

# Dual AC Dipole Excitation for the Measurement of Magnetic Multipole Strength from Beam Position Monitor Data

# Outline

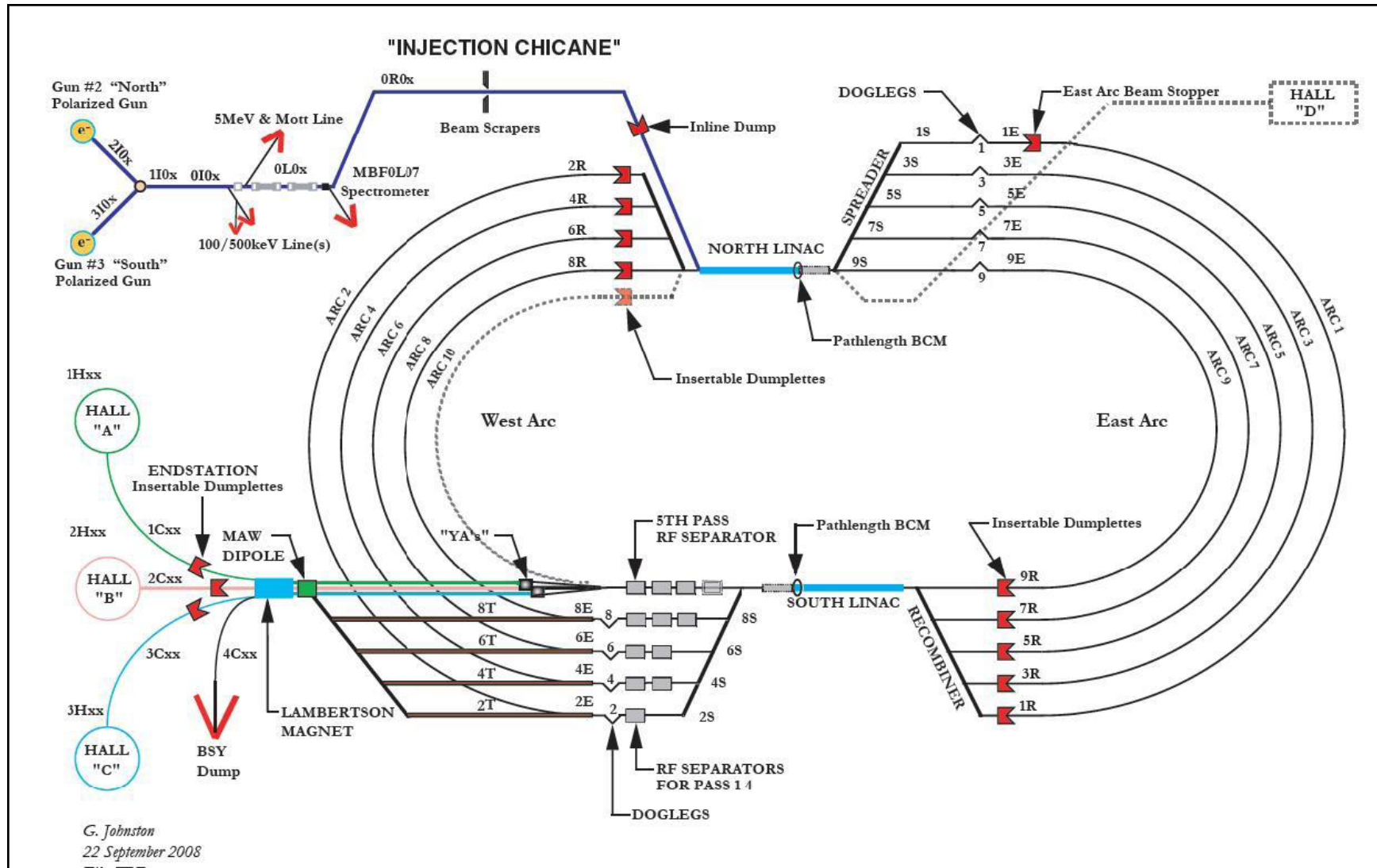
- **CEBAF Overview**
- **Analytical Model**
- **Experimental Setup**
- **ARC1 Beamline**
- **Sextupole Measurements**
- **Beam Position Monitor Nonlinearity**
- **ARC1 Dipole Measurements**
- **Future Measurements**
- **Summary**

# Aerial View

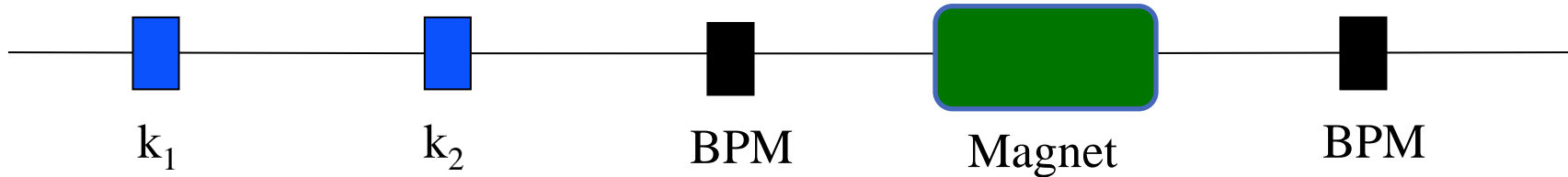
- 5-pass CW Electron Accelerator
- Three user facilities (A, B, C)
- CW Polarized Source
- >85% Polarization
- Two 1497 MHz Linacs
- Two Recirculation Arcs
- Operating at 6 GeV
- Now being upgraded to 12 GeV



# CEBAF Overview

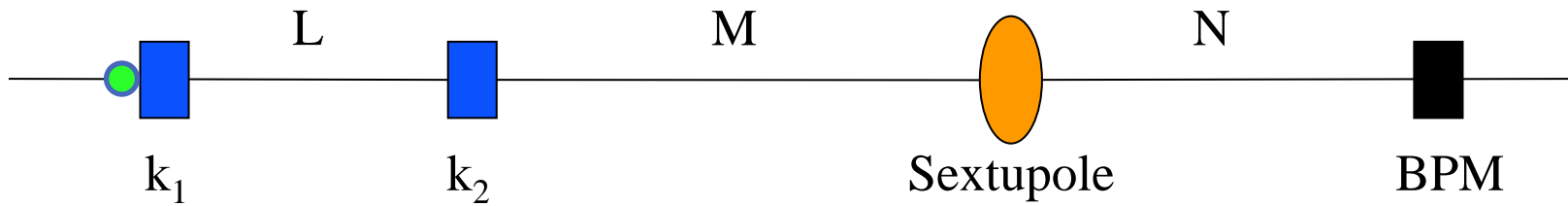


# Introduction



- Modulate the beam with two AC dipoles at two different frequencies
- Record time domain data at 8 Beam Position Monitors within a CEBAF ARC
- Perform Fast Fourier Transform of BPM data
- Linear system will just have kick frequencies with amplitudes that depend on the phase advance of the lattice
- Nonlinear fields will produce harmonics of the kick frequencies

