

# A LONGITUDINAL BEAM DYNAMICS CODE FOR PROTON SYNCHROTRON

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## Abstract

A new code for longitudinal beam dynamics design and beam simulation in proton synchrotron has been developed. In this code, the longitudinal beam dynamics design can be performed for arbitrary curve of dipole magnetic field, and for both basic harmonic cavity and dual harmonic cavity. The beam dynamics simulation with space charge effect can be done in longitudinal phase space, also for both basic harmonic cavity and dual harmonic cavity. The influence of stray fields of RF cavity, which is the higher order mode of cavity coming from the RF generator, on the beam can also be simulated by using the code.

## INTRODUCTION

In the design of Rapid Cycling Synchrotron (RCS) of China Spallation Neutron Source (CSNS/RCS) [1] [2], RAMA and ORBIT [3] are used for longitudinal beam dynamics design and beam dynamics simulation. However, RAMA does not work for the dipole field ramping deviated from sinusoidal curve, and also can't perform the longitudinal beam dynamics design with dual harmonic cavity. ORBIT can not perform the simulation with dual harmonic cavity, or with a dipole field ramping deviated from sinusoidal curve. To meet the requirement of beam dynamics design and study in CSNS/RCS, the code C-SCSIM was developed. The longitudinal beam dynamics design can be performed for arbitrary curve of dipole magnetic field, and for both basic harmonic cavity and dual harmonic cavity. The beam dynamics simulation with space charge effect can be done in longitudinal phase space, also for both basic harmonic cavity and dual harmonic cavity. The influence of stray fields of RF cavity, which is the higher order mode of cavity coming from the RF generator, on the beam, can also be simulated by using the code. The key issues on the code development are given, and the results of longitudinal beam dynamics design and beam simulation are also presented. Figure 1 shows the issues considered in the code.

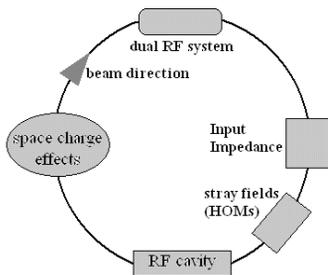


Figure 1: The simulation elements in C-SCSIM.

## BEAM DYNAMICS DESIGN WITH DUAL HARMONIC RF SYSTEM

Many proton synchrotrons are designed to use dual harmonic RF system to increase the bunching factor, as well as to decrease the transverse space charge effect. As a longitudinal beam dynamics tracking code, C-SCSIM can meet the requirement of physical design in dual harmonic RF system. The physical mechanism and program algorithm for simulation with dual RF system in C-SCSIM has been strictly tested and checked. With dual harmonic RF system, the basic longitudinal beam dynamics can be expressed as:

$$V_1 \sin \phi^{(i)} + V_2 \sin(2\phi^{(i)} - \phi_2) = \rho L \frac{dB(t)}{dt}, \quad (1)$$

where  $\rho$  is the bend radius of the dipoles,  $L$  is the circumference and  $B(t)$  is the magnetic strength of the dipole magnet.  $\phi^{(i)}$  is the accelerating phase for the  $i$ th particle and  $\phi_2$  is phase advance of the second harmonic as shown in Fig. 3.

On the other hand, when dual RF system is introduced, the “potential energy” contributed by the second harmonic should be included. The synchrotron motion of particles becomes [4],

$$\begin{aligned} \dot{\delta} &= \frac{\omega_0 e V_1}{2\pi\beta^2 E} (\sin \phi - \sin \phi_s) \\ &+ \frac{\omega_0 e V_2}{2\pi\beta^2 E} [\sin(2\phi - \phi_2) - \sin(2\phi_s - \phi_2)], \end{aligned} \quad (2)$$

$$\dot{\phi} = h\omega_0 \eta \delta.$$

Eq.2 is used to describe the longitudinal motion of the beam in the phase space in the dual harmonic RF system in the code C-SCSIM.

Figure 2 shows the simulation results using C-SCSIM for J-PARC/RCS and SNS/AR.

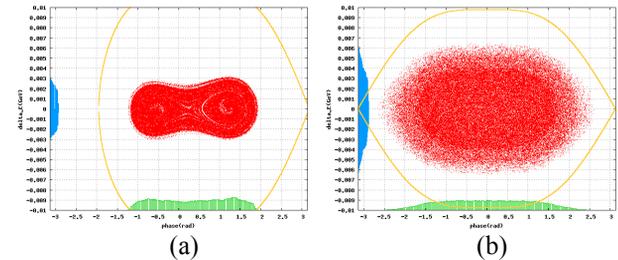


Figure 2: Simulation results with dual RF system (a) in J-PARC/RCS, (b) in SNS/AR

For beam dynamics design, the bunching factor can be calculated by the code. Figure 3 shows bunching factor for CSNS/RCS with single RF cavity and dual RF cavity. Figure 4 shows the calculation result of bunching factor for CSNS/RCS with space charge effect, compared with

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ORBIT in the same initial condition. In order to improve the bunching factor as large as possible, simulation experiment has been done with many different series of second harmonic voltages and phases under the condition of larger bunching factor. Finally, an optimized result of RF voltages has been found, which is shown in Fig. 5.

