

THE SIMULATION OF ION CLOUD BUILD-UP IN ELECTRON STORAGE RING*

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Abstract

In this paper, Monte Carlo method was used to study the ion cloud build-up in electron storage ring. Except for electron beams, there are also positive charged ions, which are produced by ionizing collisions between electron beams and residual gas. The produced ions will oscillate around closed orbit under space charge field from ion cloud and electron bunch. Initially, ions maybe trapped by space charge field of electron bunches. With increasing ion density, the space charge field of ion cloud will be stronger and ions may be unstable. Through long term tracking, equilibrium state of ion cloud would be reached, the distributions of ion cloud in transverse phase space is consistent with analytical analysis.

INTRODUCTION

In electron storage ring, positive ions are created by colliding ionization of residual gases. According to linear theory, the ions whose mass large than critical mass should be trapped by beam potential well, and do stable periodic oscillations around the beam orbit. As the number of ions increasing, the oscillation amplitude becomes so large that some ions begin to lose. In the end, the generation and loss of ions will reach dynamic equilibrium state. Then the neutralization factor would be obtained by simulation. At mean time, equilibrium distribution of ion clouds is achieved. A code based on Monte Carlo method was developed to study the ion cloud build-up process. At present time, the study was limited to ion motion in transverse phase space, ignoring the longitudinal drift. The detail of simulation study would be introduced briefly in following sections.

METHOD DESCRIPTION

In the simulation, the average beam size of the whole ring is adopted, and the following assumptions are made.

- The electron bunch is Bi-Gaussian distribution in transverse direction, and it is rigid without loss and deformation.
- Only the transverse movement of ions is considered. When an ion is created, it is stationary, and the initial transverse position is same to parental electron's position selected randomly from the bunch.
- Assuming that ions are only driven by the space charge filed which beam and ion cloud produce, externally applied field is neglected.

- The linear electric field approximation was used in tracking.

Table 1: Nominal HLS parameters

Parameter	Description	Value
E	Beam Energy	800MeV
C	Circumference	66.1308m
I	Electric Current	300mA
h	Harmonic number	45
σ_x	Av.hor.Beam Size	654.717um
σ_y	Av.ver.Beam Size	93.9149um
σ_s	RMS Bunch Length	1.47519cm
R	Vacuum Chamber Radius	0.04m

The Production of Ions-MC Approach

There are mainly two ways to calculate the number of ions generated by collision. One is the mean 'ionization rate' [1][2], the other is MC simulation method. The MC method is most efficient way to study stochastic process, such as residual gas ionization. It is based on an expression for the collision probability below [3]:

$$p_i(t) = 1 - \exp(-\sigma_i n v \Delta t) \quad (1)$$

Where $p_i(t)$ is the probability that beam electrons ionize the residual gas at time t , σ_i is the collisional ionization cross section, n is the volume density of gas, v is the beam electron's speed. For each electron in bunch, a random number $R(R \in [0,1])$ is chosen, if $R < P_i(t)$, then this electron ionizes the residual gas, producing an ion.

Table 2 gives the residual gas of HLS at 800MeV, as well as the corresponding partial pressure, volume density and collisional ionization cross section [1].

Table 2: 800MeV, the residual gas of HLS

Gas species	p(Torr)	n/m ³	σ_i (m ²)
H ₂	6.57×10^{-10}	0.212×10^{14}	2.80×10^{-23}
H ₂ O	1.20×10^{-10}	0.386×10^{13}	1.43×10^{-22}
CO	1.54×10^{-9}	0.495×10^{14}	1.60×10^{-22}
CO ₂	1.66×10^{-10}	0.534×10^{13}	2.53×10^{-22}

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The Mode of Ions

The bunch is Bi-Gaussian distribution in transverse direction, its coulomb potential can be computed numerically or analytically. In linear approximation, the transverse electric field in x and y can be given by [4] [5]

$$\vec{E} = \frac{\lambda_e e}{2\pi\epsilon_0\sigma_{x,y}(\sigma_x + \sigma_y)} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2)$$

Here, λ_e is the line density of electrons in a bunch, ϵ_0 is vacuum permittivity, e is the elementary charge.

If the ion cloud has the same distribution and size with the bunch, the ion cloud's transverse electric field can also be calculated by equation (2), just replacing λ_e with λ_i , which represents the number of ions per length. When ion distribution is different with Gaussian type, the errors introduced by (2) is limited according to others study. λ_i changes over time until it reaches an equilibrium quantity. And the neutralization factor can be calculated by the equation below [1].

$$\eta = \frac{n_i}{n_e} = \frac{\lambda_{ieq}}{I/c} \quad (3)$$

Where n_i is the number of trapped ions around the whole ring and n_e is the number of circulating particles, λ_{ieq} is the equilibrium number of the ions per length.