# Analytical Model

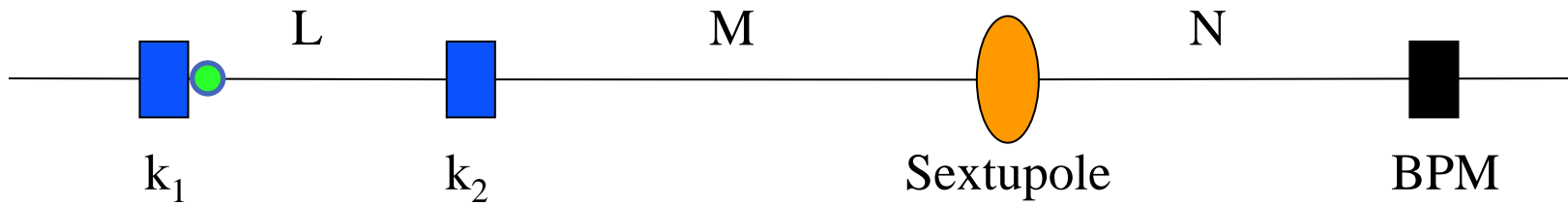


$$X_{k1-} = X'_{k1-} = 0$$

$$X_{k1+} = 0$$

$$X'_{k1+} = A_1 \cos \omega_1 t$$

# Analytical Model

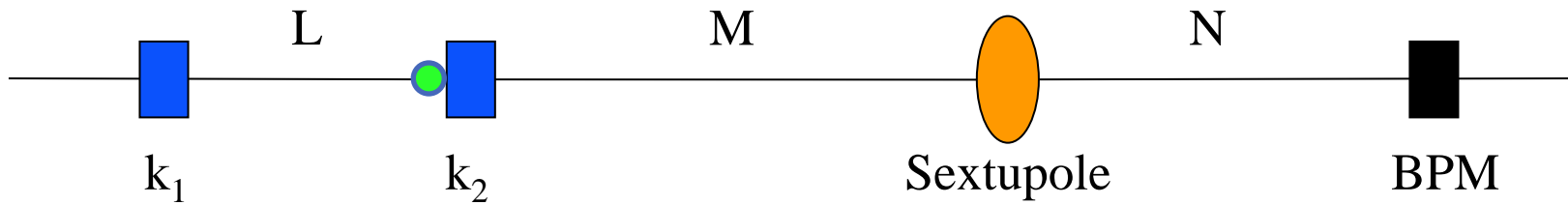


$$X_{k1-} = X'_{k1-} = 0$$

$$X_{k1+} = 0$$

$$X'_{k1+} = A_1 \cos \omega_1 t$$

# Analytical Model



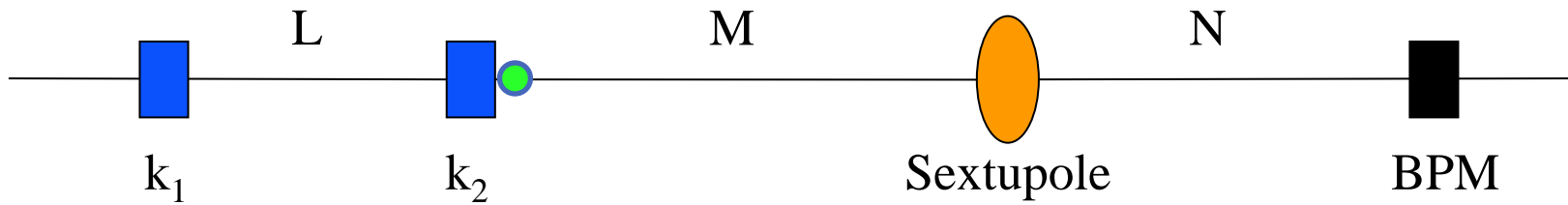
$$\begin{pmatrix} X_{k2-} \\ X'_{k2-} \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} X_{k1+} \\ X'_{k1+} \end{pmatrix}$$

$$X_{k2-} = L_{12} A_1 \cos \omega_1 t$$

$$X'_{k2-} = L_{22} A_1 \cos \omega_1 t$$



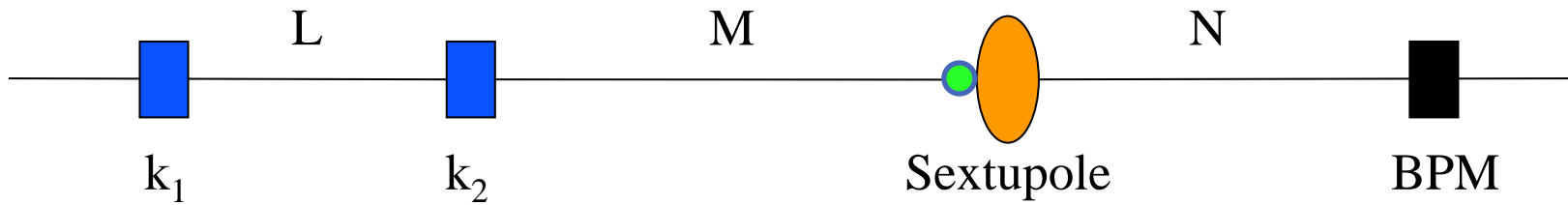
# Analytical Model



$$X_{k2+} = L_{12} A_1 \cos \omega_1 t$$

$$X'_{k2+} = L_{22} A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

# Analytical Model

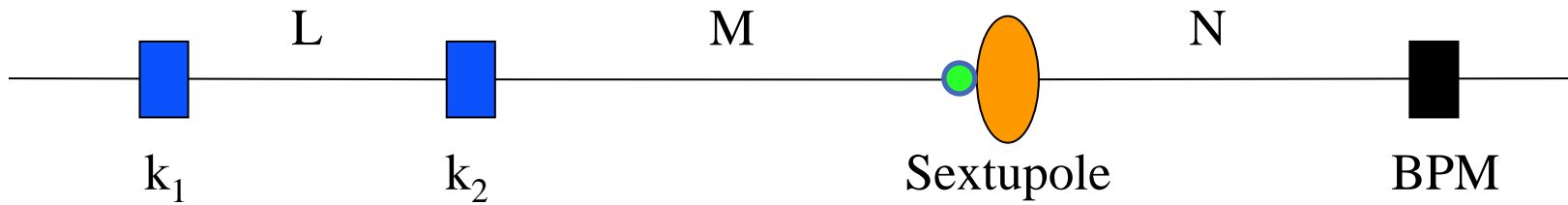


$$\begin{pmatrix} X_{s-} \\ X'_{s-} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} X_{k2+} \\ X'_{k2+} \end{pmatrix}$$

