# HIGH-INTENSITY MONOCHROMATIC CHERENKOV RADIATION IN THz RANGE BY FEMTOSECOND ELECTRON BUNCHES IN IMPURITY-DOPED SEMICONDUCTOR TUBE

Atsushi Ogata, Koichi Kan, Takafumi Kondoh, Kimihiro Norizawa, Jinfeng Yang, Yoichi Yoshida
ISIR, Osaka University, JAPAN

*Abstract*

Terahertz wakefields are generated by injection of an electron beam into a semiconductor electron-hole plasma with a plasma frequency in the terahertz regime. Wakefield radiation emitted from a beam moving along the axis of a metal-wrapped semiconductor tube will have discrete frequency components. Quasi-monochromatic radiation is obtained by making only the lowest frequency component coherent through optimization of the bunch length. Results of preliminary experiments using millimeter-sized dielectric tubes are presented.

## INTRODUCTION

This paper proposes to optimize three parameters in the excitation of the Cerenkov radiation in the terahertz (THz) range using the structure shown in Fig. 1. The radiation field is the wake of the electron bunch, which has also been used for beam acceleration. The three parameters are the size of the tube, the bunch length of the electron beam, and the plasma skin depth of the liner. If they are in the range of the radiation wavelength, the radiation is enhanced and becomes quasi-monochromatic.

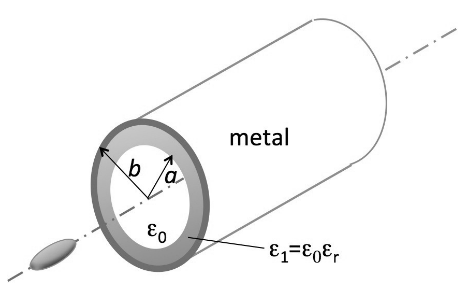


Figure 1: A hollow tube in a beam line.

The spectrum from the structure in Fig. 1 is discrete, and if the bunch length is comparable to the wavelength of the lowest component, only its intensity can be proportional to $N^2$, where $N$ is the number of electrons in a bunch. The resultant radiation thus becomes coherent and quasi-monochromatic.

To achieve a plasma frequency of $\omega_p = 10$ THz or a plasma skin depth of $c/\omega_p = 30\ \mu$m, the electron density must be $3.16 \times 10^{16}$ cm$^{-3}$. The electron density of an electron-hole plasma can be controlled by doping a semiconductor. Recent technical advances have enabled the fabrication of semiconductor waveguides with sizes of around 30 $\mu$m, and production of electron beams with bunch lengths as short as 30 $\mu$m [1].

## DISPERSION RELATION

We express the dielectric function in such a way that it corresponds to the case of an impurity-doped semiconductor:

$$\epsilon_r(\omega) = 1 - \frac{ne^2}{\epsilon_0 m^* \omega(\omega + i\gamma)} - \frac{n_0 e^2}{\epsilon_0 m(\omega^2 - \omega_I^2)}, \quad (1)$$

where $e$, $\epsilon_0$, and $m$ are the electron charge, vacuum permittivity, and electron mass, respectively. Here, $n$ is the carrier density of the semiconductor plasma, $\gamma$ is the relaxation constant of the plasma oscillation, $n_0$ is the electron density in the atomic core, and $m^*$ is the effective carrier mass. Using the Planck constant $\hbar$, the atomic band gap is expressed as $\hbar\omega_I$.

Because $\omega_I$ is usually in the petahertz range, we can neglect $\omega^2$ in the denominator of the third term in Eq. (1). Defining $\epsilon_{core}$ and the plasma frequency as

$$\epsilon_{core} = 1 + \frac{n_0 e^2}{\epsilon_0 m \omega_I^2} \quad \text{and} \quad \omega_p = \left(\frac{ne^2}{\epsilon_{core}\epsilon_0 m^*}\right)^{1/2},$$

we obtain the following dielectric function for the semiconductor:

$$\epsilon_r(\omega) = \epsilon_{core}\left[1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}\right]. \quad (2)$$

The parameters $\omega_p$ and $\gamma$ are not independent of each other [1]; $\gamma$ is related to the conductivity $\mu$ through $\mu = e/m^*\gamma$, and the empirical relations between $n$ and $\mu$ are known. Manufacturers provide data on the resistivity $\rho = (e\mu n)^{-1}$, which is usually in the range 1–20 $\Omega$ cm$^{-1}$. A sample set of values to obtain $\omega_p = 10$ THz in P-doped n-type Si ($m^* = 0.26m$ and $\epsilon_{core} = 11.7$) is $\rho = 11.5$ $\Omega$ cm$^{-1}$, $n = 10^{17}$ cm$^{-3}$, $\mu = 721$ cm$^{-2}$V$^{-1}$s$^{-1}$, and $\gamma = 9.38$ THz.

