

EIGENMODE COMPUTATION FOR FERRITE-LOADED CAVITY RESONATORS *

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Abstract

The GSI Helmholtzzentrum für Schwerionenforschung in Darmstadt is operating the heavy-ion synchrotron SIS18 for fundamental research. Within the ring two biased ferrite-loaded cavity resonators are installed. During the acceleration phase their resonance frequency has to be adjusted to the revolution frequency of the heavy ions to reflect their increasing speed. To this end, a properly chosen bias current is used to modify the differential permeability of the ferrite material which consequently enables to adjust the eigenfrequency of the resonator system.

The goal of the presented work is to numerically determine the lowest eigensolutions of accelerating ferrite-loaded cavities based on the Finite Integration Technique. Since the underlying eigenmodes depend on the differential permeability, the static magnetic field generated by the bias current has to be computed in a first step. The eigenmodes can then be determined with the help of a dedicated Jacobi-Davidson eigensolver. Particular emphasis is put on the implementation to enable high performance computations based on distributed memory machines.

INTRODUCTION

For acceleration of charged particles at the heavy-ion synchrotron at GSI two ferrite-loaded cavity resonators are installed within the ring. The main advantage of ferrite cavities is twofold [1]: On one hand, the ferrites cause a reduction of the wavelength compared to vacuum, which allows the construction of smaller accelerating structures. On the other hand, the resonance frequency can easily be tuned by properly choosing a bias current and thereby modifying the differential permeability of the ferrite material. For the SIS 18 ferrite cavity, biasing enables to alter the resonance frequency in a range of about 0.8 to 5.4 MHz. The tuning is of particular importance during the acceleration phase, in which the resonance frequency of the cavities has to be adjusted to the revolution frequency of the heavy ions in order to reflect their increasing speed.

In this paper, after briefly presenting the SIS 18 ferrite cavity, fundamental relations relevant for the calculation of eigenmodes of biased ferrite-loaded cavity resonators are summarized, followed by a description of the applied numerical approach. Finally, we conclude with a simple numerical example.

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MAIN COMPONENTS OF THE SIS 18 FERRITE CAVITY

A simplified sketch of the main components of the SIS 18 ferrite cavity is shown in Fig. 1. Inside the cavity housing 64 ferrite ring cores are installed around the beam pipe. A magnetic field is generated in these rings by means of two different current windings: Firstly, a field constant in time due to the bias current of up to a few hundred ampere, and, secondly, an additional time-harmonic component induced via the radio frequency (RF)-coupling. As a consequence of the RF-field, a time-harmonic voltage is induced at the ceramic gap in the center of the resonator structure, which provides the actual acceleration of the particle beam. Moreover, copper cooling disks are installed between the ferrite material to remove the heat introduced due to magnetic losses. A more detailed description of the SIS 18 ferrite cavity can be found in Ref. [1].

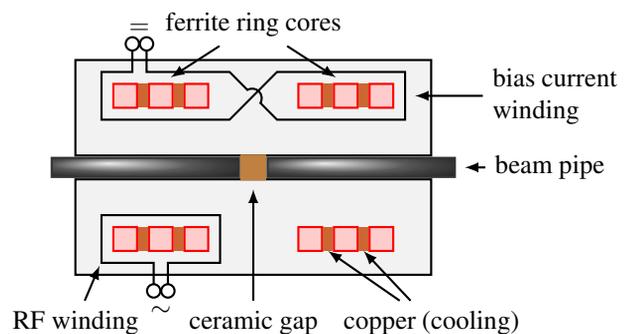


Figure 1: Simplified sketch of the main components of the SIS 18 ferrite cavity.

FUNDAMENTAL RELATIONS

As discussed before, the magnetic induction $\vec{B}(t)$ inside the accelerating cavity can be decomposed into a static field generated by the bias current and the time-harmonic part, i.e.

$$\vec{B}(t) = \mu_0 \mu_{\text{bias}} \vec{H}_{\text{bias}} + \mu_0 \overset{\leftrightarrow}{\mu}_d \text{Re} \left(\vec{H}_d \cdot e^{-i\omega t} \right). \quad (1)$$

Since the cavity is operated in a range where the bias current is much larger than the amplitude of the RF-component, the condition $|\vec{H}_d| \ll |\vec{H}_{\text{bias}}|$ is fulfilled in good approximation. Additionally, throughout this work the eigenvectors are calculated under the further assumption that effects of hysteresis are negligible. This allows

for a linearization of the nonlinear constitutive equation for the ferrite material at the current working point. By modifying the bias current, the static magnetic field is changed and hence the working point on the $B(H)$ -curve is set. As a further implication, the differential permeability at this working point is also adjusted. This directly allows for a tuning of the resonance frequency of the ferrite cavity, since, applying perfect electric boundary conditions, the eigensolutions are determined by the fundamental relations

$$\epsilon^{-1} \nabla \times \left(\mu_0 \overset{\leftrightarrow}{\mu}_d^{-1} \nabla \times \vec{E}(\vec{r}, t) \right) = \omega^2 \vec{E}(\vec{r}, t), \quad \vec{r} \in \Omega, \quad (2)$$

$$\vec{n} \times \vec{E}(\vec{r}, t) = 0, \quad \vec{r} \in \partial\Omega. \quad (3)$$

