 1 or 2 thin lenses with radial kicks $f_1(f_2)(r) = \dfrac{ar}{br^2 + c_1(c_2)}$

b) Time dependent potential $\dfrac{1}{\beta} U(\dfrac{r}{\sqrt{\beta}})$.

2. Approximate cases – J. Cary & colleagues - decoupling of x-y motion and use of 1D solutions;

3. Stable integrable motion without space charge in Laplace fields – the only known exact case is IOTA case (Danilov, Nagaitsev, *PRSTAB* 2010);

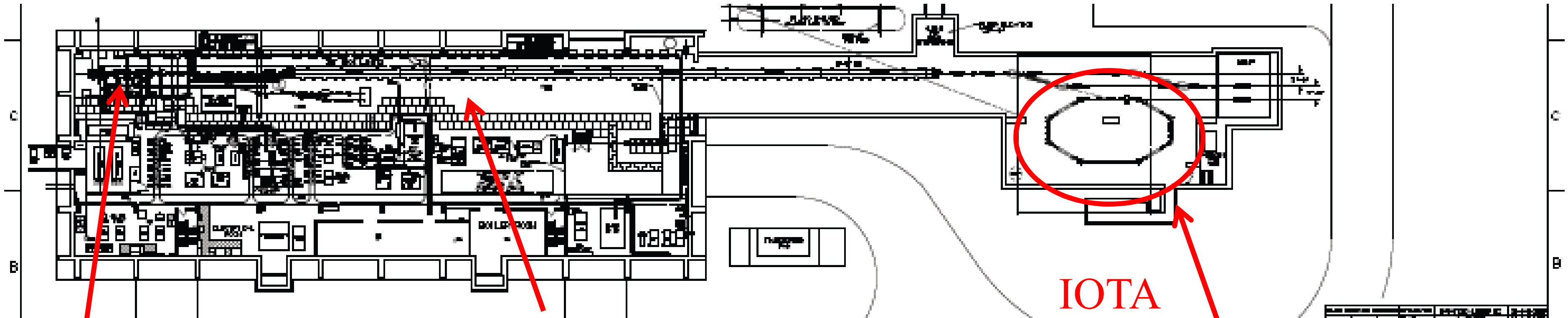
Choice of nonlinear elements

1) For large beam size accumulators and boosters – external fields can produce large frequency spread at beam size amplitudes;

2) Small beam size colliders need nonlinearities on a beam-size scale. The ideal choice is colliding beam fields (like e-lens in Tevatron)

- The IOTA ring can test both variants of nonlinearities.

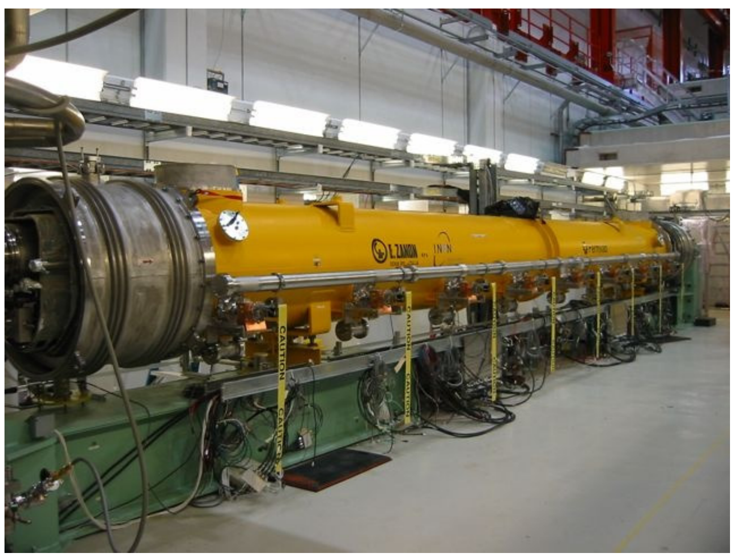
Advanced Superconductive Test Accelerator at Fermilab