$$X_{s-} = (ML)_{12} A_1 \cos \omega_1 t + M_{12} A_2 \cos \omega_2 t$$

$$X'_{s-} = (ML)_{22} A_1 \cos \omega_1 t + M_{22} A_2 \cos \omega_2 t$$

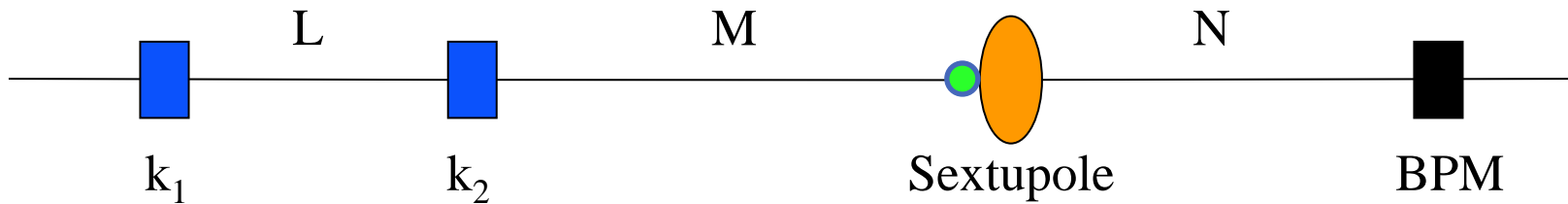
# Analytical Model



$$B_x = \frac{2B_0}{a^2} xy$$

$$B_y = \frac{B_0}{a^2} (x^2 - y^2)$$

# Analytical Model

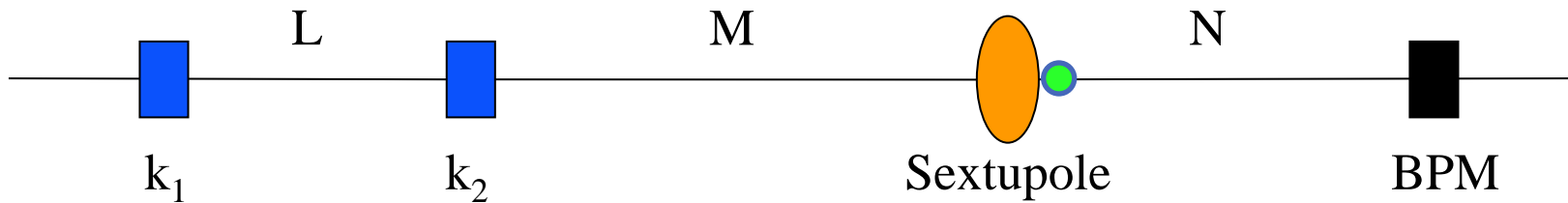


$$B_y = \frac{B_0}{a^2} x^2$$

$$\Delta\theta = 2.998 \times 10^{-4} \frac{B_y(G)L(cm)}{E(MeV)}$$

$$\Delta\theta = \frac{2.998 \times 10^{-4} B_0 L}{a^2 p_z} x^2 = SX_{s-}^2$$

# Analytical Model

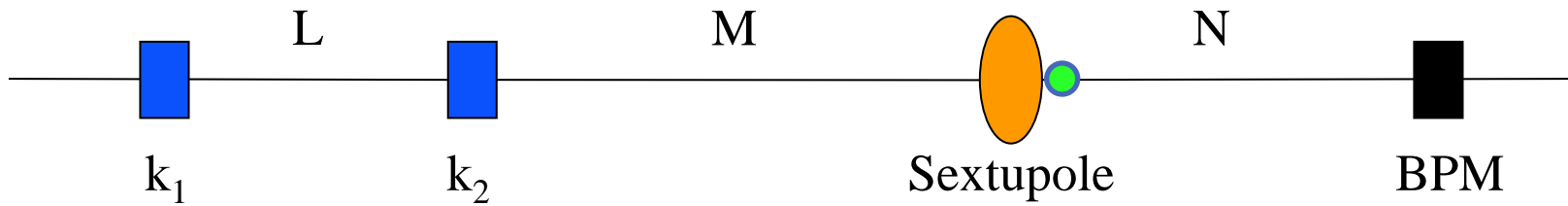


$$X_{s+} = (ML)_{12} A_1 \cos \omega_1 t + M_{12} A_2 \cos \omega_2 t$$

$$X'_{s+} = (ML)_{22} A_1 \cos \omega_1 t + M_{22} A_2 \cos \omega_2 t +$$

$$S \left[ \begin{array}{l} ((ML)_{12})^2 A_1^2 \cos^2 \omega_1 t + \\ 2(ML)_{12} (M_{12}) A_1 A_2 \cos \omega_1 t \cos \omega_2 t + \\ (M_{12})^2 A_2^2 \cos^2 \omega_2 t \end{array} \right]$$

# Analytical Model



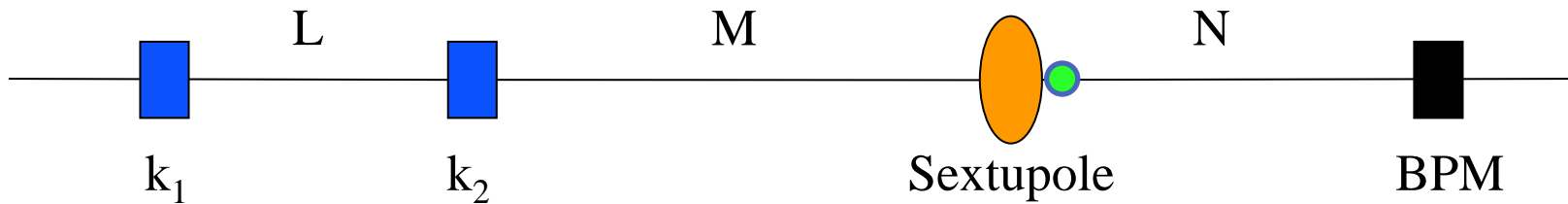
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$\cos^2 A = \frac{1}{2} [1 + \cos 2A]$$

# Analytical Model

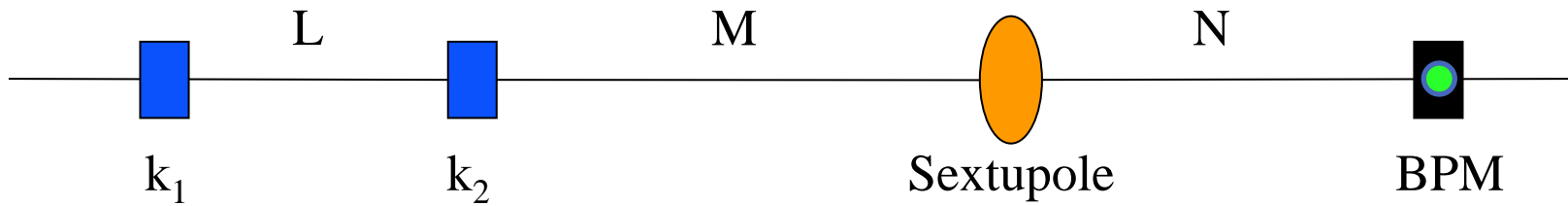


$$X_{s+} = (ML)_{12} A_1 \cos \omega_1 t + M_{12} A_2 \cos \omega_2 t$$

$$X'_{s+} = (ML)_{22} A_1 \cos \omega_1 t + M_{22} A_2 \cos \omega_2 t +$$

$$S \left[ \begin{aligned} & ((ML)_{12})^2 A_1^2 \frac{1}{2} [1 + \cos 2\omega_1 t] + \\ & (ML)_{12} (M_{12}) A_1 A_2 [\cos(\omega_1 + \omega_2)t + \\ & \cos(\omega_1 - \omega_2)t] + (M_{12})^2 A_2^2 \frac{1}{2} [1 + \cos 2\omega_2 t] \end{aligned} \right]$$

# Analytical Model



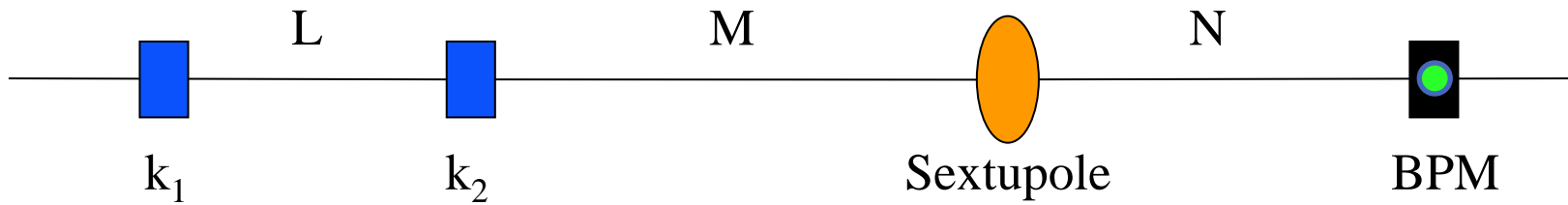
$$\begin{pmatrix} X_{bpm} \\ X'_{bpm} \end{pmatrix} = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix} \begin{pmatrix} X_{s+} \\ X'_{s+} \end{pmatrix}$$