Fig. 1 shows a tube with inner and outer radii of $a$ and $b$, respectively, consisting of a semiconductor with a permittivity of $\epsilon_1(\omega) = \epsilon_r(\omega)\epsilon_0$. Under the approximation that the electron velocity is equal to $c$, the wave numbers of the azimuthally symmetric TM-mode electromagnetic fields that can be transmitted through the tube satisfy the following equation [1]:

$$J_1(ka)Y_0(kb) - J_0(kb)Y_1(ka) = 0, \quad (3)$$



ISBN 978-3-95450-115-1

where $J_n(z)$ and $Y_n(z)$ $(n = 1, 2, ...)$ denote Bessel functions of the first and second kind, respectively. For a given $a$ and $b$, there exist an infinite number of $k$ satisfying Eq. (3), denoted as $k_n$ $(n = 1, 2, ...)$. For a given $k_n$, the corresponding frequency $\omega_n$ is given by

$$\omega_n = \frac{ck_n}{[\epsilon_r(\omega_n) - 1]^{1/2}}. \tag{4}$$

We substitute Eq. (2) into Eq. (4) and solve to obtain four roots. We choose the root with a positive real part, and hereafter, $\omega_n$ refers only to this root. The real and imaginary parts give the frequency and relaxation constant of the $n$-th component, respectively.

## WAKEFIELD

The amplitude of the wakefield produced by a single electron passing along the axis of the hollow tube is given by [1]

$$E_z(k) = \sum_n E_z(\omega_n) = -\frac{2e}{\epsilon_0 a} \sum_n \frac{B_n}{A_n}, \tag{5}$$

where

$$A_n = \epsilon_r(\omega_n) \frac{d}{dk} [J_1(ka)Y_0(kb) - J_0(kb)Y_1(ka)]_{k=k_n},$$

and $\quad B_n = J_0(k_n a)Y_0(k_n b) - Y_0(k_n a)J_0(k_n b).$

Fig. 2 shows the frequencies and absolute values of the fields in the doped (solid lines) and undoped ( dotted lines) Si tubes as a function of $b/a$ in which the static dielectric constant $\epsilon_{core}$ is equal to 11.7, under the assumption that $\gamma/\omega_p$ is equal to 1. Three $a\omega_p/c$ values are given as parameters. Figs. 2 (a) and (b) represent the real and imaginary parts of the lowest frequency component $\omega_0$ in the doped and undoped tubes, respectively. Fig. 2(c) shows plots of the absolute values $|E_z(\omega_0)| = (\mathrm{Re}[E_z(\omega_0)]^2 + \mathrm{Im}[E_z(\omega_0)]^2)^{1/2}$ of the doped (solid lines) and undoped tubes (dotted lines). The frequencies are normalized by $\omega_p$, and the lengths are normalized by $c/\omega_p$. Because $E_z(\omega_n)$ is proportional to $1/a$ and $(d/dk)^{-1}$, $E_z(\omega_n)$ is proportional to $\omega_p^2$ for this normalization.

If we do not dope the material, the dielectric function is simply $\epsilon_{core}$. By using Eq. (3) and substituting $\epsilon_r = \epsilon_{core}$ into Eqs. (4) and (5), we obtain the frequency and field corresponding to each wave number. These are plotted as the dotted lines in Figs. 2(a) and (c). The conditions are equivalent to those in experiments conducted using usual dielectric tubes [2]. Although the plasma frequency $\omega_p$ has no physical influence in this case, the values are normalized using the same $\omega_p$ value in order to ensure an accurate comparison.

A comparison between the doped and undoped results reveals that doping leads to a dependency of the minimum radiation frequency on $\omega_p$ and that the doped tubes produce higher frequencies and intensities than the undoped tubes; although, this causes decay of the fields as shown in Fig. 2(b).

