MATHEMATICAL MODEL OF CHARGED PARTICLES DYNAMICS OPTIMIZATION IN RFQ ACCELERATORS

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Abstract

Mathematical model of optimization of transverse motion of charged particles in accelerators is suggested. Problems of finding of transverse motion specifications could be formulated as complicated multi parameters optimization tasks. Problems of optimization of transverse motion to be solved effectively should be considered as step by step optimization process [1]. The main problems are focusing problem, problem of minimization of effective emittance growth and the problem of matching the parameters of the beam at the entrance to the RFQ structure [2, 3]. In the case of linear equations of transverse motion of particles it is convenient to consider envelopes as characteristics of the beam. In nonlinear case one can consider RMS characteristics of beam of trajectories to estimate the parameters of motion and to set functional of quality. Optimization algorithm based on mini-max functionals is built. Numerical results for RFQ accelerators are presented.

OPTIMIZATION PROBLEM

Let us consider the following equations to describe longitudinal and transverse motion of charged particle beam in RFQ structure [4]:

$$\frac{d^2 z}{d\tau^2} = 4\pi \frac{eUT}{W_0 L} \cos(Kz)\cos(\theta\tau + \varphi_0)x = F_z \,, \quad (1)$$

$$\frac{d^2 x}{d\tau^2} = \left(4\pi \frac{eUT}{W_0 L^2}\sin(Kz) + \frac{eU\kappa}{W_0 a^2}\right)\cos(\theta\tau + \varphi_0)x - \frac{4I/I_0}{\beta_z r_x (r_x + r_y)}x = Q_x(\tau, z, \varphi_0, r_x, r_y)x, \quad (2)$$

$$\frac{d^2 y}{d\tau^2} = \left(4\pi \frac{eUT}{W_0 L^2}\sin(Kz) - \frac{eU\kappa}{W_0 a^2}\right)\cos(\theta\tau + \varphi_0)y - \frac{4I/I_0}{\beta_z r_y (r_x + r_y)}y = Q_y(\tau, z, \varphi_0, r_x, r_y)y, \quad (3)$$

where $\tau = ct$, $\theta = 2\pi\omega/c$, U is the intervane voltage, T is accelerating effectiveness, W_0 is the charged particle rest energy, ω is the accelerating field frequency, φ_0 is the initial phase, c is the velocity of light, a is the radius of the channel, r_x and r_y are the beam envelopes, I is the beam current, I_0 - Alfven current. $K = 2\pi/L$, L - cell (period) length, $\kappa = 1 - 4T/\pi$.

Let us turn to the independent variable z. Then equation (1) could be written as follows:

$$\frac{d\varphi}{dz} = \frac{\widetilde{\omega}}{\beta}, \; \frac{d\beta}{dz} = \frac{F_z}{\beta}, \quad (4)$$

where $\varphi = \widetilde{\omega}\tau + \varphi_0$ is particle phase, $\beta = \frac{\dot{z}}{c}$.

Transverse motion equations (2), (3) then have the form:

$$\frac{d^2 x}{dz^2} = \frac{1}{\beta^2}\left(Q_x x - \frac{dx}{dz}F_z\right), \quad (5)$$

$$\frac{d^2 y}{dz^2} = \frac{1}{\beta^2}\left(Q_y y - \frac{dy}{dz}F_z\right). \quad (6)$$

Equations (5), (6) could be rewritten as the following system of equations

$$\frac{d\xi}{dz} = A_x \xi \,, \; \frac{d\eta}{dz} = A_y \eta \,, \quad (7)$$

where $\xi = (\xi_1, \xi_2)$, $\xi_1 = x$, $\xi_2 = \frac{dx}{dz}$, $\eta = (\eta_1, \eta_2)$, $\eta_1 = y$, $\eta_2 = \frac{dy}{dz}$, and matrices A_x and A_y have the form

$$A_x = \begin{pmatrix} 0 & 1 \\ Q_x/\beta^2 & -F_z/\beta^2 \end{pmatrix}; \quad A_y = \begin{pmatrix} 0 & 1 \\ Q_y/\beta^2 & -F_z/\beta^2 \end{pmatrix}.$$

Let the sets of conditions of system (7) fill in the planes $\left(x, \frac{dx}{dz}\right)$ and $\left(y, \frac{dy}{dz}\right)$ at $z = 0$ ellipses

$$\xi^* G_x^0(\varphi_0)\xi \le 1 \,, \; \eta^* G_y^0(\varphi_0)\eta \le 1 \,, \quad (8)$$

correspondingly.