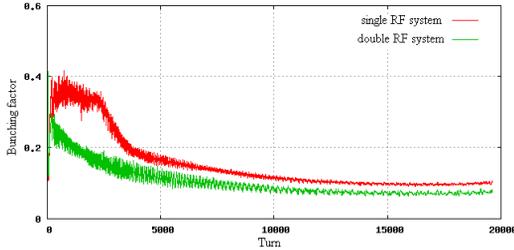


Figure 3: bunching factor for CSNS/RCS for single RF system and dual RF system.

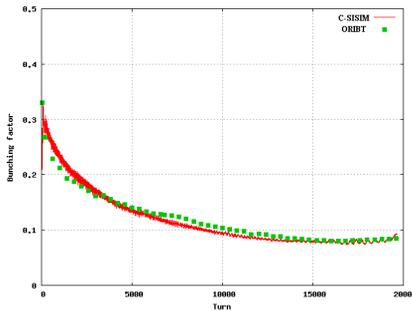


Figure 4: bunching factor for CSNS/RCS compared with ORBIT.

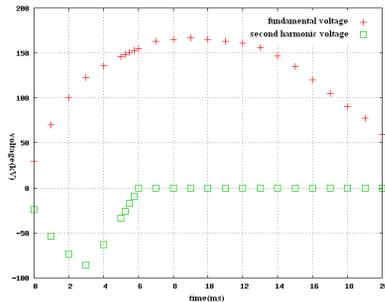


Figure 5: RF voltage optimized for dual RF system in CSNS/RCS.

## LONGITUDINAL SPACE CHARGE SIMULATION BASED ON FFT

Space charge effect is an important issue for proton synchrotron, especially when running with high beam intensity. For computer simulation, the method of Particle-In-Cell (PIC) is widely used in some of present 3-D tracking codes, with which the space charge effect of the beam is evaluated by calculating the coulomb force of each “finite size particle” in the transverse direction. However, a common space charge electromagnetic field can be used to describe the space charge force in the longitudinal direction, instead of the calculation of the

force acting on each particle [5]. Using this method, the influence of longitudinal space charge impedance and wall coupling impedance on the beam can be simulated by the tracking code. The aspect of longitudinal space charge effect in C-SCSIM is developed under this mechanism. See Fig. 6.

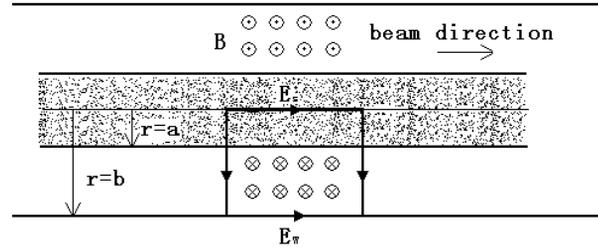


Figure 6: The distribution of the electromagnetic fields of beam in the pipe.

The energy of  $i^{th}$  particle acquired from the common longitudinal space charge electromagnetic field can be described as

$$eV^{(i)} = \sum_n I_n Z_n = \sum_n a_n |Z_n| \cos(n\phi^{(i)} - \phi_n + \chi_n) \quad (3)$$

where  $\phi^{(i)}$  is the synchrotron phase of the  $i$ -th particle and

$$\chi_n = \arctan\left(\frac{nhZ_0g_0}{2\beta\gamma^2} \frac{1}{Z_w}\right) \quad (4)$$

The FFT method is used in the code to calculate the real part and imaginary part of beam current so that the series of amplitude and phase of beam current can be obtained through transforming them. Besides, the series of amplitude is needed to be normalized so that the first term gives the correct average beam current.

It is valuable to point out how to choose the right sampling time to ensure the precision of the resolution of the frequency after FFT. In C-SCSIM, in which the 2-based FFT is realized, take  $t_s$  as the sampling time, where

$$t_s = \frac{L}{2\beta c}$$

and

$$f_0 t_s = \frac{f_0}{F} = 1,$$

where  $f_0$  and  $f_s$  is the revolution frequency and sampling frequency separately.

