COMPUTATION OF THE 2D TRANSVERSE WAKE FUNCTION OF AN ELECTRON-CLOUD FOR DIFFERENT PARAMETERS *

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Abstract

A PIC simulation of the interaction of a positive charged bunch with an e-cloud yields the wake kick from the electrons on the tail particles of the bunch. The wake is induced from a certain offset in the transverse position of the head parts of the bunch which perturbs the electron distribution. Such pre-computed wake functions of each offset part of the bunch are forming a matrix which could be used for investigating a single bunch stability under several assumptions. In this paper we investigate the linear scalability of the wake field with the offset value. Furthermore we investigate the wake field values for different realistic electron densities. Another important parameter for realizing the single bunch stability simulation is the optimal number of bunch slices in longitudinal direction. Here we study the thickness of the slices in conjunction with the mobility of the electrons around the beam axis.

INTRODUCTION

The excitation of the electron cloud by a single bunch progressing through and the reaction of the cloud on the same bunch resembles a short-range wakefield. The wakefield drives a single bunch instability of head-tail type which is manifested by a strong transverse emittance growth. In order to simulate the instability [1] the bunch particles are tracked by multiplying the 6D vector of each particle with the transformation matrices describing the lattice. Thereby the action of the e-cloud on the bunch is approximated by a wake kick which is applied on one or more points in the lattice. However the wake field from the perturbed cloud can not be treated as the wake field from a broadband resonator which is computed for constant geometries and material parameters of the beam pipe. The e-cloud as a beam environment evolves during the bunch-passage. Thus the wake field is not only a function of the distance between the excitation and the part of the bunch receiving an action, but has to take in to account the e-cloud density near the beam axis when the perturbation happens. The idea of K.Ohmi to slice the 3D bunch and compute a wake function from every longitudinal slice of the bunch (Figure 1) backwards leads to a triangular wake matrix. In order to apply the computed wake matrix for the bunch tracking we suppose properties of the wake field such as time invariance, superposition and linearity. In this paper we study the scalability of the wake field with the offset value and the e-cloud density.

*Work supported by DFG under contract number RI 814/20-2.

SIMULATION

The program MOEVE PIC Tracking [3] simulates the interaction of a single bunch with an electron cloud. The cloud is modelled as a uniform distribution of electrons in a beam pipe and is assumed to be generated by the preceding bunches. Since the perturbation of the could in the transverse plane is due to the transverse displacement of the bunch we simulate the interaction by displacing a slice of the bunch. For a flat bunch as the KEKB-LER bunch (Table 1) we use in our simulation a vertical displacement (as in Figure 1) which is typically $\Delta y = \sigma_y$.

Table 1: Bunch parameters of the low energy ring of the KEK B-factory used for modelling the interaction with the e-cloud.

Parameter	Symbol	KEKB-LER
Circumference	L	3016 m
Beam energy	E_b	$3,5$ GeV
Population	Nb	$3,3 \cdot 10^{10}$
Charge	Q	$5,28$ nC
Length (rms)	σ_z	6 mm
Transverse	σ_x	420 μm
beam size (rms)	σ_y	60 μm

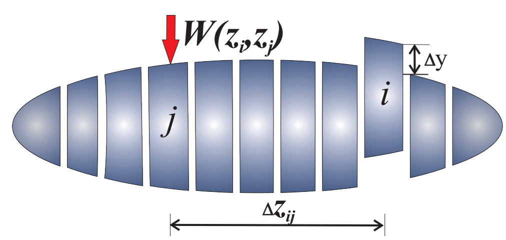

Figure 1: Slicing the 3D bunch in M longitudinal slices and introducing an offset $\triangle y$ in the transversal plane for each slice at the time.

After performing M simulations of the interaction where each of the slices $i = 1, \dots, M$ has an off-set at a time we receive the change of the vertical momentum $\Delta p_y(i,j)$ of the bunch particles averaged for each trailing slice $j = i+1, \dots, M$. Normalizing the dipole kick $\Delta p_y(i,j)$ by the number of particles N_i contained in the displaced slice i and the amount of the displacement Δy_i the entries of the wake matrix $W_1(z_j, z_i)$ write as:

$$W_1(z_j, z_i) = \frac{\gamma \Delta p_y(j,i)}{p_b r_e \Delta y_i N_i} [1/\mathrm{m}^2], \quad (1)$$

where p_b is the momentum of the bunch, γ the Lorentz factor and r_e the classical electron radius. Such a computed wake field can be converted in $[\mathrm{V/Cm}]$ by multiplying it with $1/4\pi\varepsilon_0$. The series of simulations we performed for