Then, the matrices G_x and G_y (describing the ellipses in corresponding planes) satisfy the following system of matrix equations

$$\frac{d}{dz}G_x = -A_x^{\ *}G_x - G_x A_x \,, \quad (9)$$

$$\frac{d}{dz}G_y = -A_y^{\ *}G_y - G_y A_y \,. \quad (10)$$

The system of equations (9), (10) should be solved on the interval from the entrance into the regular part of the accelerator to the end of structure, i.e. from $z = 0$ to $z = Z$. Initial conditions for the system (9), (10) are the matrices of ellipses (8) defining acceptances of the regular part of the accelerator at initial admissible control (the variation of the radius of the channel), depending on an initial phase φ_0 :

$$G_x(0, \varphi_0) = G_x^0(\varphi_0), \quad G_y(0, \varphi_0) = G_y^0(\varphi_0).$$

The minimization problem of effective emittance growth is to find a function $a(z)$, i.e. law of the radius

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change along the accelerator, providing under the conditions (8) the maximum possible overlapping of families of ellipses ($0<\varphi_0<2\pi$) at the end of accelerating structure.

And the optimization problem for the radial matching section is to find a function $a(z)$, i.e. law of the radius change along the matching sections, providing under the conditions (8) the maximum possible overlapping of families of ellipses at the entrance of the radial matching section. Here the system of equations (9), (10) should be solved on the interval from the entrance into the regular part of the accelerator to the entrance into the radial matching section, i.e. from $z=0$ to $z=-h$. (In radial matching section equations (1)-(3) could be simplified: $T=0$, $F_z=0$).

MATHEMATICAL OPTIMIZATION MODEL

Represent elements of matrices $G_x(z,\varphi_0)$ and $G_y(z,\varphi_0)$ in the following way

$$G_x=\begin{pmatrix}\frac{\Delta_x+\alpha_x^2}{\nu_x} & \alpha_x \\ \alpha_x & \nu_x\end{pmatrix},\quad G_y=\begin{pmatrix}\frac{\Delta_y+\alpha_y^2}{\nu_y} & \alpha_y \\ \alpha_y & \nu_y\end{pmatrix},$$

where $\nu_x/\Delta_x=r_x^2$, $\nu_y/\Delta_y=r_y^2$, $\Delta_x=\det G_x$, $\Delta_y=\det G_y$.

Then equations (9), (10) could be transformed in the form

$$\frac{d\nu_x}{dz}=2\left(\nu_x\frac{F_z}{\beta^2}-\alpha_x\right), \qquad (11)$$

$$\frac{d\alpha_x}{dz}=\alpha_x\frac{F_z}{\beta^2}-\nu_x\frac{Q_x}{\beta^2}-\frac{\Delta_x+\alpha_x^2}{\nu_x}, \qquad (12)$$

$$\frac{d\nu_y}{dz}=2\left(\nu_y\frac{F_z}{\beta^2}-\alpha_y\right), \qquad (13)$$

$$\frac{d\alpha_y}{dz}=\alpha_y\frac{F_z}{\beta^2}-\nu_y\frac{Q_y}{\beta^2}-\frac{\Delta_y+\alpha_y^2}{\nu_y}, \qquad (14)$$

$$\Delta_x=\Delta_y=\Delta_0\left(\frac{\beta(z)}{\beta_0}\right)^2. \qquad (15)$$

Let us introduce functional, which estimating the degree of mutual overlapping of ellipses corresponding to various initial phases at the end of accelerator (entrance of the matching section). In connection with it consider a cluster of quadratic forms generated by a pair of ellipses with the matrices G and B, and find its eigenvalues λ_1 and λ_2:

$$\chi(\lambda)=\det(G-\lambda B)=0,\quad \chi(\lambda_1)=\chi(\lambda_2)=0.$$

The value of the inverse minimum eigenvalue $\lambda=\min(\lambda_1,\lambda_2)$ characterizes the degree of mismatch pairs of ellipses. In the case of fully identical ellipses, this value is equal to unity. So always $\lambda^{-1}\geq 1$.

Consider the degree of mismatch of ellipses with matrices $G_x(Z,\varphi_0)$ and $G_y(Z,\varphi_0)$ and ellipses with given matrices B_x and B_y:

$$\lambda_x^{-1}(\varphi_0)=\lambda^{-1}(G_x(Z,\varphi_0),B_x),$$
$$\lambda_y^{-1}(\varphi_0)=\lambda^{-1}(G_y(Z,\varphi_0),B_y).$$

Matrices B_x and B_y describing the desired phase portrait of the beam at the end of accelerator (beginning of the matching section).

The quality of the accelerating structure will be estimated by the following value

$$J(a)=\max_{\varphi_0}\lambda_x^{-1}(\varphi_0)+\max_{\varphi_0}\lambda_y^{-1}(\varphi_0). \qquad (16)$$

The problem of minimizing the functional (16) is the mini-max optimization problem. Minimization of the functional can be performed by methods of directed minimization. It is shown that for the chosen form of the functional its variation can be obtained in analytical form [2, 3]. This allows on the practice to increase the number of control parameters.

OPTIMIZATION RESULTS

The analytic representations of the variations of the functionals (16) were used to find parameters of the RFQ accelerator of protons (initial energy 95keV, output energy 5 MeV, intervane voltage 100kV, RF field frequency 352 MHz, initial cell length 6.06 mm, beam current 0.1 A).