At HLS, 45 bunches are filled uniformly in the electron storage ring. A bunch and a subsequent bunch gap are grouped as one action cell that an ion experiences. Each ion's transverse position is recorded after the effect of the cell. When a bunch goes by an ion, the average focusing force on ions is

$$\vec{F} = -\frac{(\lambda_e - \lambda_i)e^2}{2\pi\epsilon_0\sigma_{x,y}(\sigma_x + \sigma_y)} \begin{pmatrix} x \\ y \end{pmatrix} \quad (4)$$

During the bunch gap, the average defocusing force is

$$\vec{F} = \frac{\lambda_i e^2}{2\pi\epsilon_0\sigma_{x,y}(\sigma_x + \sigma_y)} \begin{pmatrix} x \\ y \end{pmatrix} \quad (5)$$

The movement of a single ion can be simply described into two modes. Uniformly retarded motion when the bunch goes by, and uniformly accelerated motion during the bunch gap. One cell's the transfer matrix of an ion can be given by the equation below.

$$M_{x,y} = \begin{bmatrix} 1 + \alpha l_g & \frac{l_g}{c} \\ 2\alpha c & 1 \end{bmatrix} \begin{bmatrix} 1 - \beta\sigma_s & \frac{\sigma_s}{c} \\ -2\beta c & 1 \end{bmatrix} \quad (6)$$

With $\alpha = \frac{r_p \times \lambda_i \times l_g}{A \times \sigma_{x,y} \times (\sigma_x + \sigma_y)}$, $\beta = \frac{r_p \times (\lambda_e - \lambda_i) \times \sigma_s}{A \times \sigma_{x,y} \times (\sigma_x + \sigma_y)}$

Where l_g is the bunch gap, r_p is the classical proton radius, A is the mass number of ion.

COMPUTER SIMULATION

The Motion of a Single Ion

To study the motion of a single type ion, H_2^+ is taken as the representative. The phase space and x-t of H_2^+ are given respectively in the following two figures. In Fig. 1, the point A is the initial state, the point B is the final state.

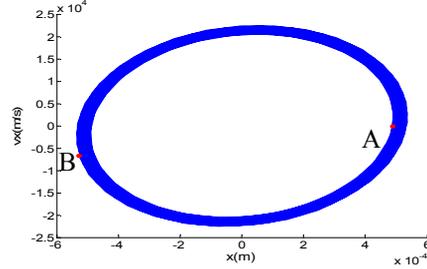


Figure 1: The phase space of a single H_2^+ in 20 laps.

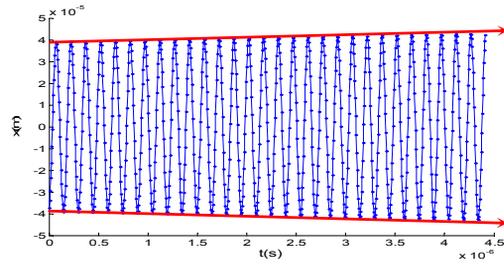


Figure 2: Transverse x vs. time of a single H_2^+ in 20 laps.

From the above two pictures, we know the ion begins to do stable periodic oscillations around the beam orbit when it was trapped by beam potential well, and the oscillation amplitude will become larger, then the ion will be missing sooner or later.

Figure 2 shows H_2^+ complete a whole oscillation every 28 cells, so the oscillation frequency in simulation is

$$f_x = \frac{28 \times C}{h \times c} = 7.2857 \times 10^6 \text{ Hz} \quad (7)$$

The Neutralization Factor

It is straightforward to extend to more complicate situation. Next, four type of residual gas are considered, which was shown in Table 2. Each ion's motion state was tracked and the number of the total ions is shown in Fig. 3.

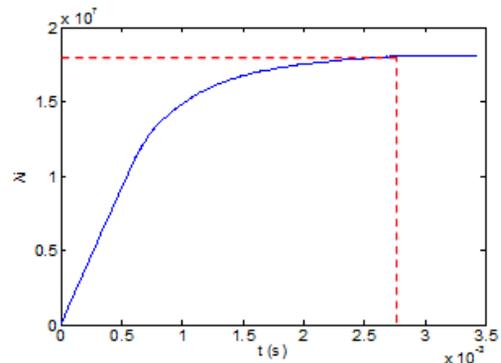


Figure 3: The number of the total ions per length vs. time.