IOTA

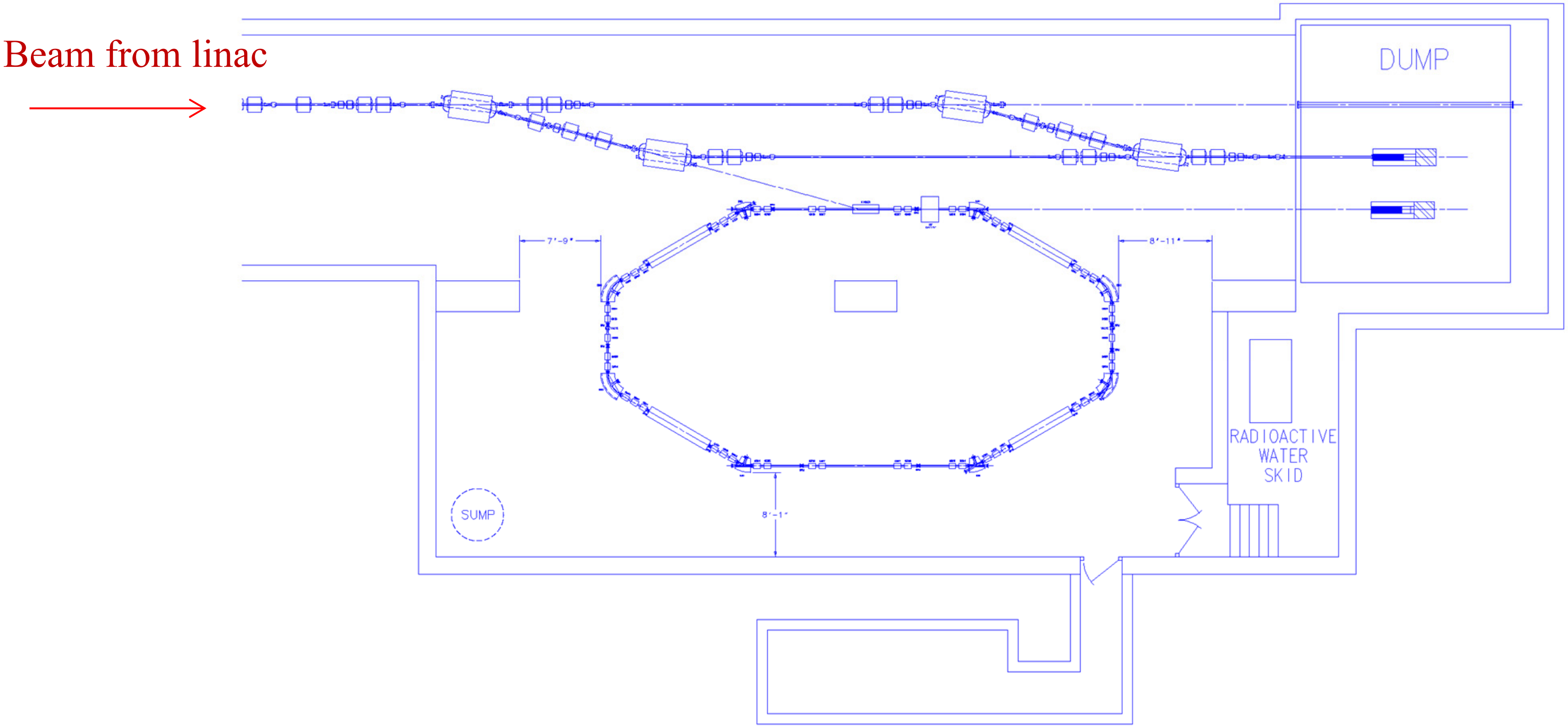
ILC Cryomodules

Photo
injector

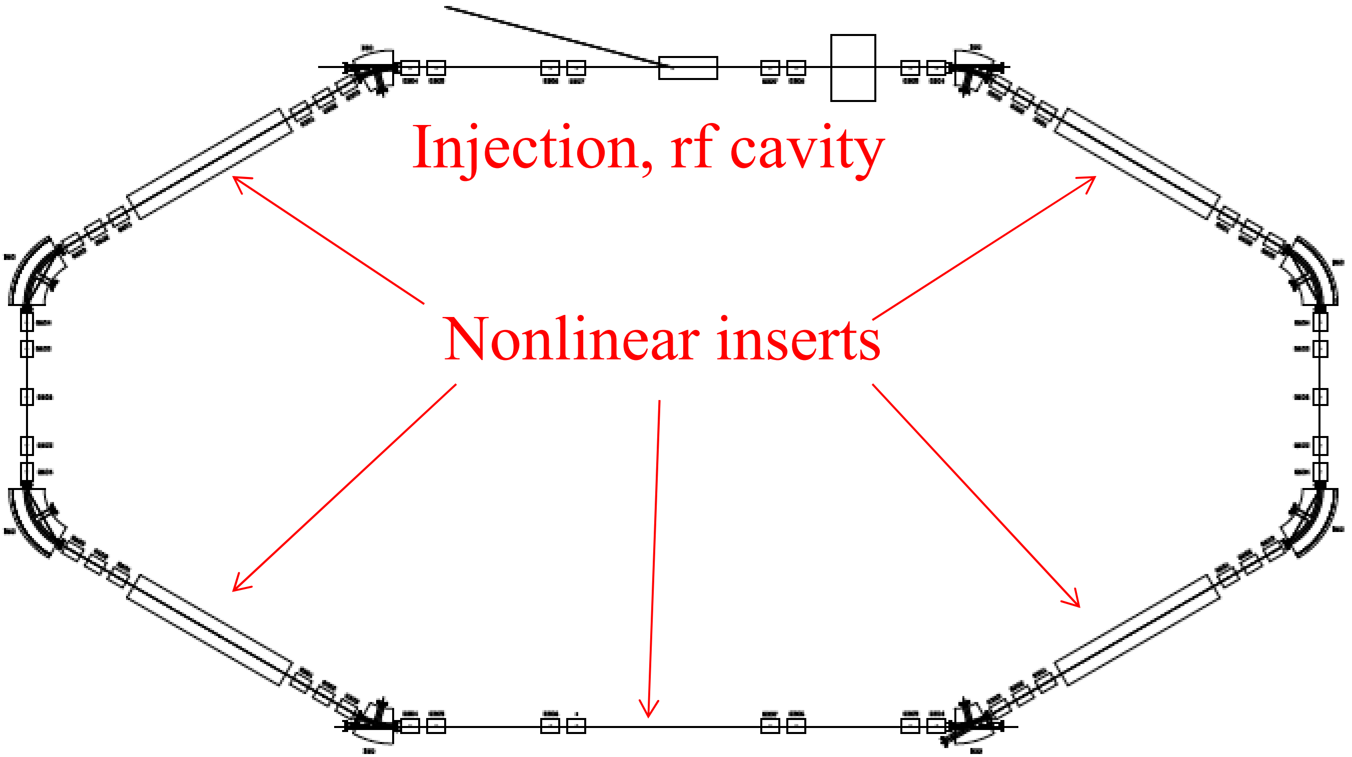


Layout

Beam from linac



IOTA schematic



- pc = 150 MeV, electrons (single bunch, 10^9)
- ~36 m circumference
- 50 quadrupoles, 8 dipoles, 50-mm diam vac chamber
- hor and vert kickers, 16 BPMs

Why electrons?

- Small size (~50 um), pencil beam
- Reasonable damping time (~1 sec)
- No space charge

- In all experiments the electron bunch is kicked transversely to "sample" nonlinearities. We intend to measure the turn-by-turn BPM positions as well as synch light to obtain information about phase space trajectories.

Proposed experiments

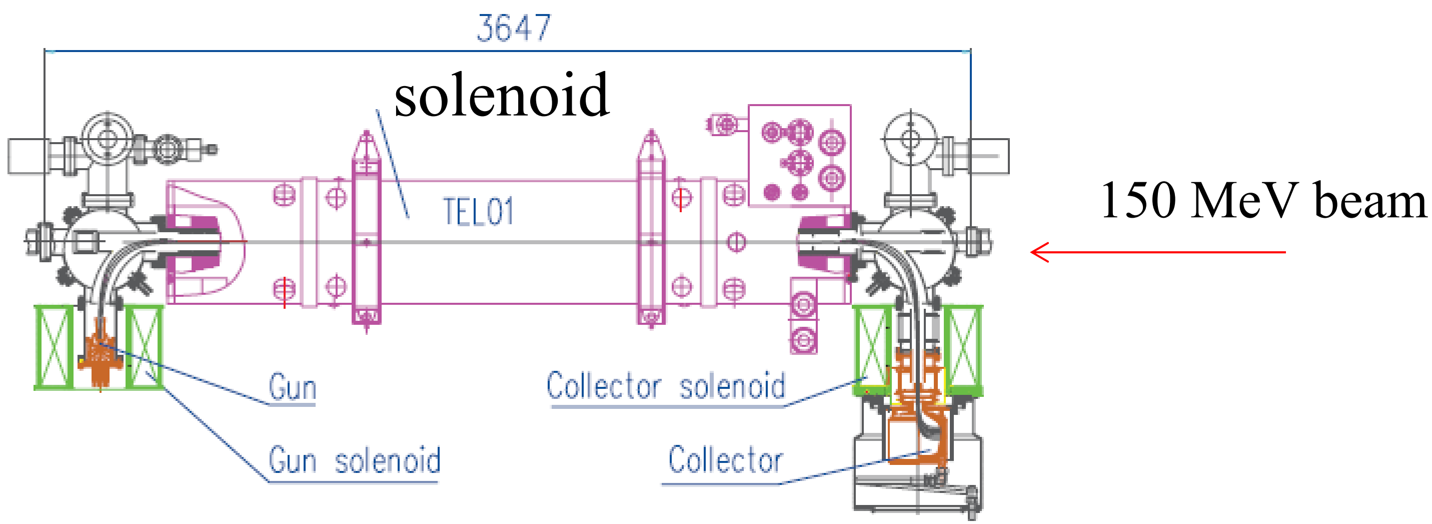
- We are proposing several experiments with nonlinear lenses
  - Based on the electron (charge column) lens