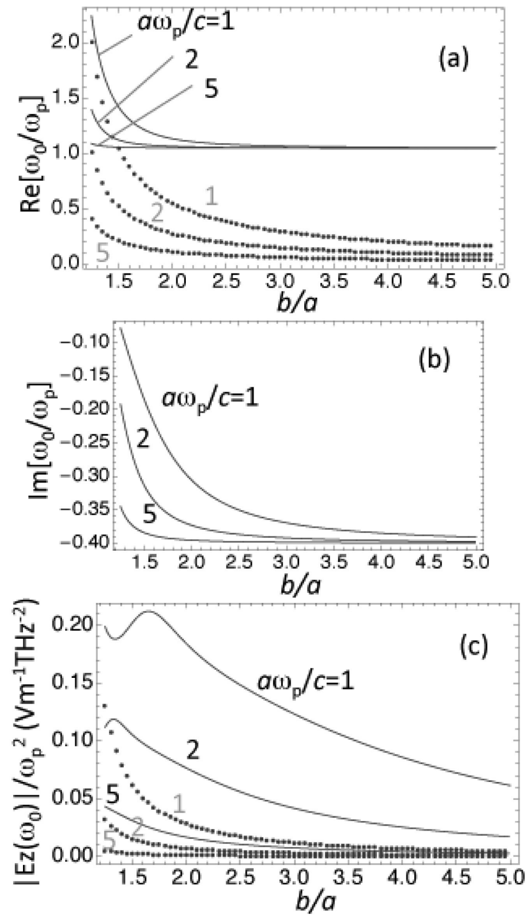


Figure 2: The $b/a$ dependencies of (a) Re$[\omega_0/\omega_p]$ and (b) Im$[\omega_0/\omega_p]$ of the doped (solid lines) and undoped tubes (dotted lines). (c) The $b/a$ dependencies of the absolute values of $E(\omega_0)/\omega_p^2$ of the doped (solid lines) and undoped (dotted lines) tubes. The numbers 1, 2, and 5 represent the inner tube radius normalized by the plasma skin depth $c/\omega_p$. The semiconductor parameters $\epsilon_{core} = 11.7$ and $\gamma/\omega_p = 1$ have been used.

## RADIATION COHERENCE AND POWER

We here assume that an electron bunch has a one-dimensional Gaussian distribution with a standard deviation of $\sigma_z$. The coherent radiation field of the bunch composed of $N$ electrons is then given by

$$[E_z(\omega_n)]_{coh} = [N + (N-1)NF(\sigma_z, \omega_n)]E_z(\omega_n) \simeq N^2 F(\sigma_z, \omega_n)E_z(\omega_n), \tag{6}$$

$$F(\sigma_z, \omega_n) = \exp\left[-\frac{\sigma_z^2 \omega_n^2}{c^2}\right], \tag{7}$$

where $F(\sigma_z, \omega_n)$ is the so-called form factor [1].

The two dotted lines in Fig. 3 show the dependence of $F(\sigma_z, \omega)$ on $\omega$ for $\sigma_z = c/\omega_p$ and $2c/\omega_p$. The five vertical lines correspond to values of $\omega_n$ $(n = 0, .., 4)$ given by Eqs. (3) and (4). The upper line connecting the five $|E_z(\omega_n)|$ values normalized by $|E_z(\omega_0)|$ corresponds to $\sigma_z = 0$. The $\sigma_z = c/\omega_p$ and $\sigma_z = 2c/\omega_p$




ISBN 978-3-95450-115-1

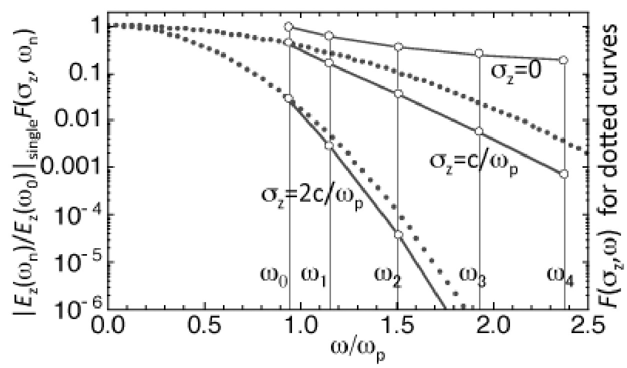


Figure 3: The factor $F(\sigma_z, \omega)$ (dotted lines) as a function of $\omega$ at $\sigma_z = c/\omega_p$ and $2c/\omega_p$. The vertical lines correspond to values of $\omega_n$ ($n = 0, .., 4$) given by Eqs. (3) and (4). The $\sigma_z = 0$ line connects the five $|E_z(\omega_n/E_z(\omega_0)|$ values. The $\sigma_z = c/\omega_p$ and $\sigma_z = 2c/\omega_p$ lines connect those values multiplied by $F(\sigma_z, \omega)$, respectively. The points are calculated under conditions of $\epsilon_{core} = 11.7, a = 2c/\omega_p$, and $b/a = 2$.

lines connect the $|E_z(\omega_n)/E_z(\omega_0)|$ values multiplied by $F(\sigma_z, \omega)$. The points are calculated under conditions of $\epsilon_{core} = 11.7, a = 2c/\omega_p$, and $b/a = 2$. The coherent fields can be calculated by multiplying the plotted values by $N^2$.

