

# OPTIMIZATION OF DRIVE-BUNCH CURRENT PROFILE FOR ENHANCED TRANSFORMER RATIO IN BEAM-DRIVEN ACCELERATION TECHNIQUES\*

F. Lemery<sup>1</sup>, D. Mihalcea<sup>1</sup>, and P. Piot<sup>1,2</sup>

<sup>1</sup> Department of Physics and Northern Illinois Center for Accelerator & Detector Development, Northern Illinois University DeKalb, IL 60115, USA

<sup>2</sup> Accelerator Physics Center, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

## Abstract

In recent years, wakefield acceleration has gained attention due to its high acceleration gradients and cost effectiveness. In beam-driven wakefield acceleration, a critical parameter to optimize is the transformer ratio. It has been shown that current shaping of electron beams allows for enhanced ( $> 2$ ) transformer ratios. In this paper we present the optimization of the pulse shape of the drive bunch for dielectric-wakefield acceleration.

## INTRODUCTION

In collinear beam-driven acceleration techniques, a “drive” electron bunch with suitable parameters propagating through a high-impedance structure or plasma medium induces an electromagnetic wake. A following “witness” electron bunch, properly delayed, can be accelerated by these wakefields. Collinear beam-driven acceleration techniques have demonstrated accelerating fields in excess of GV/m [1, 2]. The fundamental wakefield theorem [3] limits the transformer ratio – the maximum accelerating wakefield  $E_+$  over the decelerating field  $E_-$  experienced by the driving bunch – to  $\mathcal{R} \equiv |E_+/E_-| \leq 2$  for bunches with symmetric current profiles. Tailored bunches with asymmetric, e.g. linearly-ramped, current profiles can lead to  $\mathcal{R} > 2$  [4]. Achieving large transformer ratios is beneficial for beam-driven acceleration as it enables longer interaction times and increases the overall efficiency of the method; large values of  $\mathcal{R}$  however, compromise large values of  $E_+$ .

Although appealing, enhancing the transformer ratio by shaping the bunch current profile has never been attempted because of the lack of feasible shaping methods; instead, the transformer ratio was enhanced using the ramped-bunch-train technique [5, 6, 7]. Over the last few years, techniques to shape the bunch on timescales below 1 picosecond have emerged; allowing new possibilities for transformer-ratio enhancement [8, 9, 10].

To understand the trade-off between the peak accelerating field and transformer ratio, we explore several current profiles. In order to quantify the performance of the

\* This work was sponsored by the DTRA award HDTRA1-10-1-0051 to Northern Illinois University, and by the DOE contract DE-AC02-07CH11359 to the Fermi research alliance LLC.

numerically-generated current profiles to enhance beam-driven acceleration techniques, we consider a drive bunch injected in a cylindrical-symmetric dielectric-lined waveguide (DLW) [11]. The DLW consists of a hollow dielectric cylinder with inner and outer radii  $a$  and  $b$ . The cylinder is taken to be diamond (relative electric permittivity  $\epsilon_r = 5.7$ ); and its outer surface is contacted with a perfect conductor; see Fig. 1.

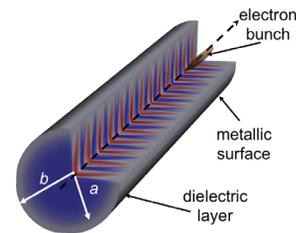


Figure 1: Geometry of the dielectric-wakefield acceleration investigated in this paper. The electron bunch passes through a dielectric cylinder producing a wakefield in the vacuum region ( $r < a$ ). The dielectric material is located in the region  $\delta = b - a$ . The cylinder’s outer surface is also coated with a conducting layer.