$$\nabla^2 U \neq 0$$

  - Based on electromagnets

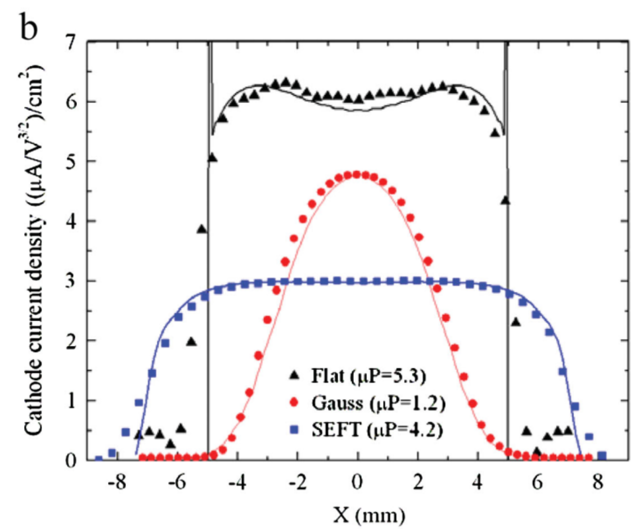
$$\nabla^2 U = 0$$

Experiments with electron lens



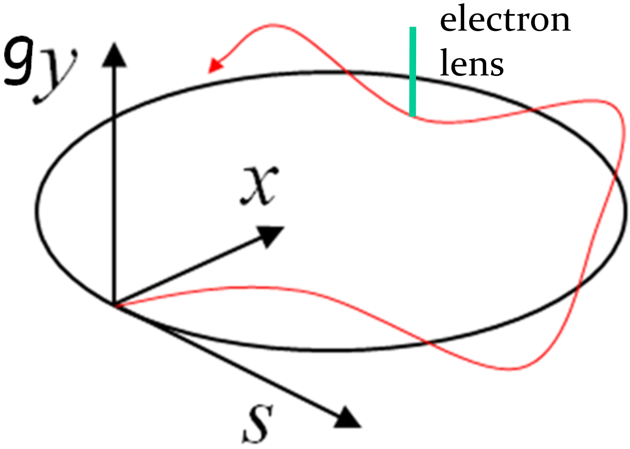
Example: Tevatron electron lens

- For IOTA ring, we would use a 5-kG, ~1-m long solenoid
- Electron beam: ~0.5 A, ~5 keV, ~1 mm radius



Experiment with a thin electron lens

- The system consists of a thin (L ‹ β) nonlinear lens (electron beam) and a linear focusing ring



- Axially-symmetric thin McMillan lens:

$$\theta(r) = \frac{kr}{ar^2 + 1}$$

  - Electron lens with a special density profile

- The ring has the following transfer matrix

$$\begin{pmatrix} cI & sI \\ -sI & cI \end{pmatrix} \begin{pmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix}$$

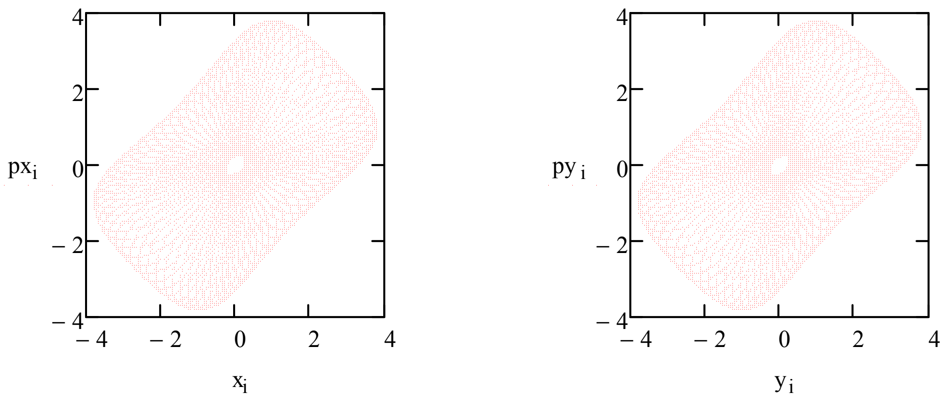
$$c = \cos(\phi)$$

$$s = \sin(\phi)$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Electron lens (McMillan – type)

- The system is integrable. Two integrals of motion (transverse):
  - Angular momentum: $xp_y - yp_x = const$
  - McMillan-type integral, quadratic in momentum



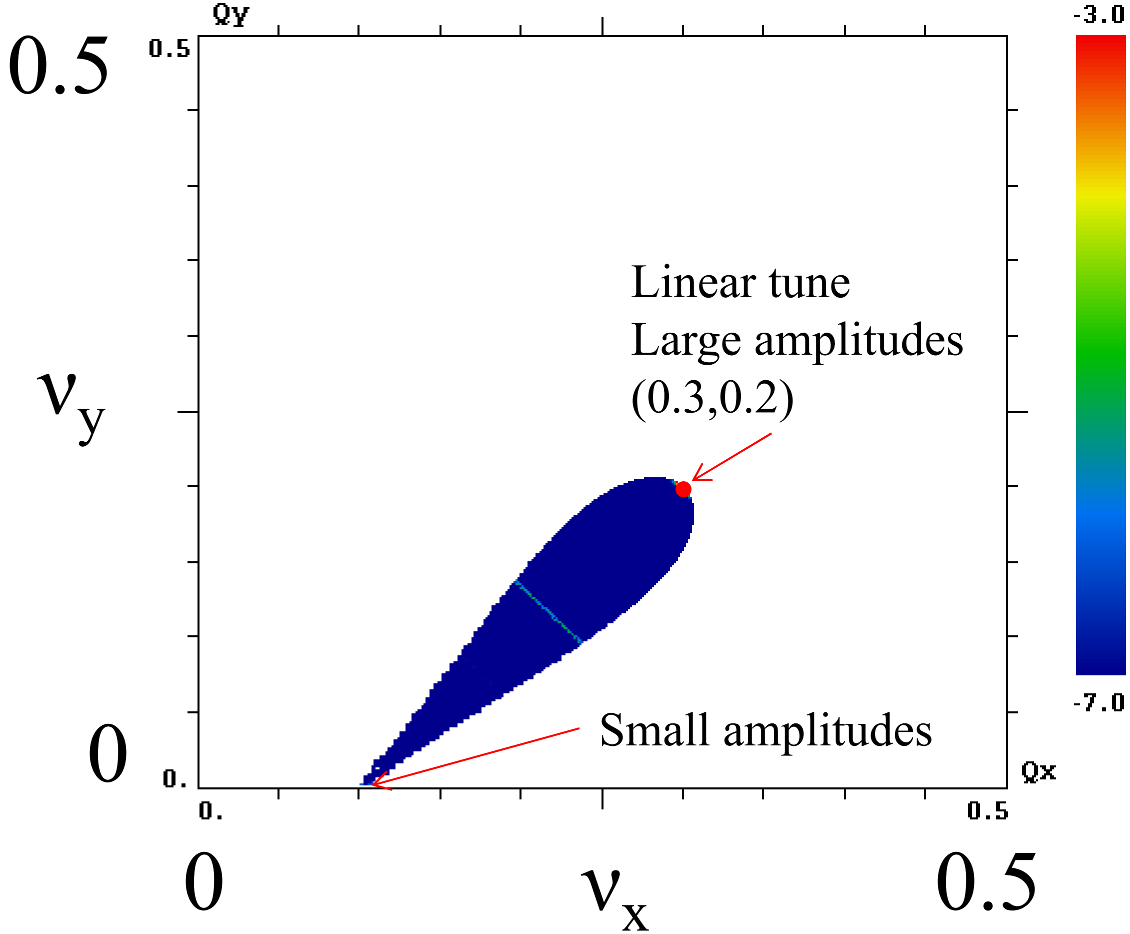
Electron lens current density:

$$n(r) \propto \frac{I}{\left(ar^2+1\right)^2}$$

- For large amplitudes, the fractional tune is 0.25
- For small amplitude, the electron (defocusing) lens can give a tune shift of ~-0.3
- Potentially, can cross an integer resonance

