ANALYSIS OF A RECTANGULAR DIELECTRIC-LINED ACCELERATING STRUCTURE WITH AN ANISOTROPIC LOADING*

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Abstract

Analysis of Cherenkov radiation generated by high current relativistic electron bunch passing through a rectangular waveguide with anisotropic dielectric loading has been carried out. Some of the materials used for the waveguide loading of accelerating structures (sapphire) possess significant anisotropic properties. In turn, it can influence excitation parameters of the wakefields generated by an electron beam. Using orthogonal eigenmode decomposition for the rectangular dielectric waveguide, the analytical expressions for the wakefields have been obtained. Numerical modelling of the longitudinal and transverse (deflecting) wakefields has been carried out as well. It is shown that the dielectric anisotropy causes frequency shift in comparison to the dielectric-lined waveguide with the isotropic dielectric loading.

INTRODUCTION

A field of perspective particle accelerators is in search of new revolutionary technologies to achieve a progress in high-energy physics experiments. Methods based on dielectric loaded wakefield accelerating structures are the most promising for future linear colliders [1, 2]. A high current electron beam or a high power RF source can excite these structures. The accelerating structure with dielectric filling is a dielectric waveguide with an axial vacuum channel for passing beam. The dielectric waveguide is placed into a conductive sleeve. The high current (up to 100 nC) short (1-2 mm) generating beam of low energy (15-150 MeV) excites Cherenkov wake field which is used to accelerate a low intensive but high energy accelerated beam. The accelerated beam is placed to a distance behind the driving bunch corresponds to an accelerating phase of the wake field.

Dielectric wakefield structures provide both high acceleration rate and ensure the control over the frequency spectrum of the structure by introducing additional ferroelectric layers [3] as well as a possibility using of perspective materials with unique properties like diamond and sapphire [4].

As a rule, the cylindrical geometry proposed for structures with dielectric loading is essential for attaining the highest accelerating gradients as well as for obtaining the maximal possible shunt impedance of the structure [2, 4]. Analytic mode analysis of such accelerating structures for the longitudinal and transverse electric field components was developed in a number of publications (see, for example, [5]). At the same time, structures with a rectangular cross section and dielectric loading were also considered in some cases [6–15] in view of technological difficulties in preparing cylindrical structures with stringent requirements to tolerances for geometrical parameters and uniformity of the permittivity of the filling along the structures [3], as well as their possible application for generating a sheet electron beam. Rectangular structures can be used for test experiments in analysis of new accelerating systems [15] and for studying the properties of materials effective for producing high acceleration gradients of the structure (diamond, sapphire) [4]. Such structures are also considered (along with cylindrical structures) for generating terahertz radiation and producing wakefield acceleration in the frequency range 0.5–1.0 THz [4].

Theoretical analysis of dielectric accelerating structures of rectangular geometry has been carried out in a number of publications [6–9, 13–15]. To determine the amplitudes of individual Cherenkov radiation modes excited in a rectangular waveguide with a dielectric loading, the impedance matching technique was used earlier [7–9]. When such formalism was used instead of direct solution of the nonhomogeneous system of Maxwell equations (which is a standard analytic approach in analysis of wake fields in cylindrical structures [5]), the amplitudes of wake fields had to be expressed in terms of the shunt impedance (or integrated loss factor) for each mode of the structure. Such an approach involves certain approximations, while direct solution of the non homogeneous system of Maxwell's equations without indirect constructions is always preferable for analyzing the problems of generation in waveguide structures.

A method of the first order transverse operator as applied to waveguide problems was worked out in [10–12]. In [13], the generalized orthogonality relation between the LM and LE modes was derived. However, the bilinear form introduced in [13] is not the scalar product in the L_2 space, which requires a substantiation of the possibility of application of this relation for describing orthogonality between the components of the electric and magnetic field vectors. In [14, 15], analysis was performed on the basis of the construction of second order differential equations for transverse field components (based on direct solution of the Maxwell equations) of the two channel rectangular structure with dielectric filling; this structure was developed for increasing the energy conversion factor from the leading beam to the beam being accelerated. In [16] a strict theory of beam

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excitation of rectangular waveguide structure with isotropic dielectric loading was developed.