Here ϵ is the permittivity, $\overset{\leftrightarrow}{\mu}_d$ the differential permeability, ω the eigenfrequency of the corresponding RF mode and \vec{n} a normal vector on the cavity boundary $\partial\Omega$. It is worth emphasizing that the differential permeability tensor $\overset{\leftrightarrow}{\mu}_d$ is involved on the left hand side of equation (2). This tensor has the following properties: Firstly, it is a fully occupied (3×3)-tensor, which for a bias magnetic field aligned with the z -axis reduces to the well-known Polder tensor [2]

$$\overset{\leftrightarrow}{\mu}_d = \begin{pmatrix} 1 + \chi & i\kappa & 0 \\ -i\kappa & 1 + \chi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (4)$$

Secondly, this tensor is non-Hermitian due to the non-vanishing imaginary part of the parameters κ and χ in case that magnetic losses are taken into account. This results in complex eigenvalues. Moreover, κ and χ are functions of both the bias magnetic field and the frequency ω . Consequently, the magnetic field generated by the bias current has to be determined at first to solve the nonlinear eigenproblem. In order to meet these requirements a new solver is developed, which is described in more detail in the following section.

COMPUTATIONAL MODEL

The current implementation is based on the Finite Integration Technique (FIT) [3] using a hexahedral staircase mesh. For modeling the resonator structure as well as meshing and visualization of the simulation results CST STUDIO SUITE[®] [4] is used. The field solutions are obtained by a dedicated solver implemented in C / C++ and built on PETSc (Portable, Extensible Toolkit for Scientific Computation) [5]. The solver includes two main subcomponents (cf. Fig. 2): On one hand, a magnetostatic solver supporting nonlinear material for the computation of the magnetic field due to the bias current, which enables to linearize the constitutive equation for the ferrite material at the current working point; on the other hand, an eigensolver for the subsequent solution of the nonlinear eigenvalue problem.

Magnetostatic Solver

For the solution of the magnetostatic field problem

$$\nabla \times \vec{H} = \vec{J}, \quad (5)$$

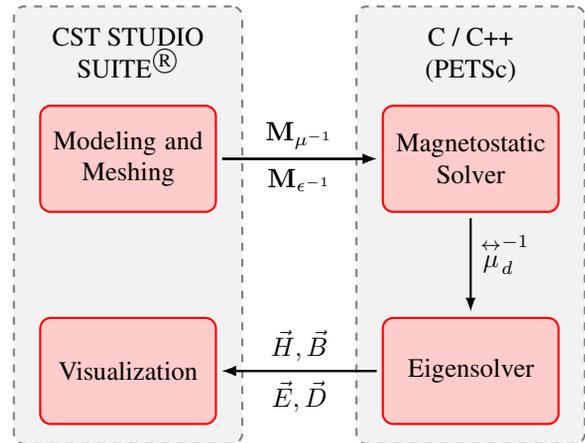


Figure 2: Strategy for the computation of eigenmodes of ferrite-loaded cavity resonators.

$$\nabla \cdot (\mu \vec{H}) = 0, \quad (6)$$

where \vec{J} is the (bias) current density, the so-called H_i -algorithm [6] is employed, which is based on the Helmholtz decomposition. According to this fundamental theorem, the magnetic field can be resolved into the sum of two fields

$$\vec{H} = \vec{H}_i + \vec{H}_h \quad (7)$$

$$\text{with } \nabla \times \vec{H}_i = \vec{J} \text{ and } \vec{H}_h = -\nabla\varphi, \quad (8)$$

where φ is a scalar potential. The H_i -algorithm hence starts with the construction of the field \vec{H}_i . As this vector field may be chosen arbitrarily and may, in particular, be unphysical, a solution as simple as possible should be preferred. For this purpose, firstly the smallest subvolume containing all current paths is selected. Having done this, the vector components of \vec{H}_i are set such that the rotational equation is trivially fulfilled on its surface. After dividing this volume into two subvolumes, the vector components are also set accordingly on the new surface. This is repeated until all vector components are set.