Ideal McMillan round lens

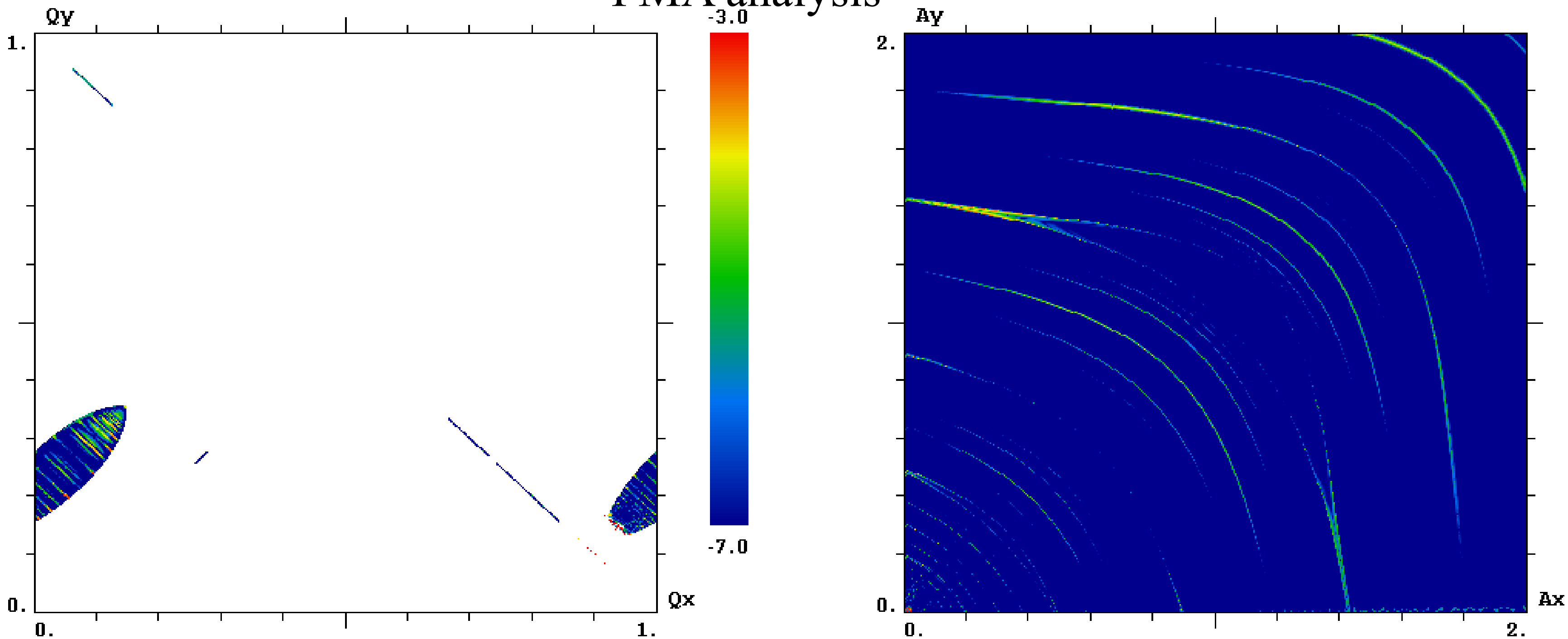
FMA fractional tunes



Practical McMillan round lens

e-lens (1 m long) is represented by 50 thin slises. Electron beam radius is 1 mm. The total lens strength (tune shift) is 0.3

FMA analysis



All excited resonances have the form $k \cdot (\nu_x + \nu_y) = m$

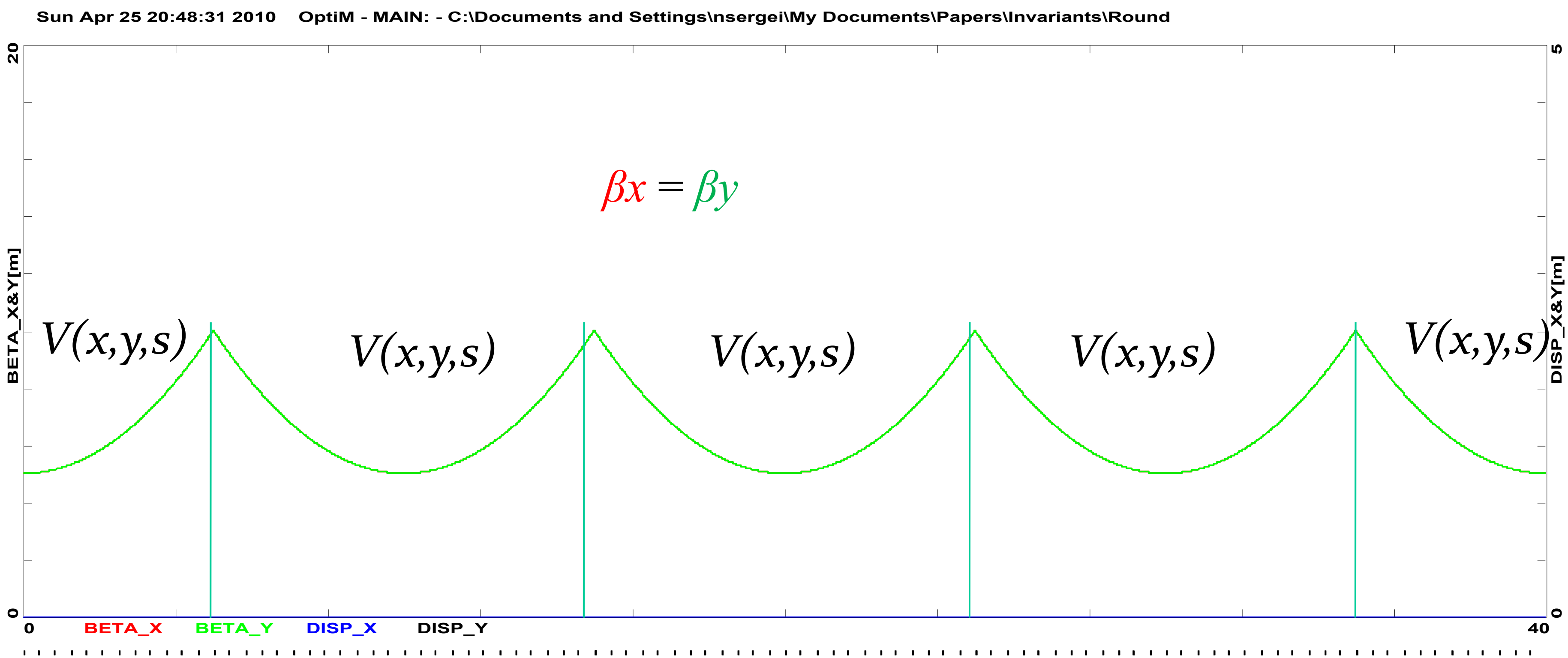
They do not cross each other, so there are no stochastic layers and diffusion

See: Phys. Rev. ST Accel. Beams 13, 084002 (2010)

Start with a round axially-symmetric LINEAR focusing lattice (FOFO)