$$X_{bpm} = N_{11}X_{s+} + N_{12}X'_{s+}$$

$$X'_{bpm} = N_{21}X_{s+} + N_{22}X'_{s+}$$



# Analytical Model



Kick frequencies for these tests were chosen to be 1 Hz and 21 Hz

## Sextupole Frequencies

$$\omega_1 = 1 \text{ Hz}$$

$$\omega_2 = 21 \text{ Hz}$$

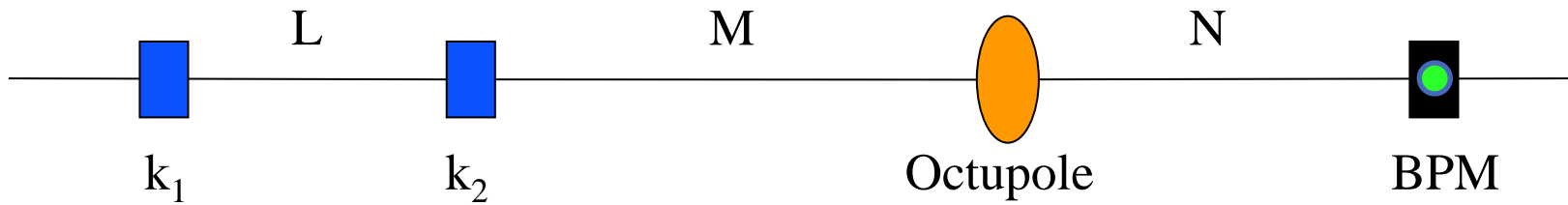
$$2\omega_1 = 2 \text{ Hz}$$

$$2\omega_2 = 42 \text{ Hz}$$

$$\omega_1 - \omega_2 = 20$$

$$\omega_1 + \omega_2 = 22 \text{ Hz}$$

# Analytical Model

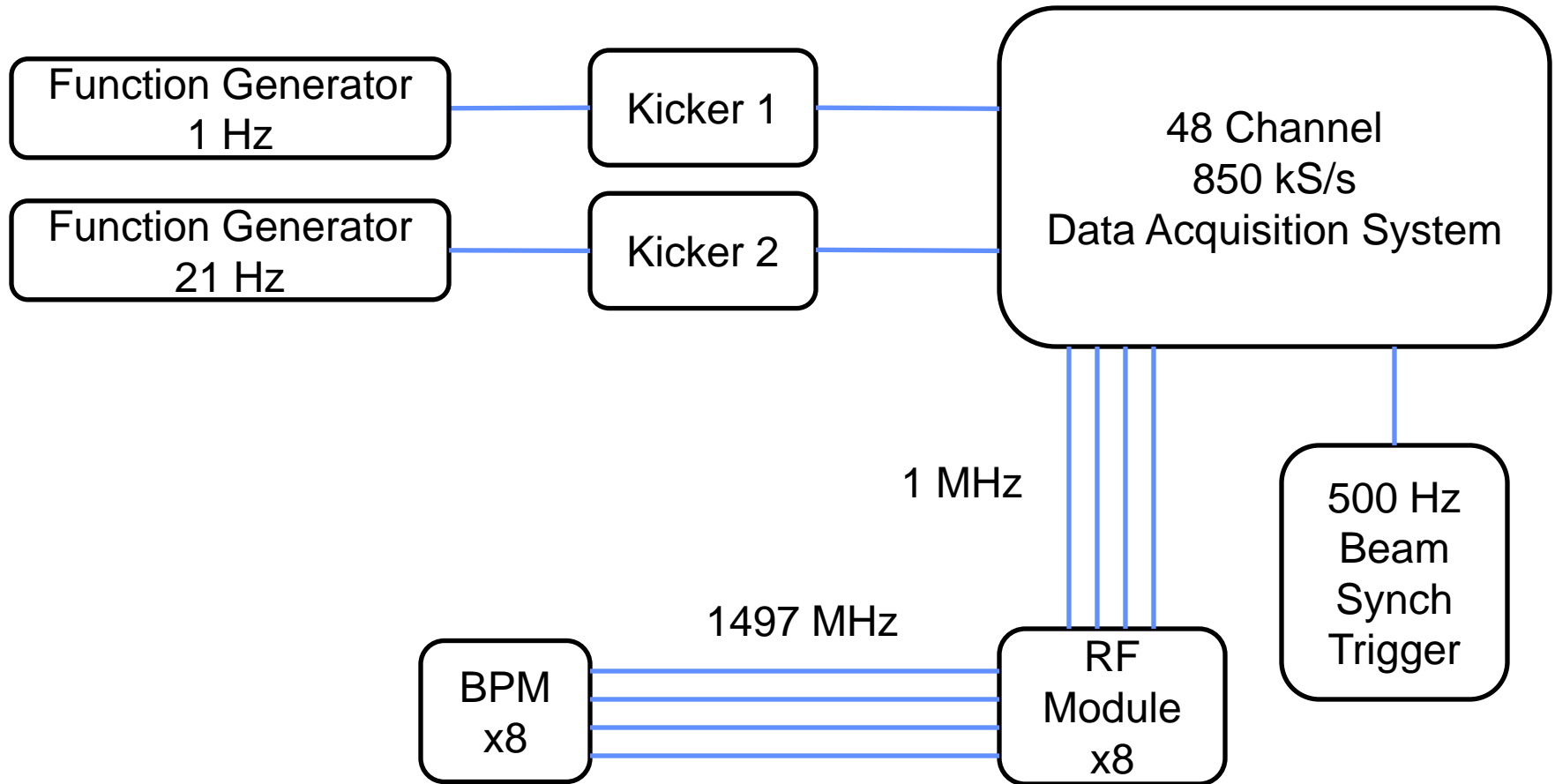


Kick frequencies for these tests were chosen to be 1 Hz and 21 Hz

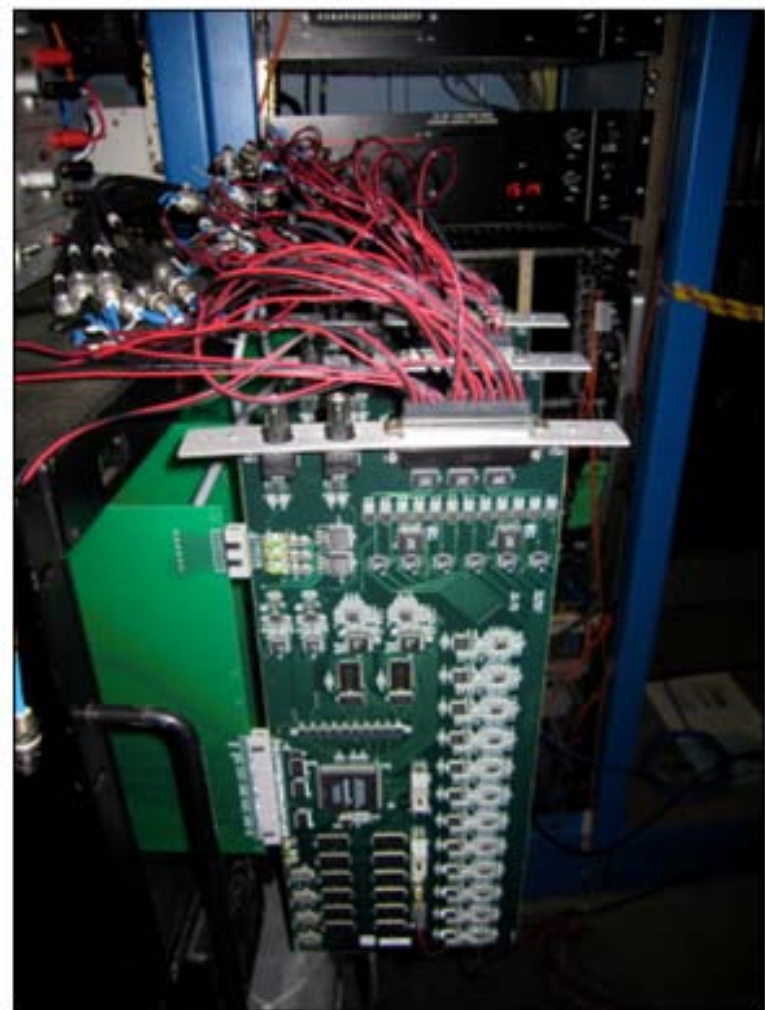
## Octupole Frequencies

$\omega_1 = 1 \text{ Hz}$	$\omega_2 = 21 \text{ Hz}$
$3\omega_1 = 3 \text{ Hz}$	$3\omega_2 = 63 \text{ Hz}$
$\omega_2 + 2\omega_1 = 23$	$\omega_2 - 2\omega_1 = 19$
$2\omega_2 + \omega_1 = 43$	$2\omega_2 - \omega_1 = 41$