In the subsequent step the scalar potential is computed by solving equation (6), which for the ansatz (7), (8) takes the form

$$\nabla \cdot (\mu \nabla \varphi) = \nabla \cdot (\mu \vec{H}_i). \quad (9)$$

In order to reflect its nonlinear character, an iterative approach, in this work either a simple successive substitution or the Newton method, is applied until convergence of the value for the permeability is observed. The obtained field solution defines the working point, at which also the permeability tensor $\overset{\leftrightarrow}{\mu}_d$ is constructed. Note that this linearization of the constitutive equation for the ferrite material enters in the subsequent eigenvalue solver.

Eigenvalue Solver

The nonlinear eigenvalue equation (2), (3) is iteratively solved as a sequence of linearized eigenproblems, whose eigenvectors are calculated with a solver of Jacobi-Davidson type [7] suited for the computation of interior

eigenvalues. After each (nonlinear) iteration the permeability tensor $\overleftrightarrow{\mu}_d$ is reconstructed for the current estimate of the eigenfrequency. Since the eigensolutions of subsequent iterations differ only slightly, all obtained eigenvectors of the previous iteration step are reused as start vectors in the next step, which significantly reduces the computational effort.

The convergence of eigenvalues for the Jacobi-Davidson solver strongly depends on the preconditioner used for the Jacobi-Davidson correction equation. In this work, the (computationally expensive) LU decomposition is calculated at the first time the correction equation is to be solved. Yet, for all subsequent steps satisfactory convergence is still observed when the same preconditioner is kept, even when used in different nonlinear iterations, if only a few eigenvalues are desired.

PARALLEL COMPUTING

Because of the clear demand of precise calculations, the implementation particularly aims at efficient computing based on distributed memory machines. To this end, the degrees of freedom are arranged such that the topological matrices and the permeability tensor have only few non-zero components in the far off-diagonal regions of the matrix. This directly leads to reduced communication between distinct processes and thus to a higher computation to communication ratio. For instance, the inverse of the permeability tensor $\overleftrightarrow{\mu}_d$ can be constructed fully locally without any communication between different processes. Furthermore, all degrees of freedom which are zero in the FIT because they are allocated on elements outside the computation domain (including perfect electric conductor cells) or due to boundary conditions are completely removed beforehand, which positively affects load balancing.

NUMERICAL EXAMPLE

Since the realization of the magnetostatic solver for nonlinear material has successfully been tested independently, here we focus on the verification of the nonlinear eigensolver. To this end, a lossless, ferrite-filled cylindrical cavity resonator (radius 1 m, length 2 m) longitudinally biased by a homogeneous static magnetic field is considered. Assuming that its magnetic properties can be described by the Polder tensor (cf. equation (4)), a characteristic equation determining the resonance frequencies can be formulated analytically [8]. For a relative permittivity of $\epsilon_r = 1$, a relative permeability of $\mu_r = 7$ and a bias magnetic field strength of $H_{\text{bias}} = 2750$ A/m, the values for the lowest eigenfrequencies are compared with the ones obtained numerically with the new implementation of the nonlinear eigensolver described before. Eigenvalues are accepted if the residual norm is below 10^{-9} in each linearized step and the relative change of the eigenvalue of two subsequent nonlinear iterations does not exceed 10^{-6} . As suggested by a convergence study (cf. Fig. 3), good accordance of the numerical values with the analytical results is evident.

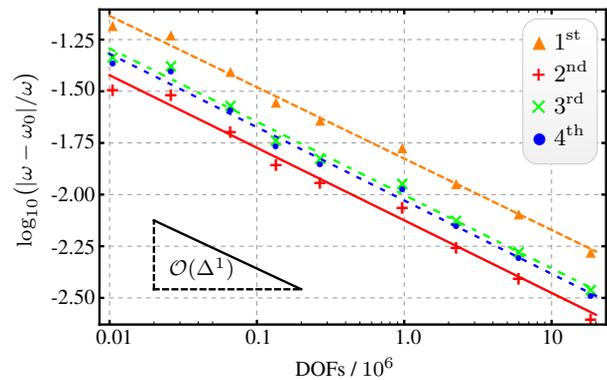


Figure 3: Relative deviation of the numerically obtained value ω to the analytical result ω_0 as a function of the degrees of freedom (DOFs) for the four lowest eigenfrequencies for a lossless, ferrite-filled cylindrical cavity resonator.

SUMMARY AND OUTLOOK

The goal of this work is the computation of eigenmodes of biased ferrite-cavities discretized by the FIT. To this end, the static magnetic field generated by the bias current is calculated at first by means of a nonlinear magnetostatic solver. Moreover, since the frequency dependence of the permeability tensor results in a nonlinear eigenvalue problem, a dedicated solver for parallel computation on distributed memory machines has been developed. First numerical results have been presented for a lossless, longitudinally biased, ferrite-filled cylindrical cavity resonator. Future work will mainly focus on the support of lossy material and the application of the solver to the SIS 18 ferrite cavity.

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