## CURRENT PROFILES

The maximization of  $\mathcal{R}$  has been well studied and can be achieved by making the decelerating field constant over the drive bunch as discussed in Ref. [4]. In contrast, the maximization of  $E_+$  can be achieved by maximizing the peak current. This however, produces a correspondingly large  $E_-$  which effectively reduces  $\mathcal{R}$ . We therefore explore the relationship between  $E_+$  and  $\mathcal{R}$  for different current shapes. The six shapes considered are as follows:

- Gaussian distribution:  $g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$ , where  $\mu$  and  $\sigma$  are the mean and root-mean-square (rms) length,
- Linearly-ramped:  $g(z) = mz + b$ , with  $m$  and  $b$  as parameters,
- Fourier distribution:  $g(z) = \sum_{n=1}^5 b_n \sin(nz)$  where  $b_n$ ’s are free parameters guided by the saw-tooth wave,

- Double triangle distribution [12] as a combination of two linearly-ramped distribution parametrized by  $l_n$  and  $h_n$ , the horizontal and vertical position of the  $n$ -th vertice respectively.
- Exponential distribution:  $g(z) = e^{-\alpha z}$  for  $z \in [0, L]$  with  $\alpha$  and  $L$  as parameters,
- Skewed gaussian distribution:  $g(z) = \frac{2}{\omega} \phi\left(\frac{z-\xi}{\omega}\right) \Phi\left(\alpha\left(\frac{z-\xi}{\omega}\right)\right)$ , where  $\xi$  is a shift,  $\omega$  the characteristic length, and  $\alpha$  the skew,  $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ , and  $\Phi(x) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$ ,
- “Realistic” current achievable with longitudinal-phase-space manipulation [10]  $g(z) = \int d\tilde{z} I_f^u(\tilde{z}) \exp\left[-\frac{(z-\tilde{z})^2}{2\sigma_u^2}\right]$ , where  $I_f^u(z) = \frac{\hat{I}_0}{\Delta^{1/2}(z)} \exp\left[-\frac{(a_f + \Delta^{1/2}(z_f))^2}{8b_f^2\sigma_{z,0}^2}\right] \times \Theta[\Delta(z)]$ , with  $\Delta(z) \equiv a_f^2 + 4b_f z$  and  $\Theta(\cdot)$  is the Heaviside function. The final current shape is therefore controlled via the parameters  $a_f$  and  $b_f$ . Here we take  $\sigma_u = 0.05$ .

Examples of shapes associated to these functions are displayed in Fig. 2. Once a set of parameters is selected, the corresponding distribution is normalized to i.e. unity as  $\bar{g}(z) = \frac{g(z)}{\int_{-\infty}^{+\infty} g(z) dz}$ . The bunch charge is then set to  $Q = 1$  nC to yield the current profile  $I(z) = Qc\bar{g}(z)$  where  $c$  is the velocity of light.

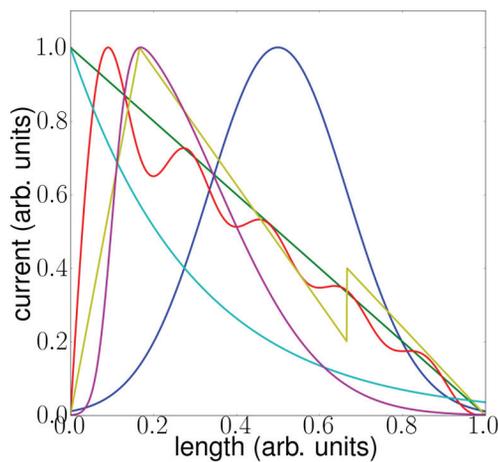


Figure 2: Shapes considered in our study: Gaussian (blue), linearly-ramped (green), exponential (cyan), Fourier (red), double triangle (yellow), and skewed gaussian (magenta). The axis are normalized to unity for clarity, e.g., the traces shown correspond to  $g[z/\max(z)]/\max[g(z)]$ .