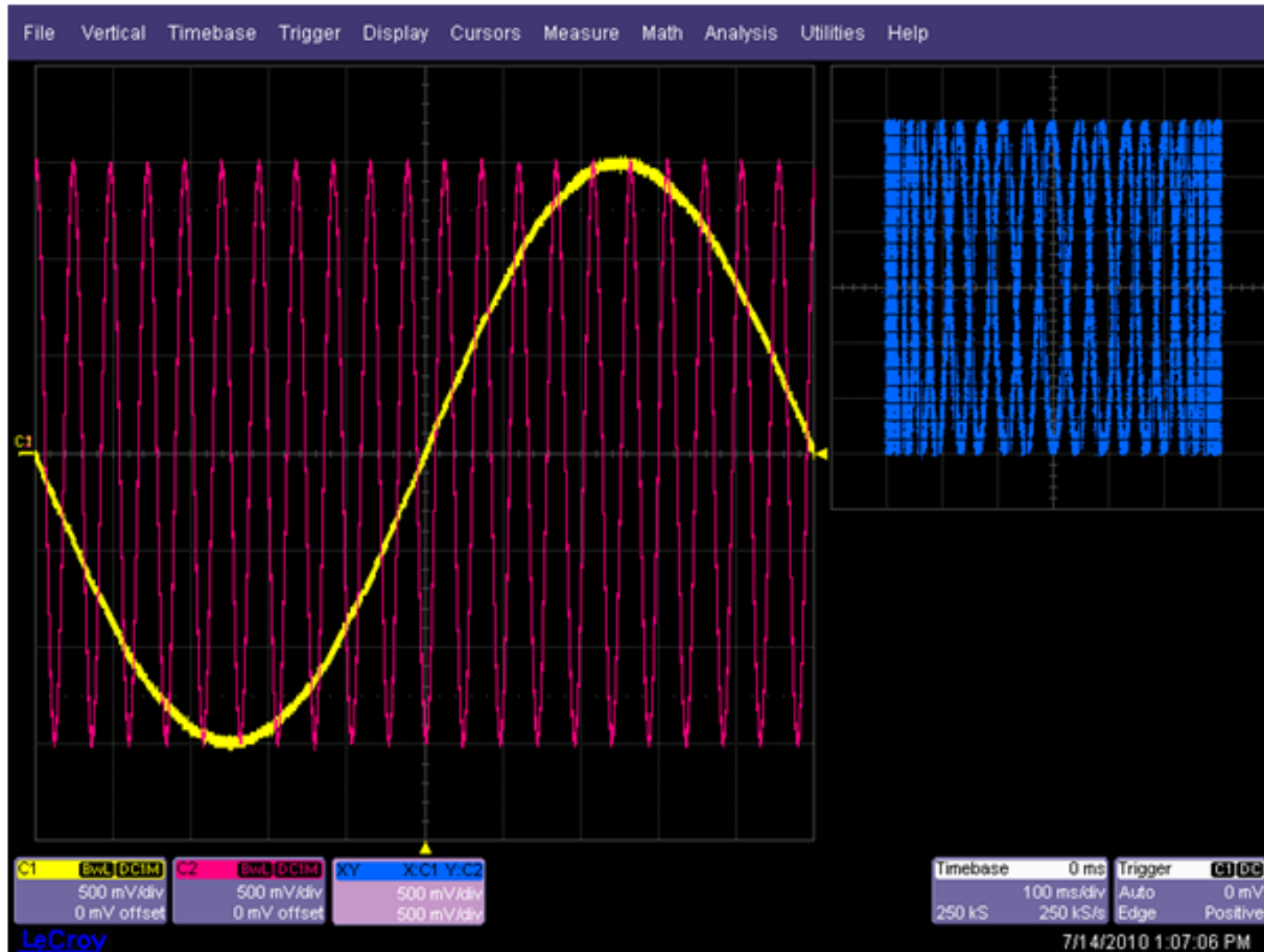
# Experimental Equipment



# Data Acquisition System



# Kicker Magnets

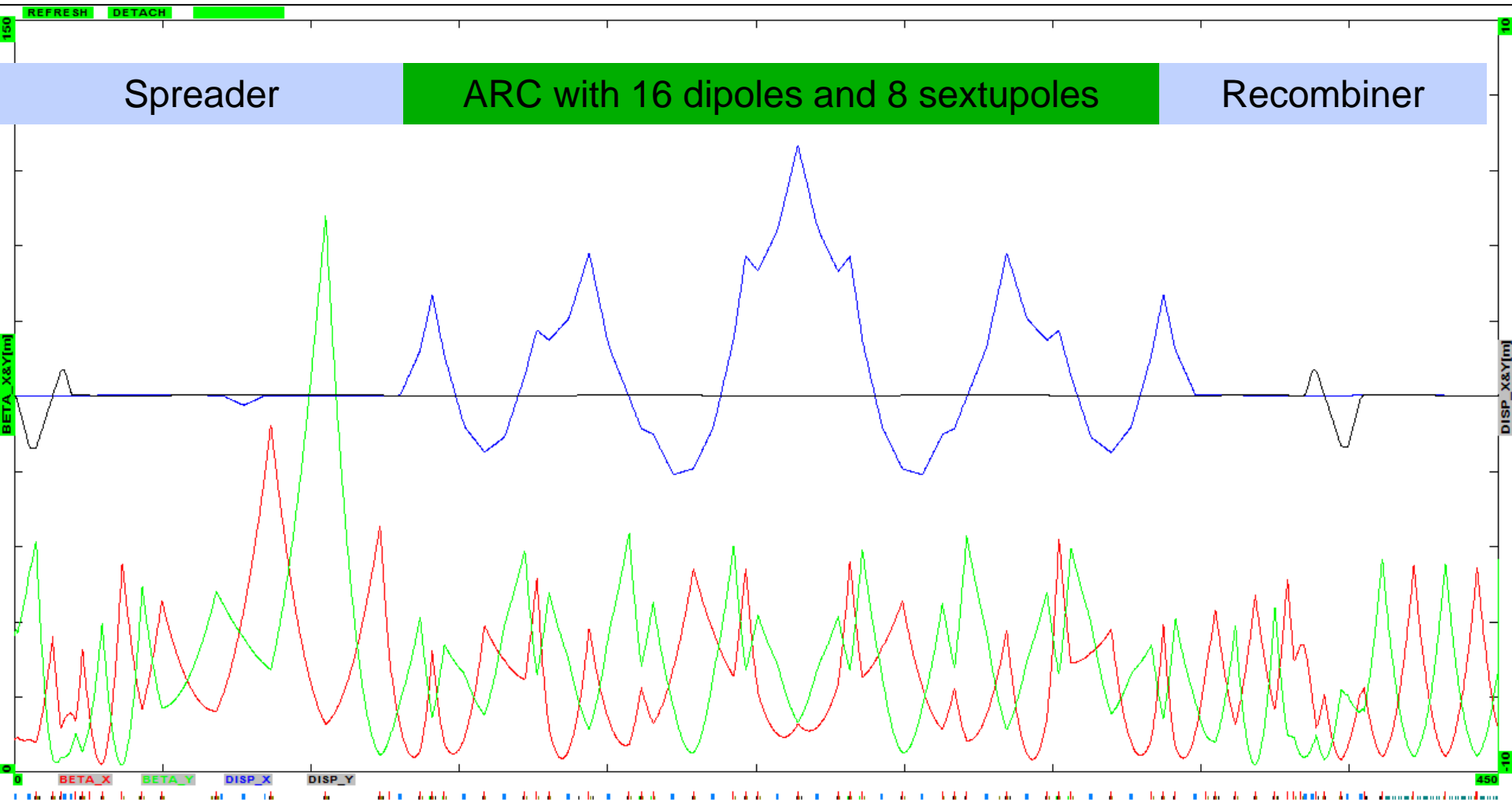




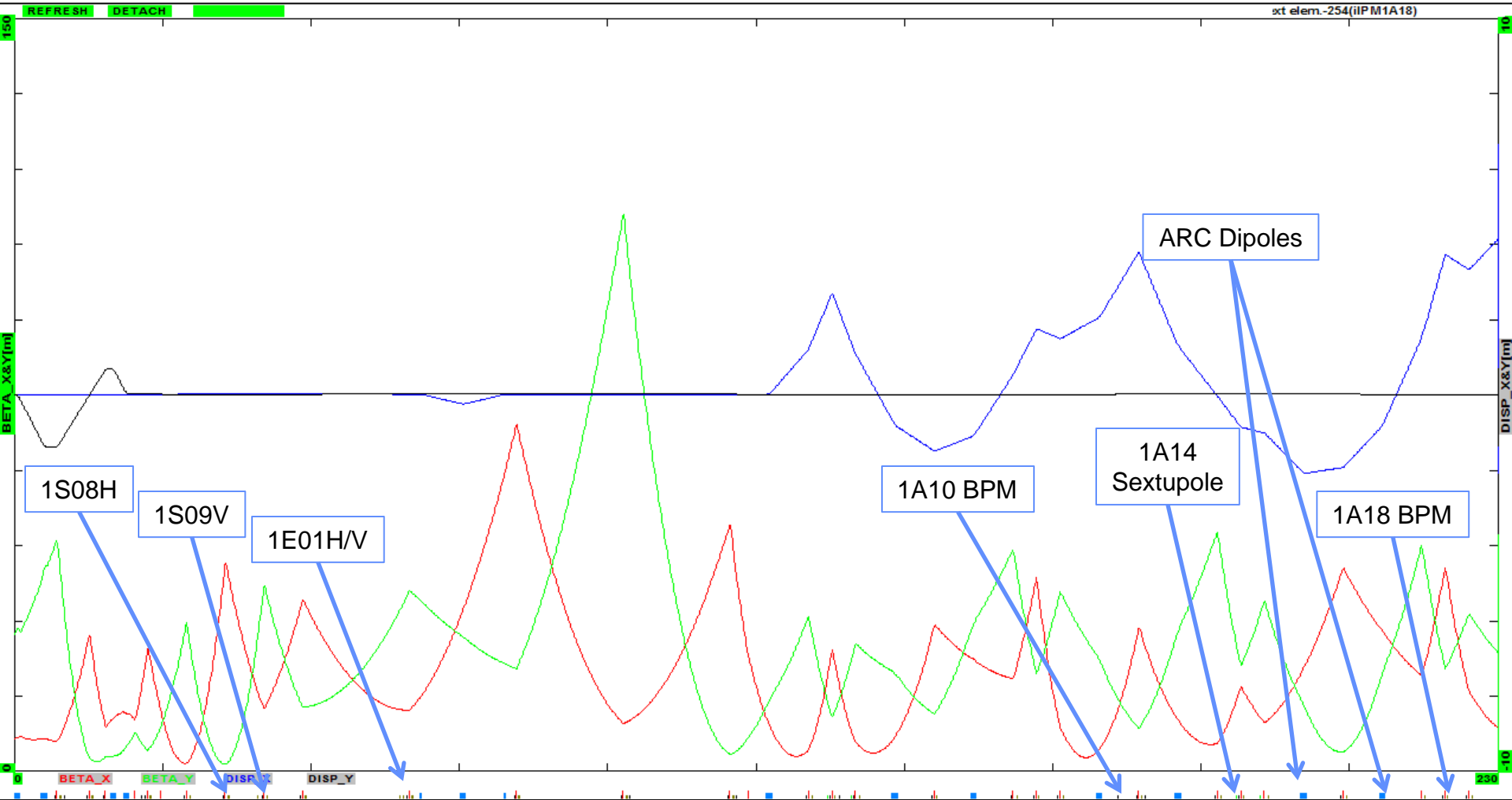