In the code,  $2^n$  bins are taken averagely in the range of  $2\pi$  in longitudinal phase space. A series of  $2^{n-1}$  values of amplitude in frequency domain can be generated through FFT.

Two of samples are in Figure 7, showing the results with and without the longitudinal space charge at 0.5015 ms when injection process has just finished in J-PARC/RCS.

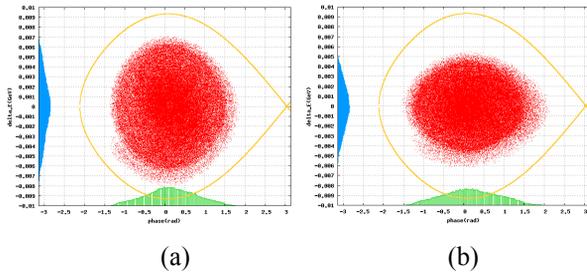


Figure 7: Particle tracking (a) without space charge, (b) with at 308<sup>th</sup>turn with space charge.

Besides, if the real part and imaginary part of each harmonic of some kind of longitudinal impedance is given, particles can be tracked to simulate the influence of the longitudinal impedance on the beam.

### BEAM SIMULATION WITH STRAY FIELD IN FERRITE LOADED RF CAVITY

Coaxial cavities are often used in this kind of proton synchrotron. As to the rapid cycling synchrotron (RCS), ferrite-loaded cavity is needed to synchronize the resonance frequencies to the revolution frequencies. There often exist many stray fields besides the fundamental field only which is used to accelerate. The influence of these stray fields to the beam behaviour is valuable to study and simulate by computer program because the stray fields can probably affect the beam strongly in the condition the stray fields resonate with the synchrotron sideband.

The equation of synchrotron motion for the stray field element is:

$$\delta_{n+1} = \delta_n + \frac{e}{\beta^2 E} \sum_{m=2} V_m [\sin(m\varphi - \varphi_m) - \sin(m\varphi_s - \varphi_m)] \quad (5)$$

where the  $n$  represent the  $n$ th particle, the  $m$  is the order of the stray fields.  $\varphi_s$  and  $\varphi_m$  are the synchrotron phase and the stray field phase respectively. Treating the stray fields in the RF cavity as “another RF cavity”, the simulation code, C-SCSIM, can evaluate reasonably the influence of these stray fields on the beam. Just like the method of calculating the space charge effects, the particles experience a “small cavity” on behalf of the influence of the stray fields besides the ideal accelerating cavity. See Figure 1.

These stray fields in RF cavities can be excited by the RF power supply which is not a pure signal and can be seen by the spectrum analyzer. The values of the frequency and amplitude of each order of stray fields can be acquired by FFT from the initial data output from oscilloscope in the experiment.

Some simulations have been done to evaluate the influence of the stray fields on the beam and optimize the RF cavity for CSNS/RCS.

As is shown in Table 1, there exists one busbar mode, which could be resonated with some order of stray field, can not be ignored.

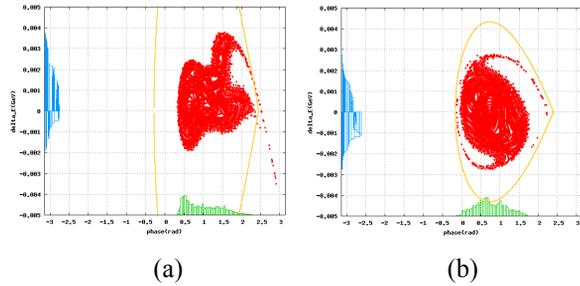


Figure 8: Particle tracking (a) before optimization, (b) after optimization.

Table 1: Comparison Before and After Optimization

	Before	After
<b>Resonance time(ms)</b>	14-16	7.5-8.5
<b>Resonance frequency(MHz)</b>	7.05	7.75
<b>Resonance order</b>	1.1856-1.2073	0.9599-1.0185
<b>Maximal Amplitude</b>	1/5.6	1/13
<b>Phase</b>	varying	varying

After optimization, the resonance order has changed from 3<sup>rd</sup> to 4<sup>th</sup>. The result from the simulation shows that the beam loss is much less. The shape of the bunch seems to more regular than that before optimization, see Fig. 8.

### CONCLUSION

Based on the longitudinal physical model and reasonable algorithm, the code C-SCSIM is a new particle-tracking code whose simulation results have been checked strictly and compared with other world wide used tracking code. So it can be used for proton synchrotron design and longitudinal parameter optimization. The functions comprises of the basic particle tracking in the given proper voltages or synchrotron phase, the dual RF system simulation, longitudinal space charge effects and stray fields optimization. C-SCSIM has been already used in the design of the CSNS/RCS.

### ACKNOWLEDGEMENTS

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