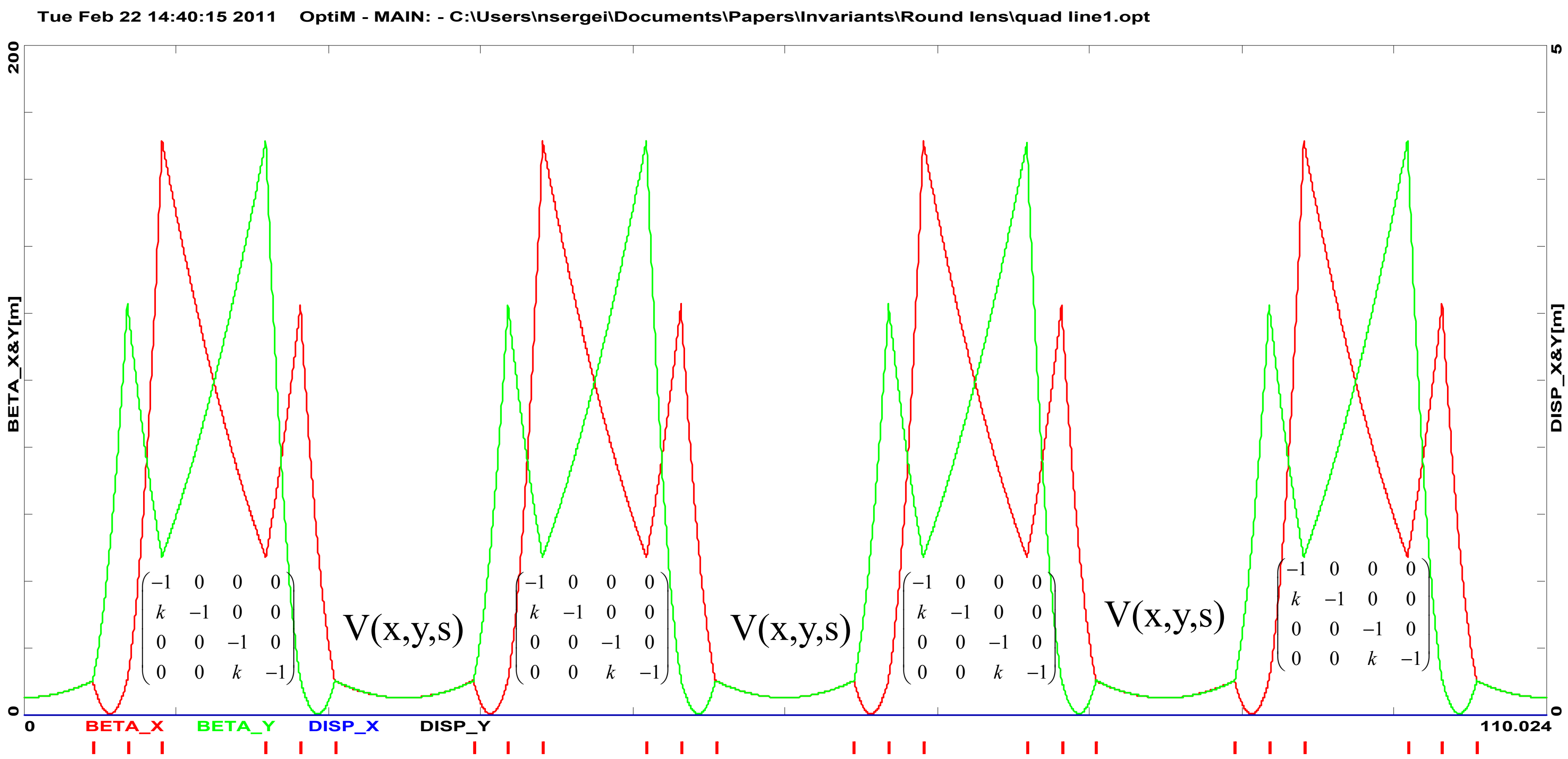
Add special non-linear potential *V(x,y,s)* such that

$$\Delta V(x,y,s) \approx \Delta V(x,y) = 0$$



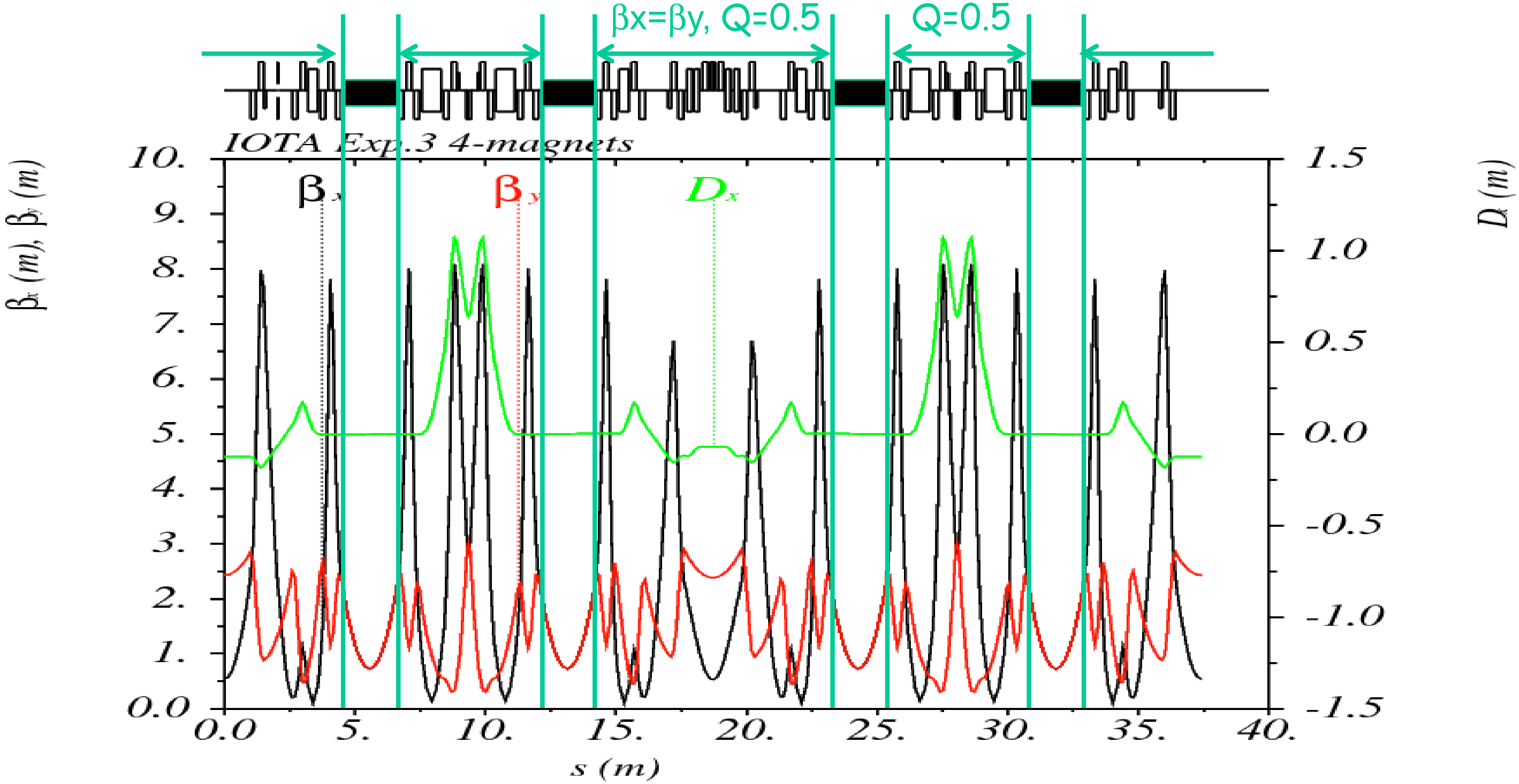
Fake thin lens inserts

Example only!