We can then make a rough estimation of the power from the wake. Assuming $\omega_p = 10$ THz, $a = 2c/\omega_p$, and $b/a = 2$, Eq. (5) gives $E_z = 5.30$ Vm$^{-1}$ for each electron with an associated power of $ceE_z = 0.254 \times 10^{-9}$ W. An electron bunch with a charge of 10 pC ($N = 62.5 \times 10^6$) then gives a wakefield of 331 MeV/m with a power of 15.9 mW without taking the coherent effects into account. If the bunch length is $2c/\omega_p = 60$ $\mu$m, the coherent form factor $F$ is 0.0034, and the power is enhanced to 33.7 kW. If the bunch charge is increased to 1 nC, the power reaches 330 MW.

## PRELIMINARY EXPERIMENTS

In the experiments, 27 MeV, 200 pC, and $\sigma_z/c = 200$ fs electron bunches were injected into $\epsilon_r = 3.8$ silica tubes wrapped in copper [2]. The THz signal processed by a Michelson interferometer was measured by a 4.2 K silicon bolometer.

The frequency spectra obtained in two tubes are shown in Fig. 4; the first tube (a) has an inner radius of 5 mm, an outer radius of 7 mm, and is 150 mm in length. The second tube (b) has respective dimensions of 1 mm, 1.5 mm, and 150 mm. The crosses in the figure show the frequency components given by Eq. (3). We can identify the $TM_{03}$, $TM_{04}$, and $TM_{09}$ modes in the first tube and the $TM_{03}$ and $TM_{05}$ modes in the second tube. The radiated power of the $TM_{03}$ mode in case (b) is experimentally estimated to be ~200 W.

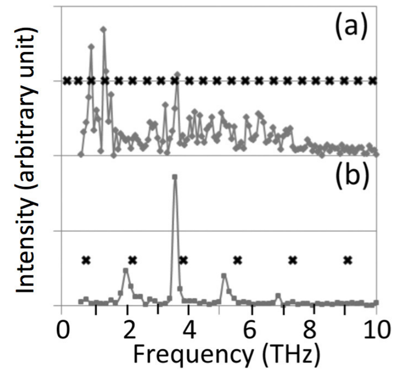


Figure 4: The frequency spectra obtained in two tubes with inner and outer radii and lengths of (a) 5 mm, 7 mm, and 150 mm, and (b) 1 mm, 1.5 mm, and 150 mm, respectively. The crosses indicate the frequency components given by the theory.

## DISCUSSION

The wakefield in a homogeneous plasma has the same velocity as its driving beam. In the present case, however, Eq. (4) gives a phase velocity $v_p$ less than $c$. The duration of the coherent radiation is roughly $c/[\omega_p(c - v_p)]$ but this time span is comparable to the decay time of the plasma wave as shown in Fig. 2(b). We infer from this that the pulse width of the radiation is determined by the decay time before the radiation loses its coherency.

Although the associated power is large, the pulse energy of the radiation is modest in this scheme. Most of the energy in the wakefield is dissipated in the semiconductor wall in a time span of $\sim 1/\gamma$. The wakefield breakdown limit is in the range ~10 GeV/m [1] in dielectric tubes. However, the situation could be more severe in the semiconductor structure.

In the present analytical model studies, we have assumed a one-dimensional distribution. However, electron bunches are actually also distributed radially. The longitudinal electric field generated by an off-axis electron would appear as finite bandwidths in the radiation spectra [1].

The relative positions of the electrons are frozen in the present studies. In a real situation, the trailing electrons are either accelerated or decelerated by the wake of the preceding electrons. Consequently, the mutual distance changes, which in turn modifies the wake. To clarify the effects of both the change in mutual distance and the finite transverse bunch size, further simulation studies are required.

The semiconductor tube will be useful also as a wakefield accelerator structure.

## REFERENCES

[1] K. Kan, J. Yang, K. Norizawa, T. Kondoh, Y. Yoshida, and A. Ogata, Radiation Phys. Chem. **80** 1323 (2011) and references therein.

[2] K. Kan, J. Yang, A. Ogata, T. Kondoh, K. Norizawa, and Y. Yoshida, Appl. Phys. Lett. **99** 231503 (2011).