## SIMULATION METHODS

In order to explore the performances of the current profiles in beam-driven dielectric-wakefield acceleration,

we consider a diamond DLW with parameters listed in Table 1. Given the current profile  $I(z)$ , the axial electric field is computed as the integral

$$E(z) = \sum_n \int_{-\infty}^z I(z - \tilde{z}) W_n(\tilde{z}) d\tilde{z} \quad (1)$$

where  $W_n(z)$  are the Green’s functions associated to the  $n^{\text{th}}$  mode; see Ref. [13]. For our calculation we limit the summation to  $n = 4$  modes. Once the axial field is obtained, the decelerating field is computed as  $E_- = \max[E(z)]$  for  $z$  within the bunch and  $E_+ = \min[E(z)]$  for  $z$  behind the bunch.

Table 1: Parameters Associated to the Dielectric Structure used in the Wakefield Simulations

Distribution	Parameters	Units
inner radius a	165	$\mu\text{m}$
outer radius b	195	$\mu\text{m}$
relative permittivity $\epsilon_r$	5.7	-
fundamental frequency $f_0$	0.83	THz

These semi-analytical simulations were imbedded in a genetic optimizer [14]. For each current profile, the associated parameters were varied. The two goals of the optimizer are to find parameters that maximize  $E_+$  and  $\mathcal{R}$ .

## RESULTS

Each shape was optimized over the parameters listed above respectively. Bunch shapes with a small number of parameters (e.g. gaussian and ramped bunch) converged more quickly than more complicated bunch shapes (e.g. double triangle, skewed gaussian, “realistic” parameterization). On average, approximately 15,000 runs were done per shape. The best achieved values for  $\mathcal{R}$  and  $E_+$  are summarized in Fig. 3.

We clearly see a trade-off between  $\mathcal{R}$  and  $E_+$ : as expected, current profiles resulting in large  $\mathcal{R}$  are restricted to smaller values of  $E_+$  and vice versa. The data presented in Fig. 3 was generated using a 1-nC electron bunch; increasing the charge would result in higher  $E_+$  without affecting  $\mathcal{R}$ . Interestingly, none of the asymmetric shapes investigated stand out as a best candidate. A linear regression of the best cases provides an empirical limit for the maximum value of  $\mathcal{R}$ :  $\max[\mathcal{R}] \sim 400 \times E_+^{-0.8}$ . For the chosen structure parameters, values of  $\mathcal{R} > 10$  are achieved with  $E_+ \approx 100$  MV/m. Again, increasing the charge to, e.g., 5-nC would result in 0.5-GV/m field or alternatively could enable reaching higher transformer ratio values for 100-MV/m fields as done in Ref. [12]. In DLW-based acceleration, larger  $E_+$  values could also be reached by reducing the aperture of the structure.

In Figures. 4 and 5 we present an example of optimized Fourier and skewed gaussian distributions, both have  $\mathcal{R} > 6$  and  $E_+ > 150$  MV/m. An important aspect

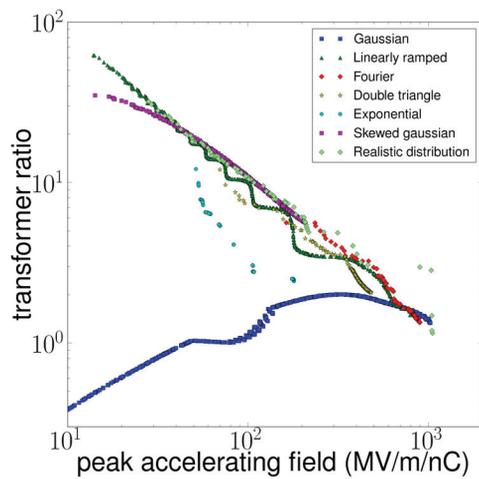


Figure 3: Trade-off curve between  $\mathcal{R}$  and  $E_+$  for the different shapes shown in Fig. 2 (with same color coding).