# ARC1 Beamline



# Arc1 Optics



# Arc1 Optics

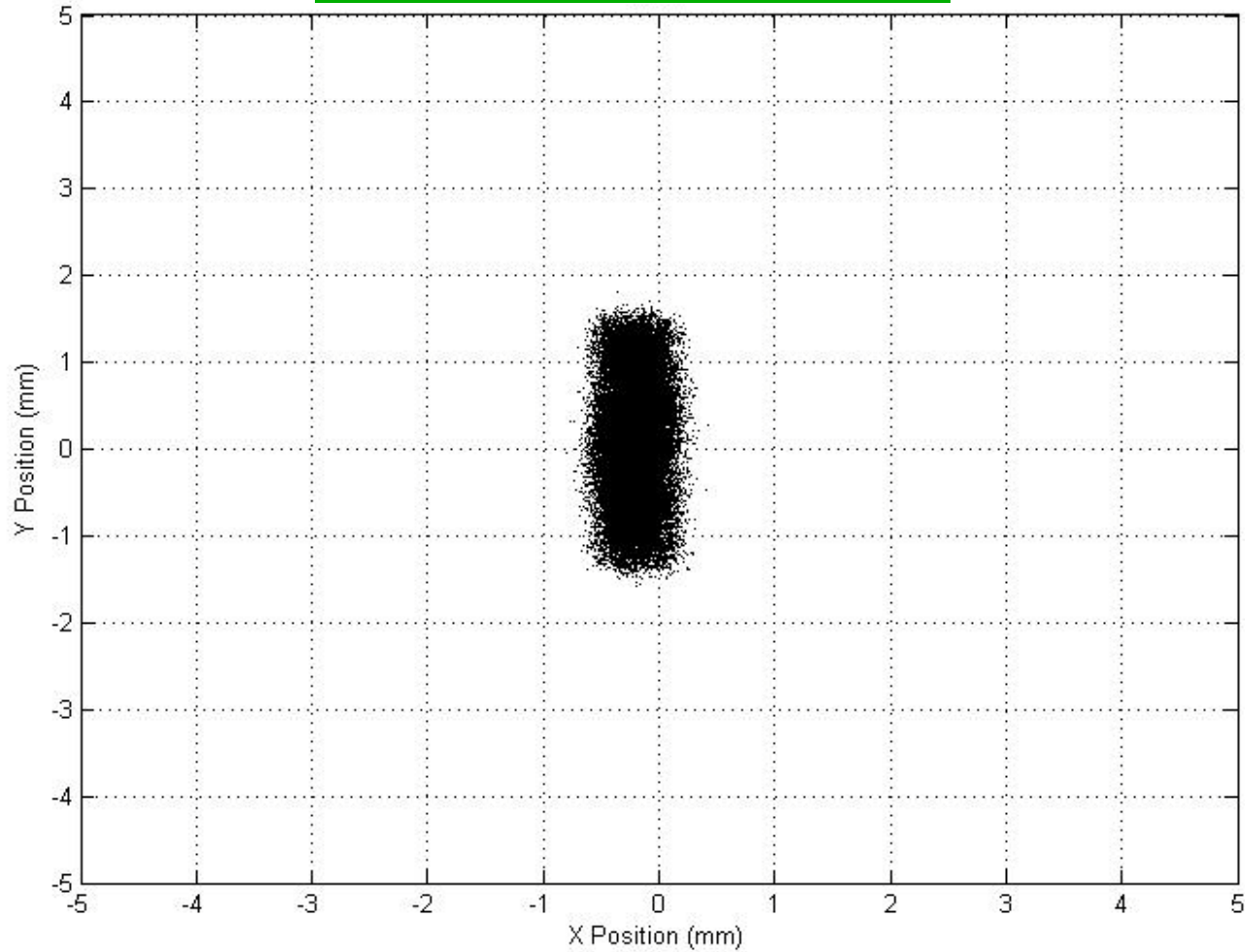


20 m



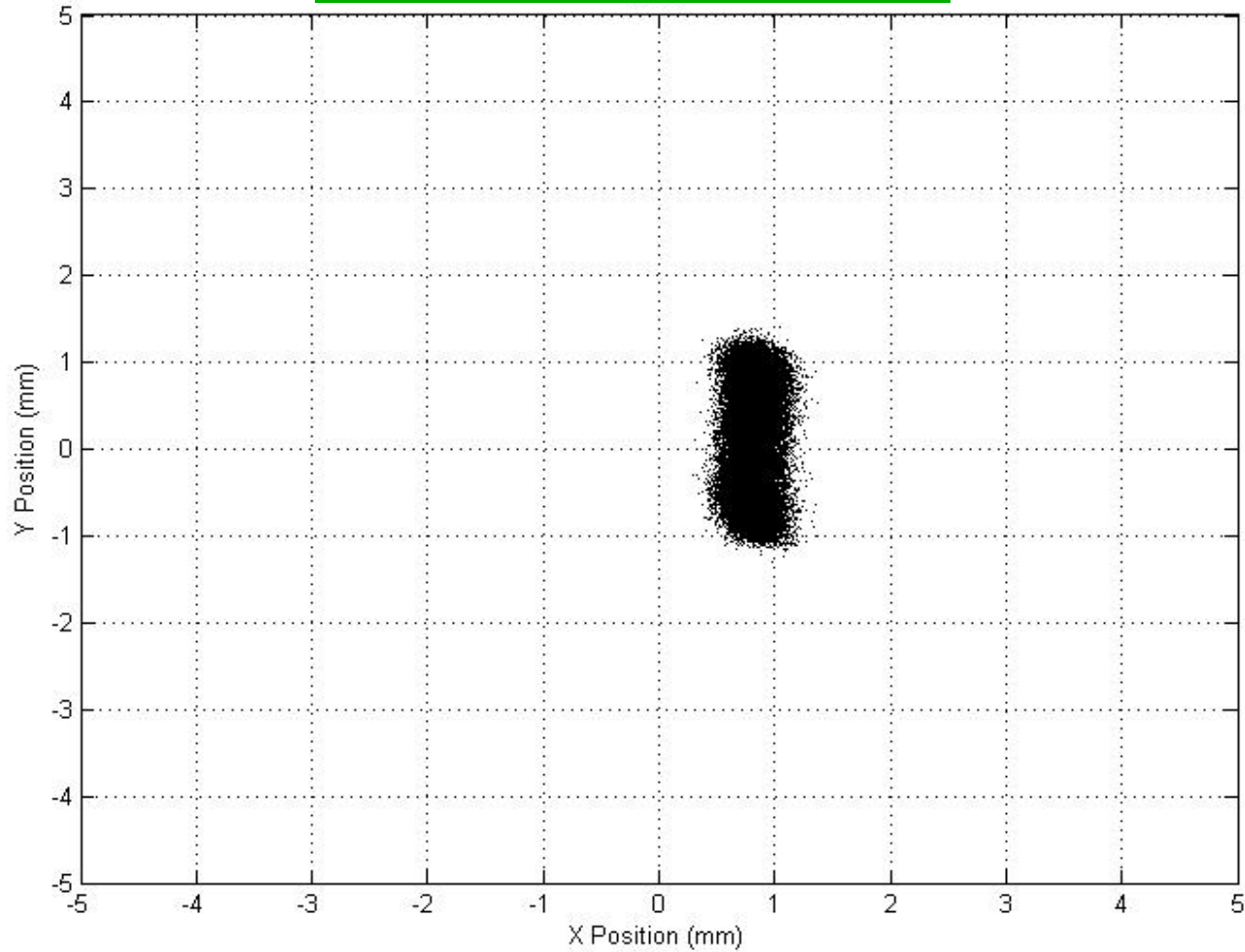
# X-Y Beam Profiles

1A10 BPM Before Sextupole