4-Magnet Lattice (Exp. 3-4)

- Equal beta-functions, $Q_x=5.0+0.3\times4$, $Q_y=4.0+0.3\times4$
- Dispersion=0 in the Nonlinear Magnet
- Maximum Vertical amplitude in the NM=11 mm
- $\alpha=0.015$



Main ideas

1. Start with a time-dependent Hamiltonian:

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + \beta(\psi)V\left(x_N\sqrt{\beta(\psi)}, y_N\sqrt{\beta(\psi)}, s(\psi)\right)$$

2. Chose the potential to be time-independent in new variables

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N)$$

3. Find potentials *U(x, y)* with the second integral of motion and such that $\Delta U(x, y) = 0$

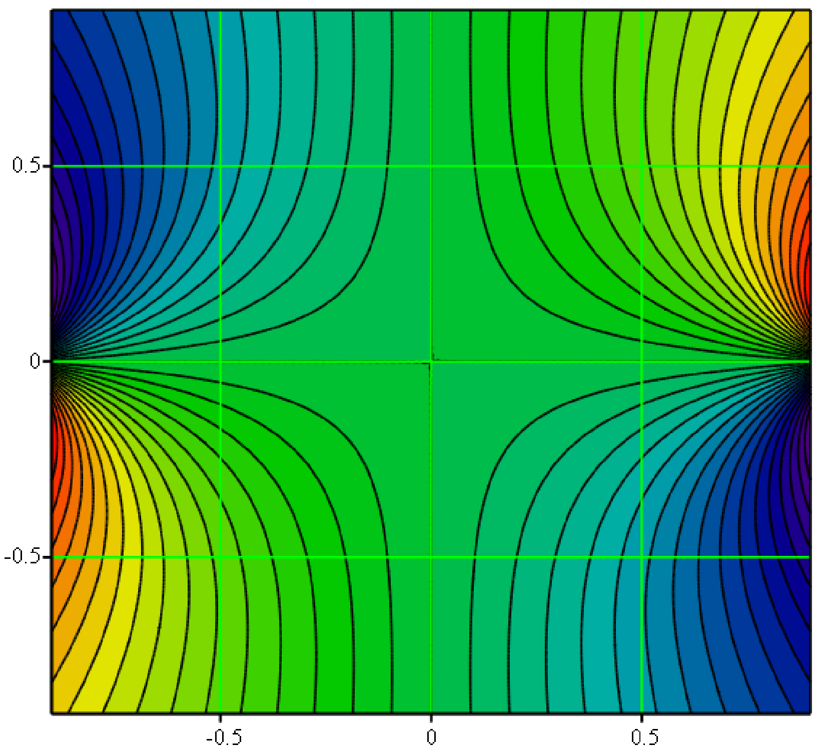
Nonlinear integrable lens

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{x^2 + y^2}{2} + U(x, y)$$

This potential has two adjustable parameters: t – strength and c – location of singularities

Multipole expansion :

For $|z| < c$ $$U(x,y) \approx \frac{t}{c^2} \mathrm{Im}\left((x+iy)^2 + \frac{2}{3c^2}(x+iy)^4 + \frac{8}{15c^4}(x+iy)^6 + \frac{16}{35c^6}(x+iy)^8 + \dots \right)$$



For $c = 1$
$|t| < 0.5$ to provide linear stability for small amplitudes

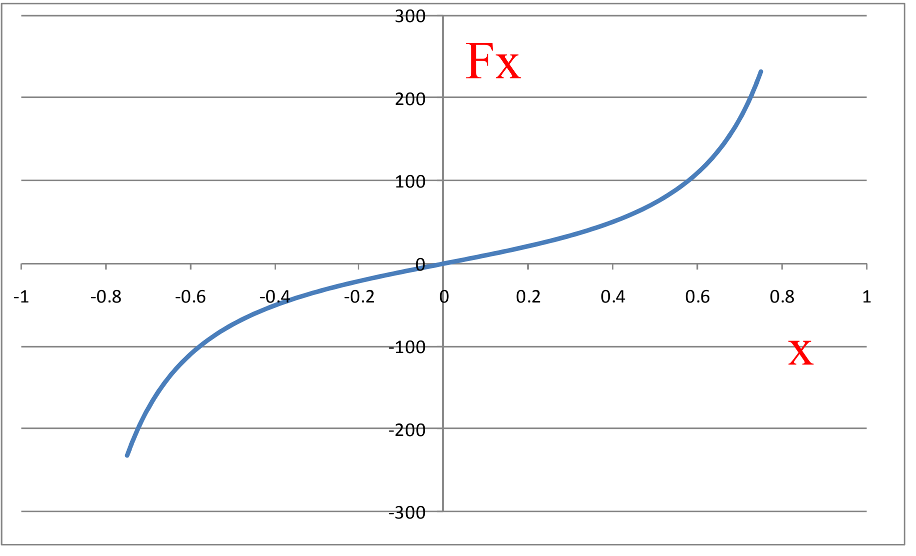
For $t > 0$ adds focusing in x

Small-amplitude tune s:

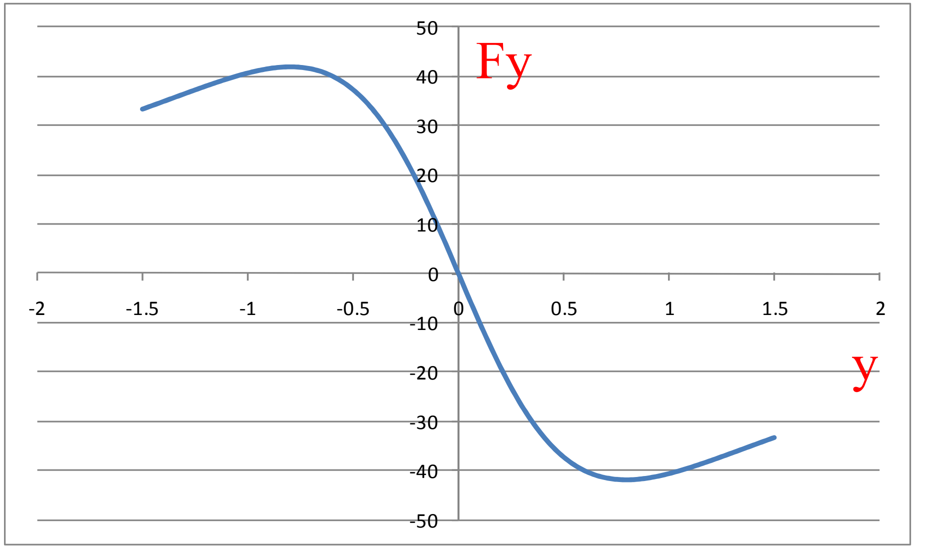
$$\nu_1 = \sqrt{1 + 2t}$$
$$\nu_2 = \sqrt{1 - 2t}$$