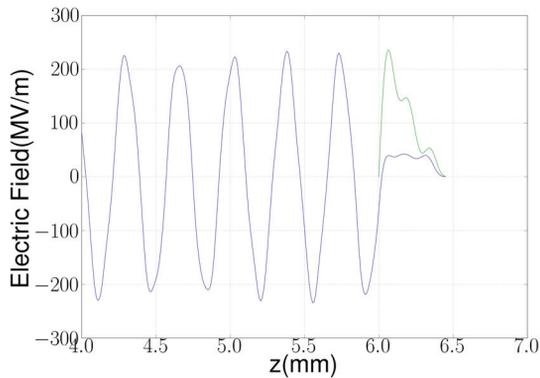


Figure 4: Fourier distribution (green trace) and associated wakefield (blue trace).

of the skewed gaussian distribution, is its ability to provide a relatively smooth decelerating field over the drive bunch as well as a slightly flattened accelerating field over the prospective location of a witness bunch. This latter feature, also observed for the double-triangle distribution, would reduce the energy spread imparted on the witness bunch. Finally, we present an example of a wakefield generated by the “realistic” bunch in Fig. 6. Such a bunch shape is achievable using a dual-frequency linear accelerator [10] which will be used in a forthcoming experiment to demonstrate beam-driven acceleration with an enhanced transformer ratio [15, 16].

## REFERENCES

- [1] I. Blumenfeld et al., Nature 445 (2007) 741.
- [2] M.C. Thompson et al., Phys. Rev. Lett. 100 (2008) 21.
- [3] R. D. Ruth et al., Part. Accel. 17 (1985) 171.

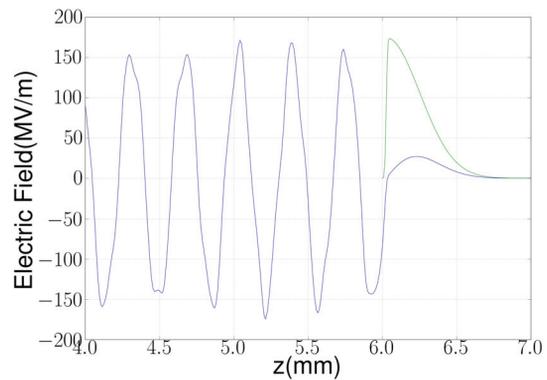


Figure 5: Skewed-Gaussian distribution (green trace) and associated wakefield (blue trace).

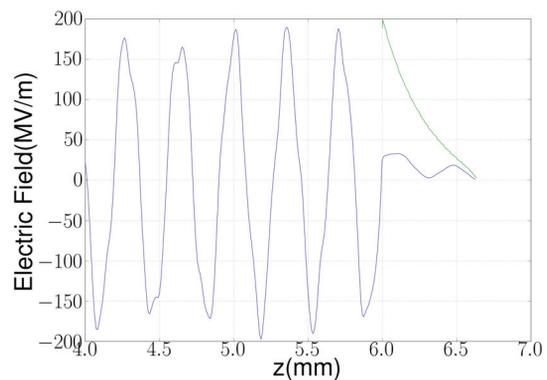


Figure 6: “Realistic” distribution (green trace) and associated wakefield (blue trace).

- [4] K. L. F. Bane et al., IEEE Transactions on Nuclear Science NS-32 (1985) 5.
- [5] P. Schütt et al., Proc. 2nd All-Union Conference on New Methods of Charged Particle Acceleration (Springer, New York, 1989).
- [6] V. Tsakanov, Nucl. Instr. Meth. A 532 (1999) 202.
- [7] C. Jing et al., Phys. Rev. Lett. 98 (2007) 144801.
- [8] R. J. England et al., Phys. Rev. Lett. 100 (2008) 214802.
- [9] P. Piot et al., Phys. Rev. ST AB 14 (2011) 022801.
- [10] P. Piot et al., Phys. Rev. Lett. 108 (2012) 034801.
- [11] W. Gai et al., Phys. Rev. Lett. 61 (1988) 2756.
- [12] B. Jian et al., Phys. Rev. ST AB 15 (2012) 011301.
- [13] M. Rosing and W. Gai, Phys. Rev. D. 42 (1990) 5.
- [14] M. Borland and H. Shang, geneticOptimizer, private communication (2009).
- [15] F. Lemery et al., Proc. IPAC11, 2781 (2011).
- [16] F. Lemery et al., paper WEEPPB01, these proceedings.