THEORETICAL ANALYSES OF RECTANGULAR WAVEGUIDE EXCITATION

Let us consider a rectangular waveguide with a symmetric filling in the form of dielectric transversal isotropic layers parallel to the *x* axis and with a vacuum channel at the centre (Fig. 1).

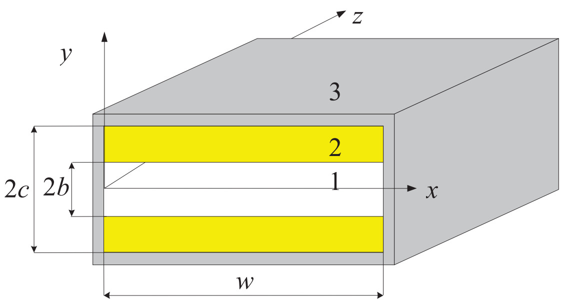

Figure 1: Rectangular waveguide.

In this case, the filling in the direction of the *y* axis is inhomogeneous, and the permittivity and permeability tensors are functions of y: $\hat{\varepsilon} = \hat{\varepsilon}(y)$ and $\hat{\mu} = \hat{\mu}(y)$.

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{\parallel}(y) & 0 & 0 \\ 0 & \varepsilon_{\perp}(y) & 0 \\ 0 & 0 & \varepsilon_{\parallel}(y) \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} \mu_{\parallel}(y) & 0 & 0 \\ 0 & \mu_{\perp}(y) & 0 \\ 0 & 0 & \mu_{\parallel}(y) \end{pmatrix}.$$

Let us transform initial Maxwell equations (1)–(4) combined with material relations for this case. Equation (4) gives biorthogonality of the eigenfunctions and similarity of the operator to a self adjoint operator.

Maxwell equations can be transformed to equations for normal to layer plate electric and magnetic field components.

$$\frac{\partial^2 E_y}{\partial \zeta^2} + \hat{T}_E E_y = \frac{-e}{\varepsilon_0 \left(1 - \varepsilon_{\perp}\mu_{\parallel}\beta^2\right)} \frac{\partial}{\partial y}\left(\frac{n}{\varepsilon_{\parallel}}\right),$$

$$\frac{\partial^2 H_y}{\partial \zeta^2} + \hat{T}_H H_y = \frac{-ev}{1 - \varepsilon_{\parallel}\mu_{\perp}\beta^2}\left(\frac{\partial n}{\partial x}\right),$$

where

$$\hat{T}_E = \frac{1}{\left(1 - \varepsilon_{\perp}\mu_{\parallel}\beta^2\right)}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y}\left(\frac{1}{\varepsilon_{\parallel}}\frac{\partial[\varepsilon_{\perp}\cdot]}{\partial y}\right)\right],$$

$$\hat{T}_H = \frac{1}{\left(1 - \varepsilon_{\parallel}\mu_{\perp}\beta^2\right)}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y}\left(\frac{1}{\mu_{\parallel}}\frac{\partial[\mu_{\perp}\cdot]}{\partial y}\right)\right].$$

Solutions of these differential equations are:

$$E_y = \sum_{n,m} \frac{q}{\varepsilon_0}\frac{\varepsilon_{1\perp}}{\varepsilon_{1\parallel}} \frac{\Psi_{E\,n,m}(x,y)}{\left\langle \Phi_{E\,n,m}, \Psi_{E\,n,m}\right\rangle} \frac{\partial \bar{\Phi}_{E\,n,m}(x_0,y_0)}{\partial y} F_{n,m}(\zeta),$$