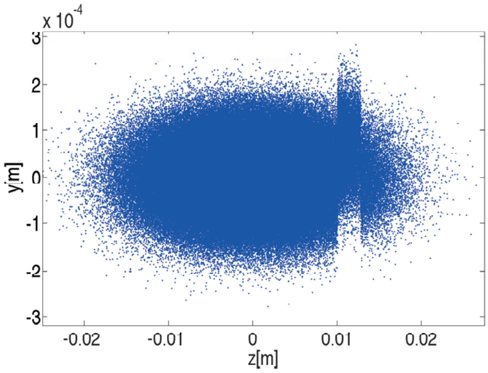

Figure 2: The bunch is represented by 10^6 macro-particles. In longitudinal direction the bunch is sliced in $M = 20$ slices. The 5th slice from the head of the bunch is displaced by $\Delta y_5 = \sigma_y$.

the KEKB bunch as illustrated in Figure 2. The beam pipe has a small radius of 5 mm and a uniform electron distribution fills a length of 10 mm. The positron bunch is represented by 10^6 macro-particles whereas the cloud, at least for lower densities, is represented by unit charges. In each direction the bunch particles have a Gaussian distribution, longitudinally the bunch spreads from $-4\sigma_z$ to $+4\sigma_z$ and is virtually sliced in $M = 20$ slices. The thickness of the slices correlates with the vertical velocity of the electrons near the beam axis. Here the thickness in the lab frame corresponds to the time the electrons need to change their vertical position for one σ_y in the proximity of the beam axis. The time step used for the interaction simulation is 1 ps.

Wake Function vs. Displacement

We compute the wake field for various displacements $\Delta y = 0.1 : 0.1 : 1\sigma_y$ of the 5th slice. The wake field is expressed in (1) as a linear coefficient of the dipole kick $\Delta p_y(i,j)$ and the dipole perturbation $\Delta y_i N_i$. Figure 3 shows a linear growth of the dipole kick with the displacement. The wake field in Figure 4 shows opposite ordering of the lines due to the growing denominator with the displacement in expression (1). The conclusion that the wake field saturates for displacement amplitudes comparable to the beam size corresponds with the results in [2].

Wake Function vs. Electron Density

In the simulations the density of the electron cloud was varied below and above the registered threshold value for incoherent single bunch instability 10^{12} electrons/m^3 at KEKB. Figure 6 shows that the amplitude of the wake forces grows with the growing density of the e-cloud. The blue line from Figure 6 for $\rho_e = 10^{11}$ electrons/m^3 is plotted once again in Figure 5. Figure 5 shows the presence of the characteristic dipole wake kick (from the electrons

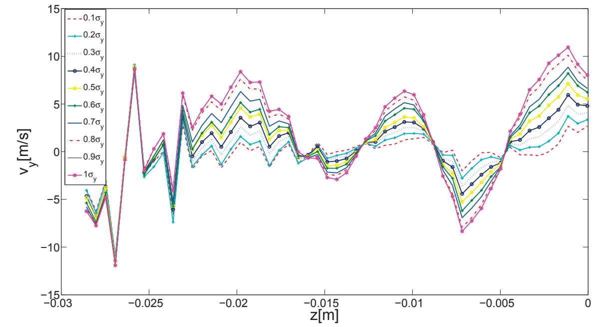

Figure 3: Average vertical dipole kick $\Delta p_y(5,j)$ on the bunch slices $j = 6,\ldots,M$, simulated for displacements of the 5th slice from 0.1σ to 1σ.

perturbed by the 5th slice) also for lower electron densities below the instability threshold. The oscillations of the wake kick are closely correlated to the oscillations of the electrons near the beam axis.

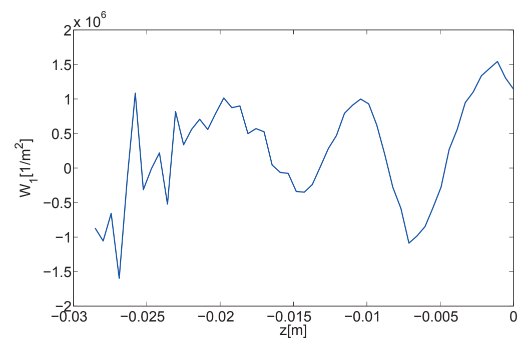

Figure 5: Average vertical wake field $W_1(5,j)$ on the bunch particles of slices $j = 6,\ldots,M$, computed for an e-cloud density of $\rho_e = 10^{11}$ electrons/m^3.

CONCLUSION

The computed wake field for different displacements of the bunch parts shows a linear scalability. Further verification is needed especially to evaluate the role of the beam pipe size which changes the influence of the mirror charges. Never the less the results show that the wake matrix could be used for the assessment of the single bunch stability.