Transverse forces

Focusing in x

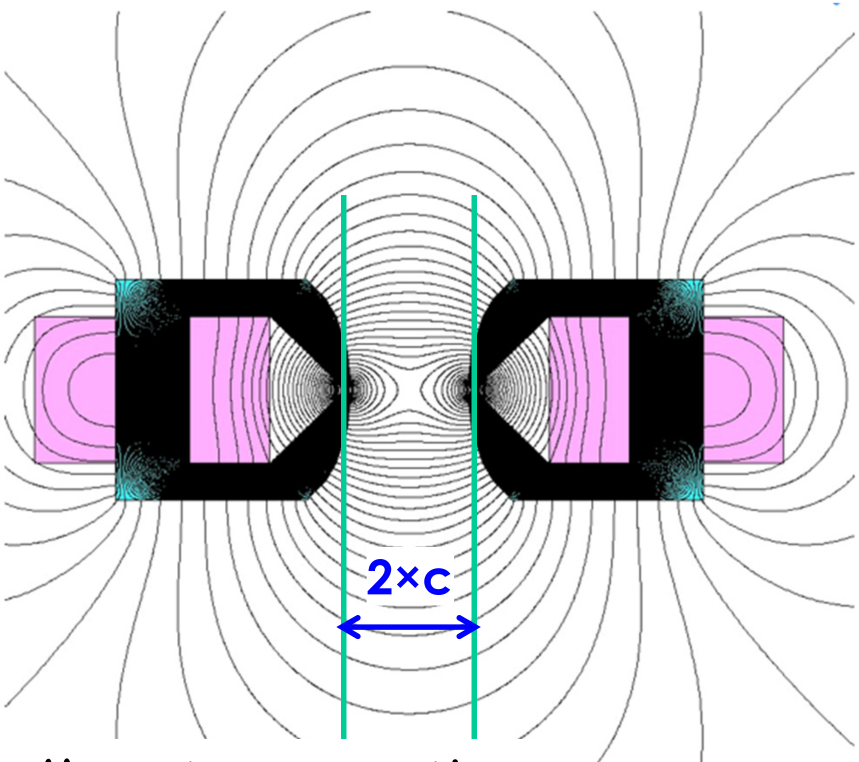


Defocusing in y



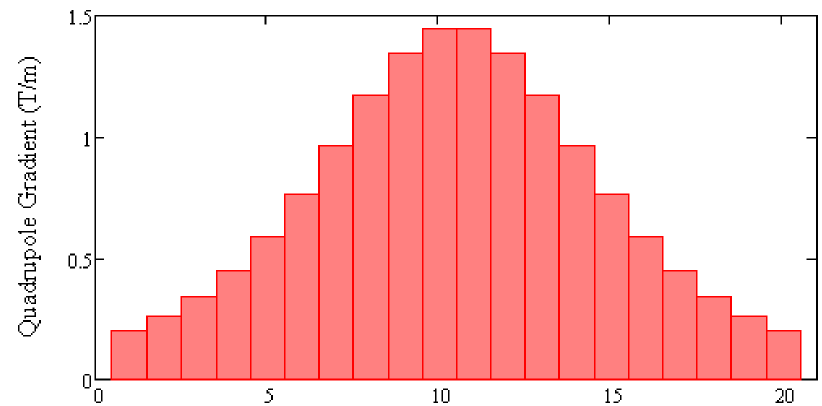
Nonlinear Magnet

- Practical design – approximate continuously-varying potential with constant cross-section short magnets