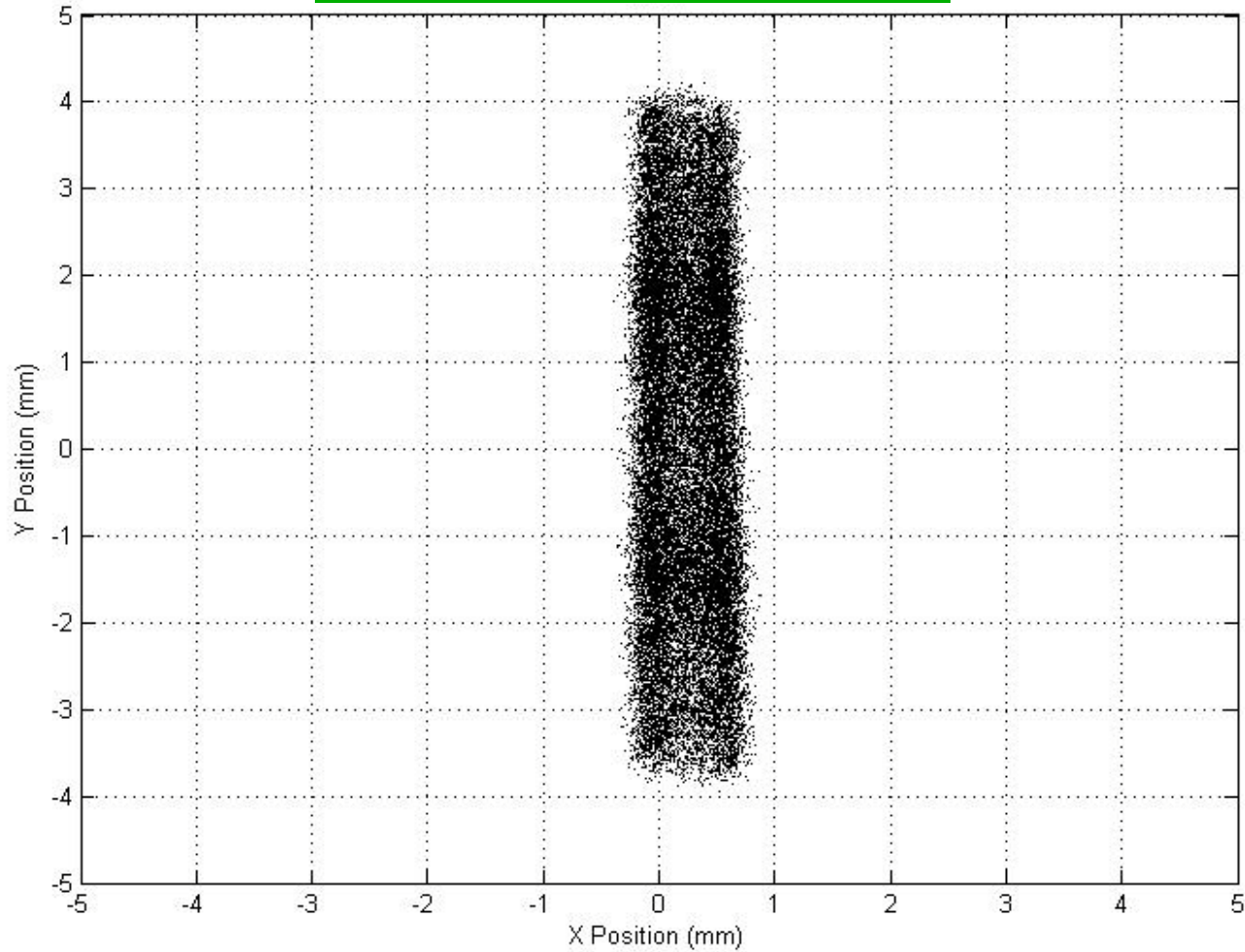
# X-Y Beam Profiles

1A11 BPM Before Sextupole



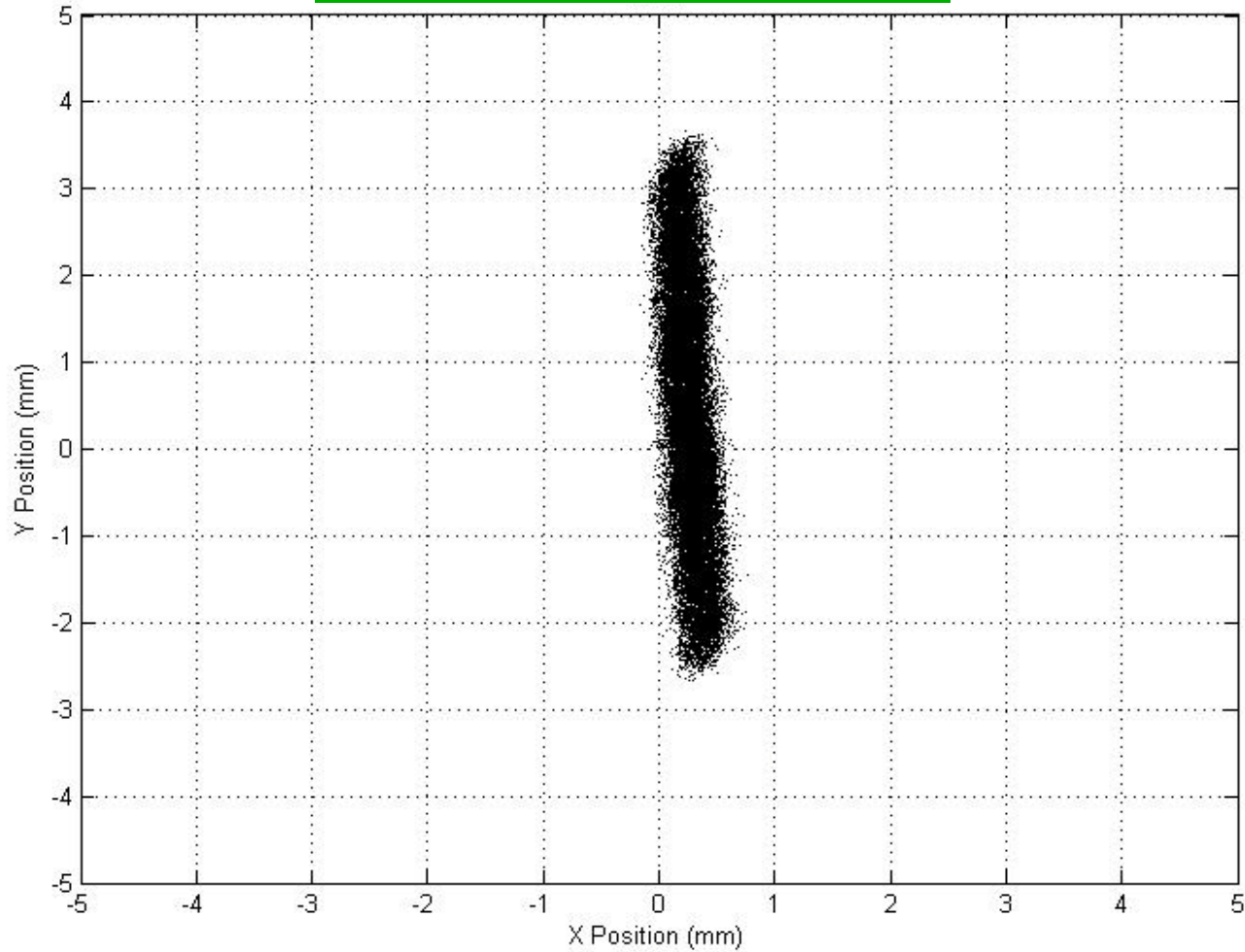
# X-Y Beam Profiles

1A13 BPM Before Sextupole