Initial (before optimization) emittances of the beam in the planes (x,x') and (y,y') corresponding to starting choice of the law of variation of the channel radius along the accelerator are presented in Figures 1 - 2.

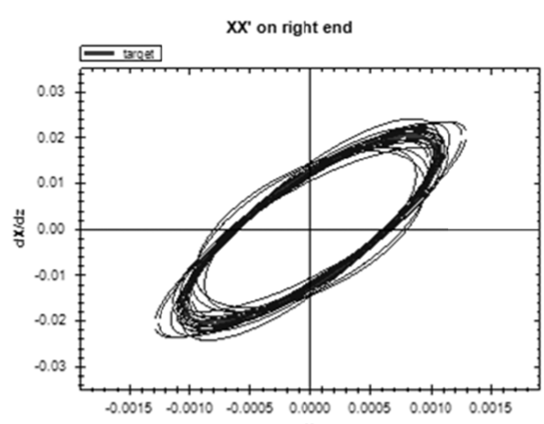

Figure 1: Initial emittances in the plane (x,x').

ISBN 978-3-95450-115-1

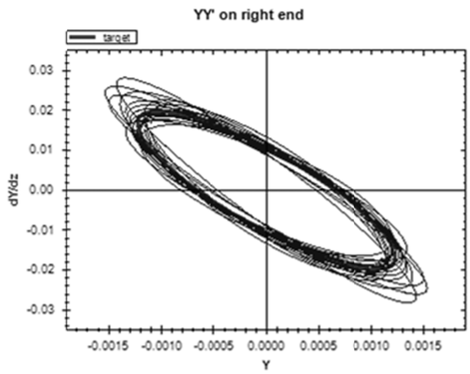

Figure 2: Initial emittances in the plane (y, y').

The illustrations of the optimization effect are shown in Figure 3-4.

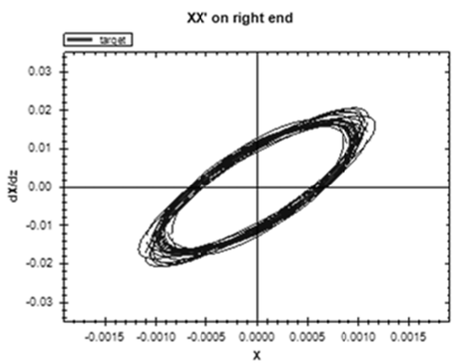

Figure 3: Optimized emittances in the plane (x, x').

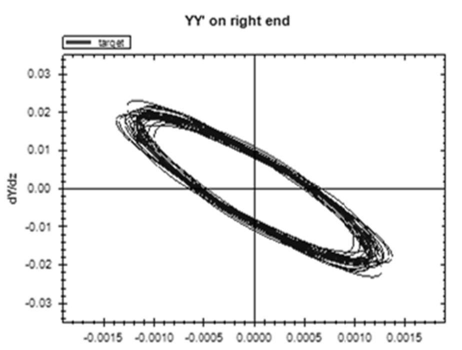

Figure 4: Optimized emittances in the plane (y, y').

Below starting radius of the channel and the result of optimization are presented on Figures 5 and 6 correspondingly.

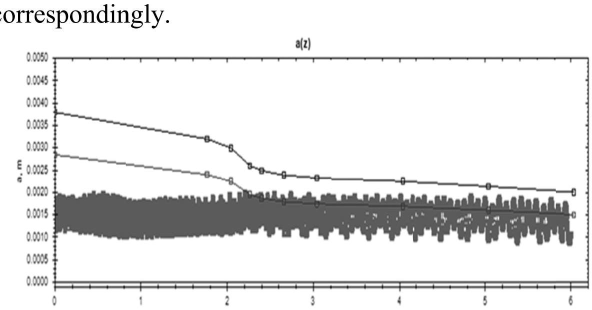

Figure 5: Initial radius of the channel and beam envelopes.

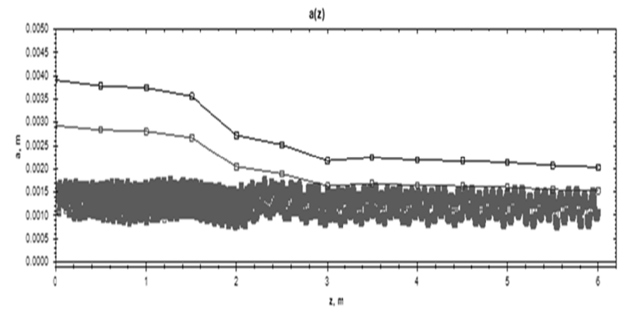

Figure 6: Optimized radius of the channel and beam envelopes.

ACKNOWLEDGMENT

Author is grateful to Dmitri Ovsyannikov and Anton Kolobov for their help in obtaining numerical results.

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