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ISBN 978-3-95450-115-1

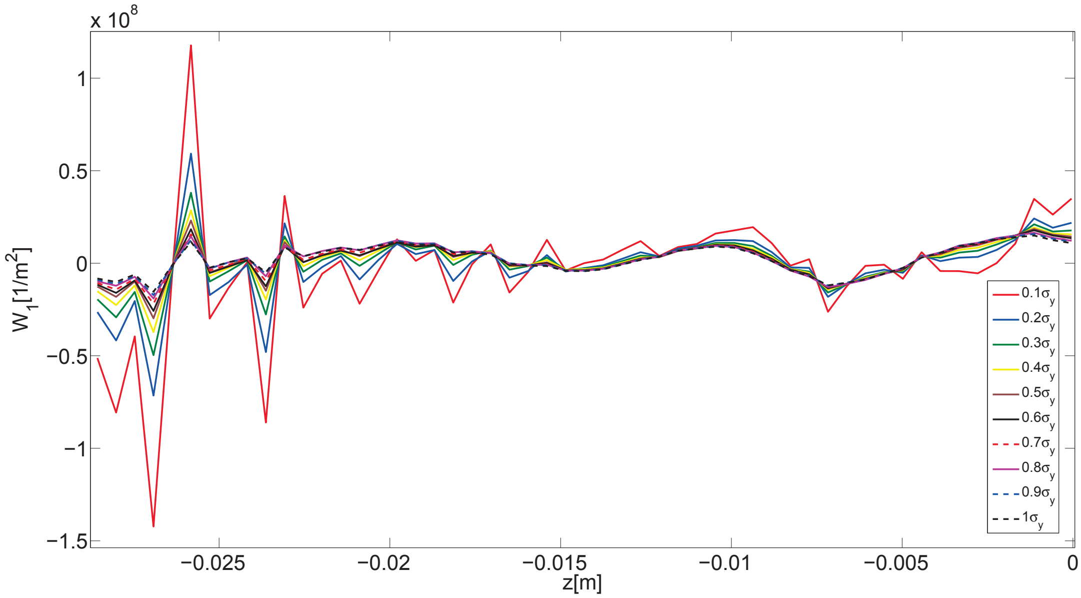

Figure 4: Average vertical wake field $W_1(5, j)$ on the bunch particles of slices $j = 6, \ldots, M$, computed for displacements of the 5th slice from 0.1σ to 1σ.

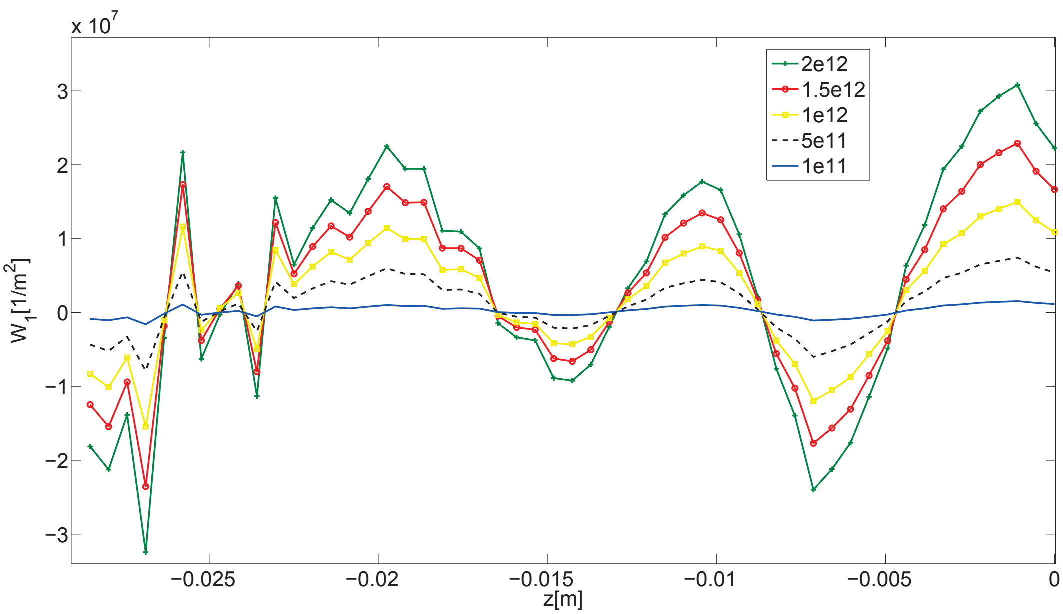

Figure 6: Average vertical wake field $W_1(5, j)$ on the bunch particles of slices $j = 6, \ldots, M$, computed for e-cloud densities $\rho_e = 1.0 \cdot 10^{11}, 5.0 \cdot 10^{11}, 1.0 \cdot 10^{12}, 1.5 \cdot 10^{12}$ and $2.0 \cdot 10^{12}$ electrons /m^3.

ISBN 978-3-95450-115-1