$$H_y = q v \mu_{1\perp} \sum_{n,m} \frac{\Psi_{H\,n,m}(x,y)}{\left\langle \Phi_{H\,n,m}, \Psi_{H\,n,m}\right\rangle} \frac{\partial \bar{\Phi}_{H\,n,m}(x_0,y_0)}{\partial x} F_{n,m}(\zeta),$$

where $\Psi_{n,m}$ and $\Phi_{n,m}$ are the eigenfunctions of the transverse operators and the adjoint operators, respectively, $F(\zeta) = -\sin(k_z\zeta)/k_z$ if $\lambda_{E\,n,m} > 0,\ k_z = \sqrt{\lambda_{E\,n,m}}$; $F(\zeta) = \exp(-k_z\zeta)/(2k_z)$ if $\lambda_{E\,n,m} < 0,\ k_z = \sqrt{\left|\lambda_{E\,n,m}\right|}$.

The expression for remaining electric and magnetic field components can be determined from the Maxwell equations.

CALCULATION RESULTS

The expressions derived above were used for analyzing the wakefields generated by a Gaussian relativistic electron bunch with parameters of the Argonne Wakefield Accelerator in the sapphire-based rectangular accelerating structure [4]: $w = 11$ mm, $b = 1.5$ mm, $c = 2.39$ mm, $\varepsilon_{2\perp} = 11.5$, $\varepsilon_{2\parallel} = 9.4$ (Fig. 1), which corresponds to a frequency of 25.0 GHz of the accelerating LM mode of the structure. For comparison a waveguide with isotropic dielectric filling with the same parameters but $\varepsilon_2 = 11.5$ corresponds to the base frequency of 23.25 GHz, $\varepsilon_2 = 10.45$ corresponds to the base frequency of 24.23 GHz, $\varepsilon_2 = 9.4$ corresponds to the base frequency of 25.36 GHz.

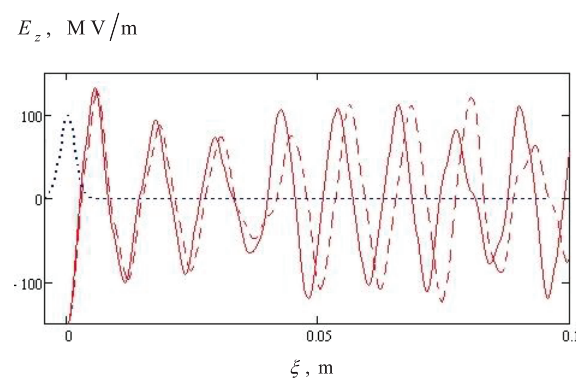

Figure 2: Longitudinal wake field.

As a source of Cherenkov radiation, a generator electron bunch with a Gaussian charge distribution and energy $W = 15$ MeV, charge $q = 100$ nC and bunch length $\sigma_z = 1.5$ mm was considered. The dependence of the longitudinal electric field component E_z produced by the bunch on the distance $\xi = z - vt$ behind it is shown in Fig. 2 (the bunch is located at point $x_0 = w/2$, $y_0 = 0$, $\xi_0 = 8$ cm; the coordinates of the observation point are $x = w/2$, $y = 0$, $\xi = z - vt$); the high accelerating gradient (exceeding 100 MV/m) of wake radiation behind the bunch is worth noting. Solid line corresponds to anisotropic sapphire with $\varepsilon_{2\perp} = 11.5$, $\varepsilon_{2\parallel} = 9.4$, dashed line corresponds to isotropic filling with $\varepsilon_2 = 10.45$. It is visible that sapphire anisotropy leads to shift of a frequency range of the waveguide essential to wakefield acceleration, but with a little influence on a wake field amplitudes.

Thus, we have proposed an analytic method for calculating wake fields of Cherenkov radiation in a rectangular accelerating structure with anisotropic dielectric loading. Using this method for the AWA accelerator parameters, we have analyzed the sapphire based dielectric structure with a rectangular cross section, in which accelerating gradients higher than 100 MV/m can be demonstrated.

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