Analyzing data, even in the absence of cleaning devices, the total number of ions will not exceed a particular value. At about 2.75×10^{-3} s, the generation and loss of ions have reached the dynamic equilibrium, and after that the number of ions is maintained at about 1.8098×10^7 (CO_2^+ :75.92%, CO_2^+ :20.27%, H_2^+ :0.4%, H_2O^+ : 3.4%). The neutralization factor of the ring is

$$\eta \approx 0.29\% \tag{8}$$

The Equilibrium Distribution of the Ions

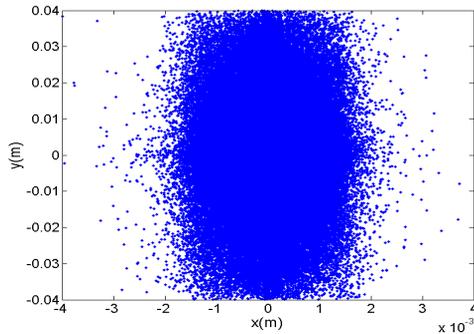


Figure 4: The transverse distribution of ions in the beam pipe.

Figure 4 gives the real transverse distribution of ions in the beam pipe. As can be seen, ions spread over in y direction, but relatively concentrate in x direction. The reason is $\sigma_y < \sigma_x$, so the force acted on the ion in y direction is bigger than x direction. Performing in the trajectory of the ion, the moving range in y direction is larger than x direction. Figure 5 gives a brief explanation of the transverse distribution.

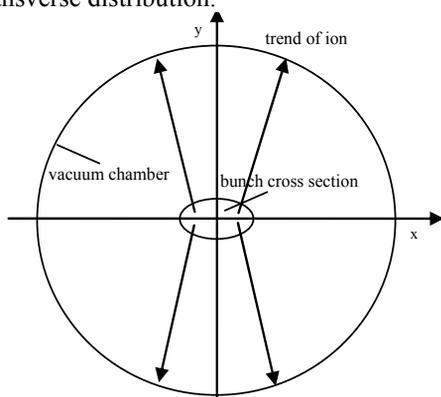


Figure 5: the theory diagram of transverse distribution.

In Figure 5, the inside ellipse represents the cross section of a bunch, the outside circle represents the cross section of the vacuum chamber, the initial point of the arrow gives the initial transverse position of a ion, the arrow shows the ultimate trend of the ion, the envelope formed by the four arrows shows a view of transverse distribution of the ions roughly. It can be seen the simulation result is consistent with the theoretical analysis.

The Statistical Distribution of the Ions

The following two figures show the transverse statistical distribution of ions, and the statistical results were fitted at the same time.

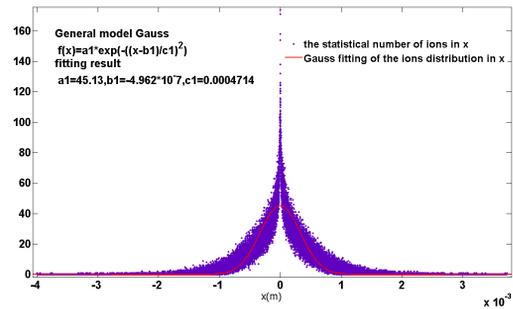


Figure 6: The statistical distribution of ions in x.

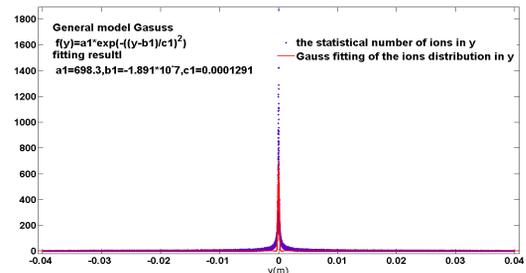


Figure 7: The statistical distribution of ions in y.

If Gaussian distribution is used to describe ion cloud, the fitting RMS size of ion cloud is (333.3um,91.287um), which is different from the size of an electron bunch (654.717um,93.9149um). Checking the detail of simulation results, it is not surprising that the ion's transverse distribution is different than Gaussian type, as presented in other's theoretical analysis.

SUMMARY AND OUTLOOK

Monte Carlo method was used to simulation the build-up process of ion cloud in electron ring. Up to now, two assumptions are limiting the performance of developed code, linear electric field approximation and rigid electron beam assumption. A code including strong-strong interaction between electron bunch and ion cloud and full space charge force calculation is under developing. And the beam performance with influence of ion cloud would be obtained accurately.

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