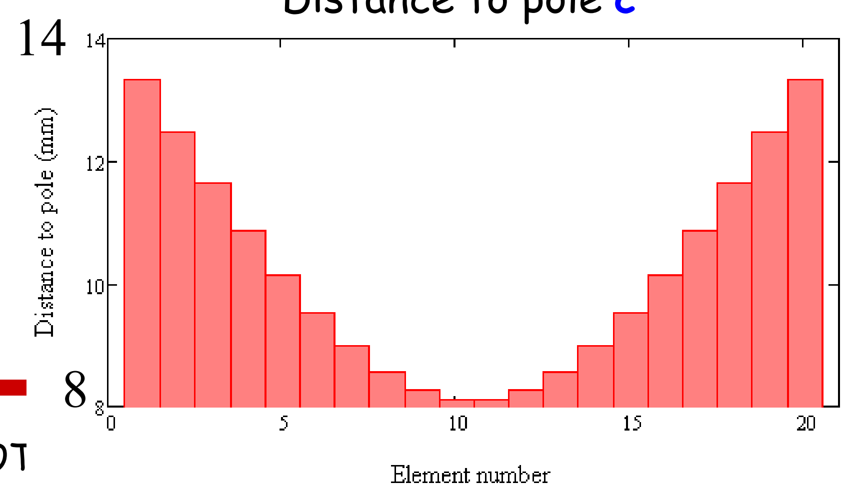


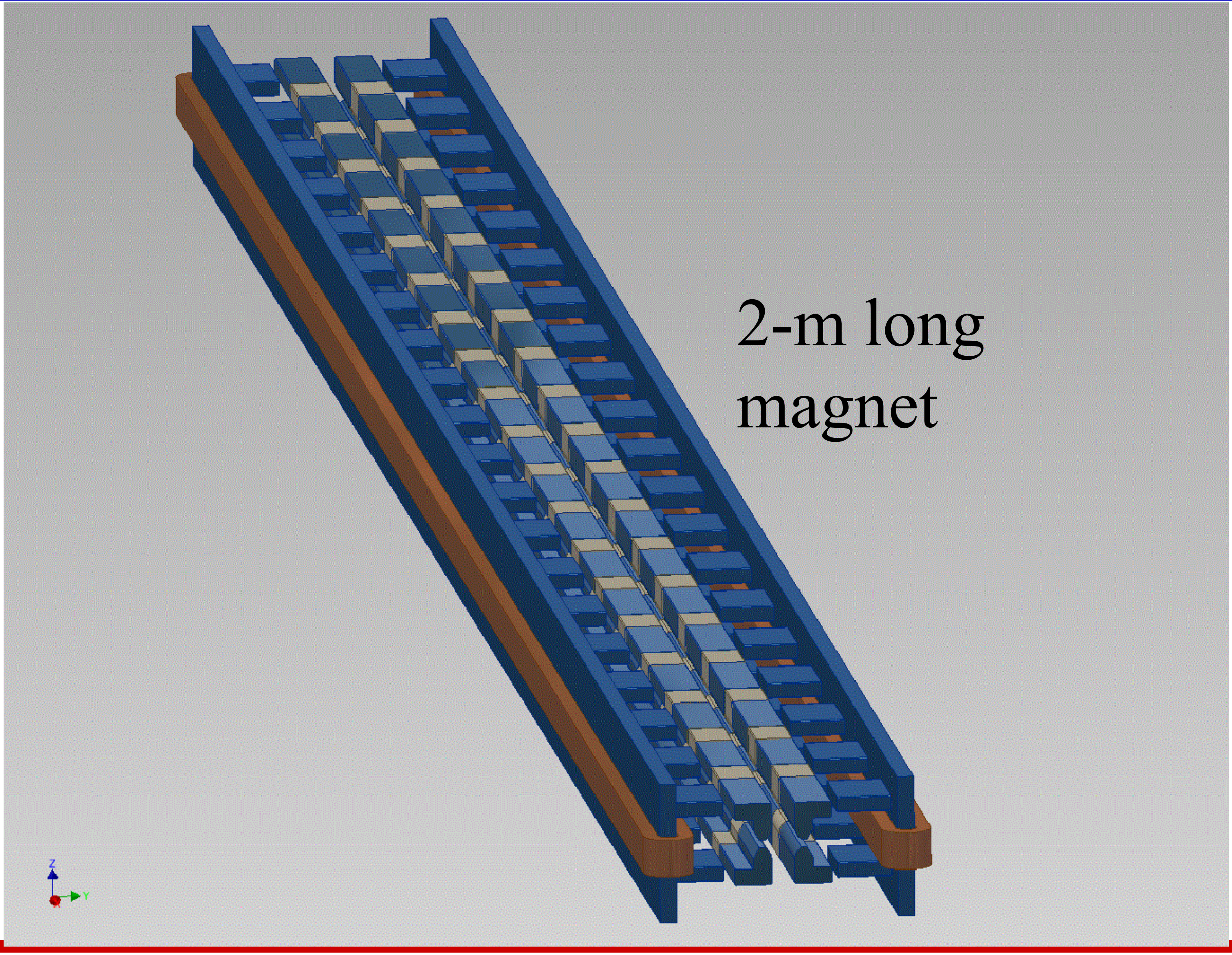
Magnet cross section
V.Kashikhin

Quadrupole component of nonlinear field

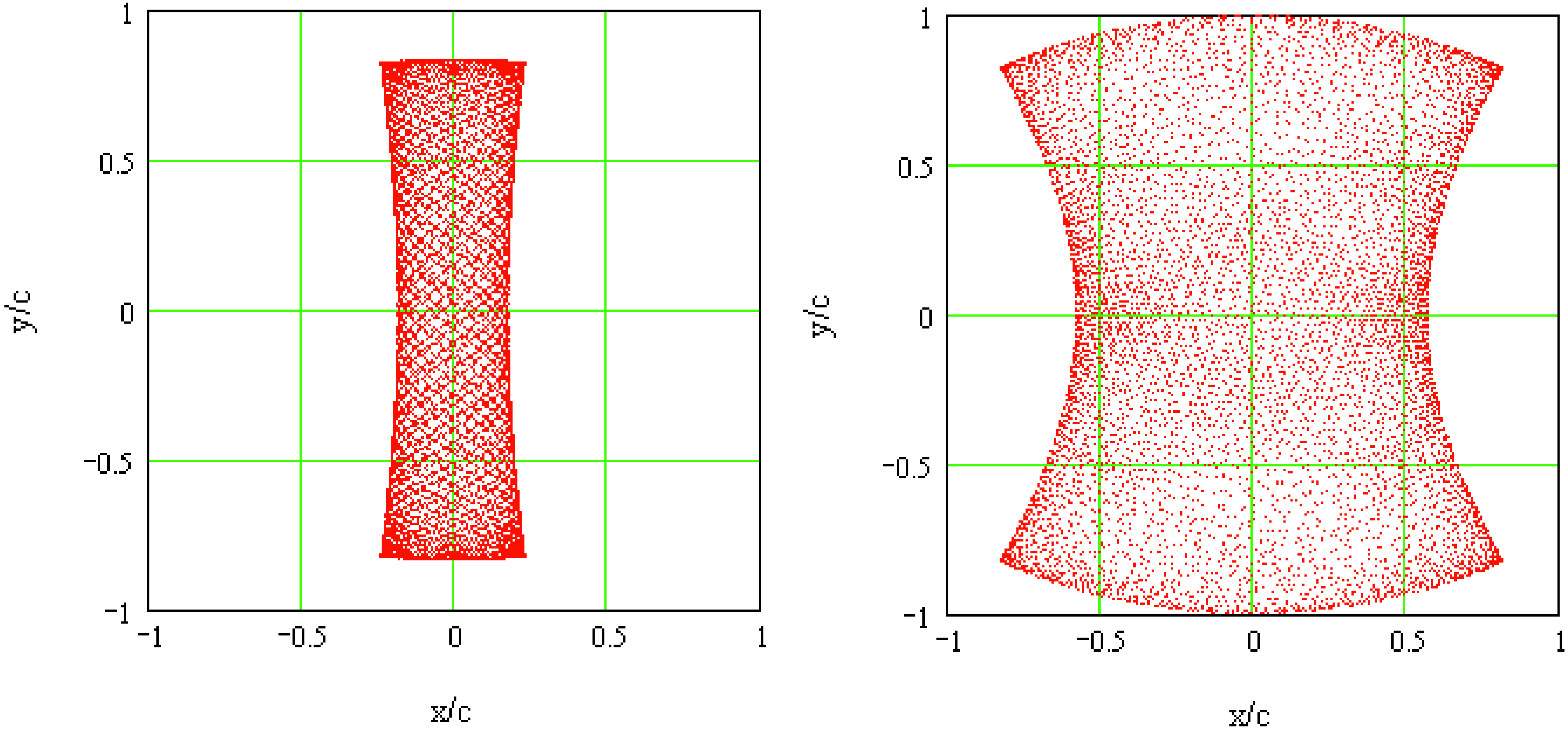


Distance to pole c



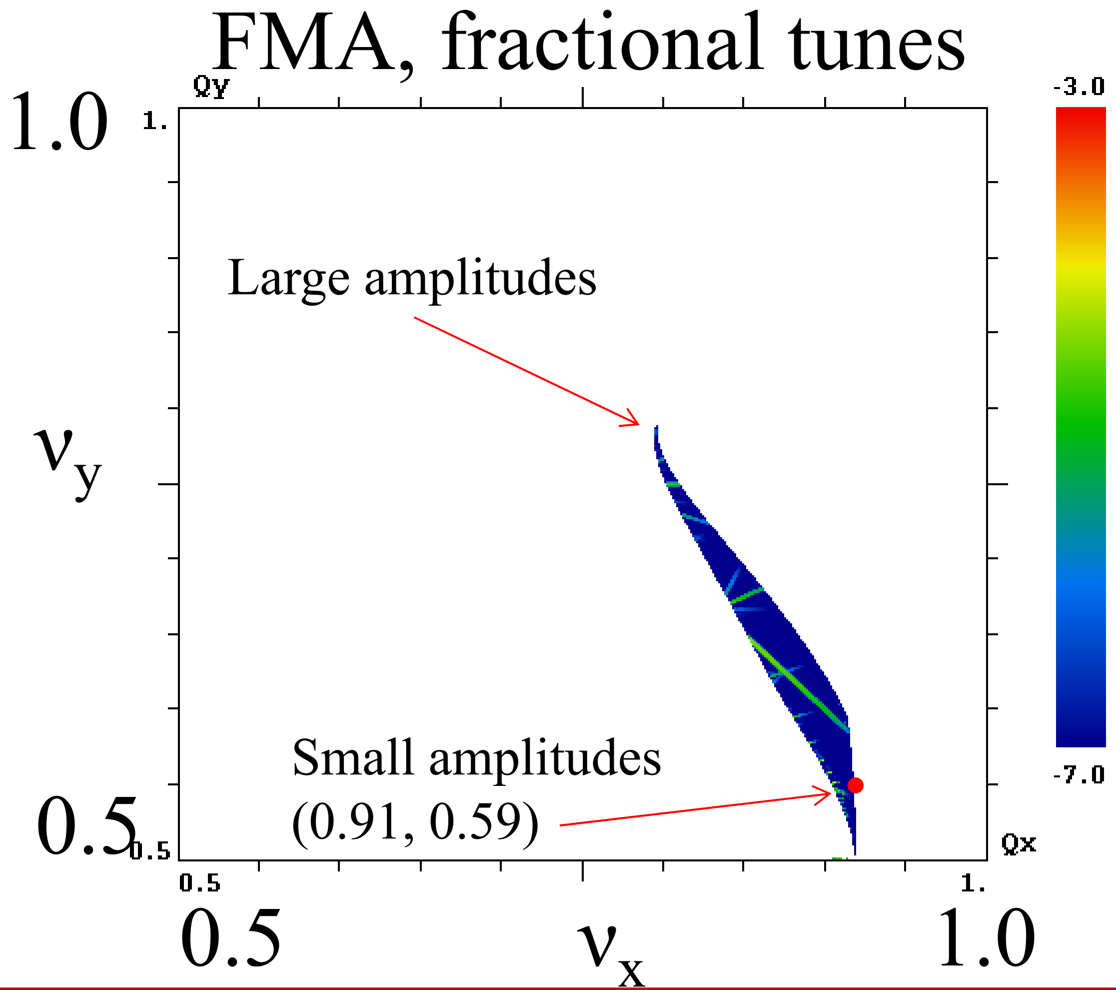

2-m long
magnet

Examples of trajectories



Ideal nonlinear lens

- A single 2-m long nonlinear lens creates a tune spread of ~0.25.



Experimental goals with nonlinear lenses

- ***Overall goal is to demonstrate the possibility of implementing nonlinear integrable optics in a realistic accelerator design***

- Demonstrate a large tune shift of ~1 (with 4 lenses) without degradation of dynamic aperture
  - minimum 0.25

- Quantify effects of a non-ideal lens

- Develop a practical lens design.

Summary

- We have found first (practical) examples of completely integrable non-linear optics.

- We have explored these ideas with modeling and tracking simulations.

- The Integrable Optics Test Accelerator (IOTA) ring is now under construction. Completion expected in 2014.
  - Poster: TUPPC090 "Beam Physics of Integrable Optics Test Accelerator at Fermilab"
- The ring can also accommodate other Advanced Accelerator R&D experiments and/or users
  - Current design accommodates Optical Stochastic Cooling