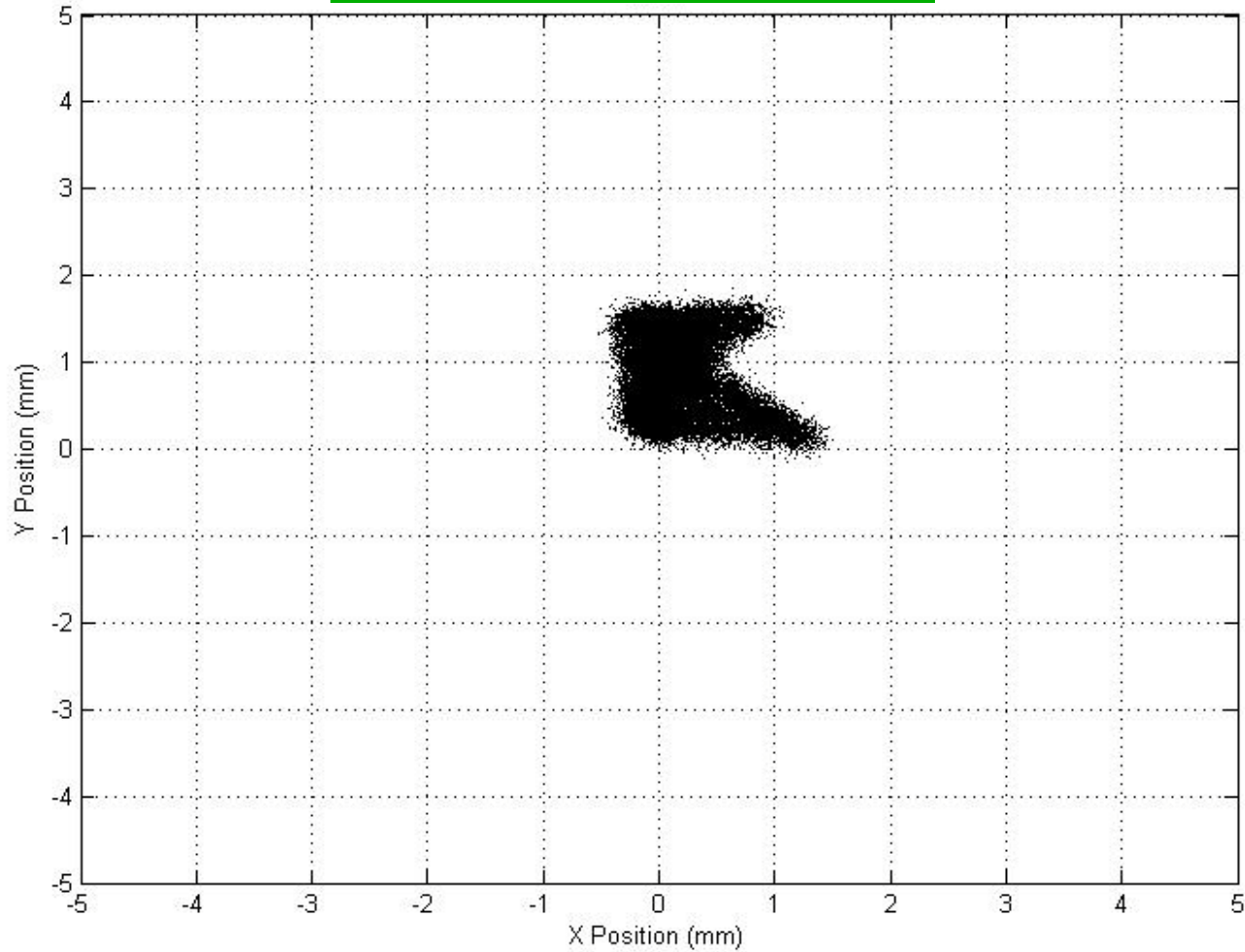
# X-Y Beam Profiles

1A14 BPM Before Sextupole



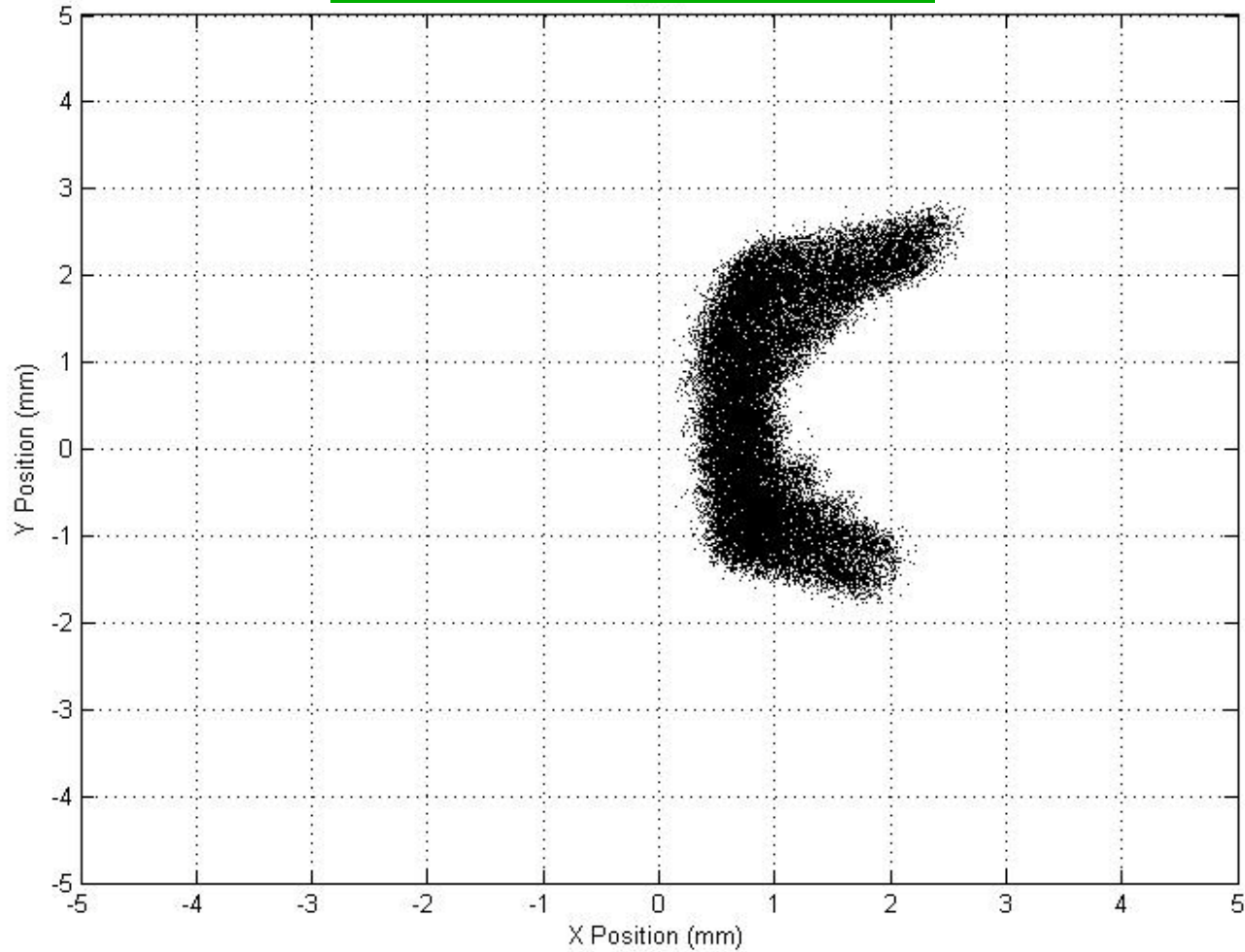
# X-Y Beam Profiles

1A16 BPM After Sextupole



# X-Y Beam Profiles

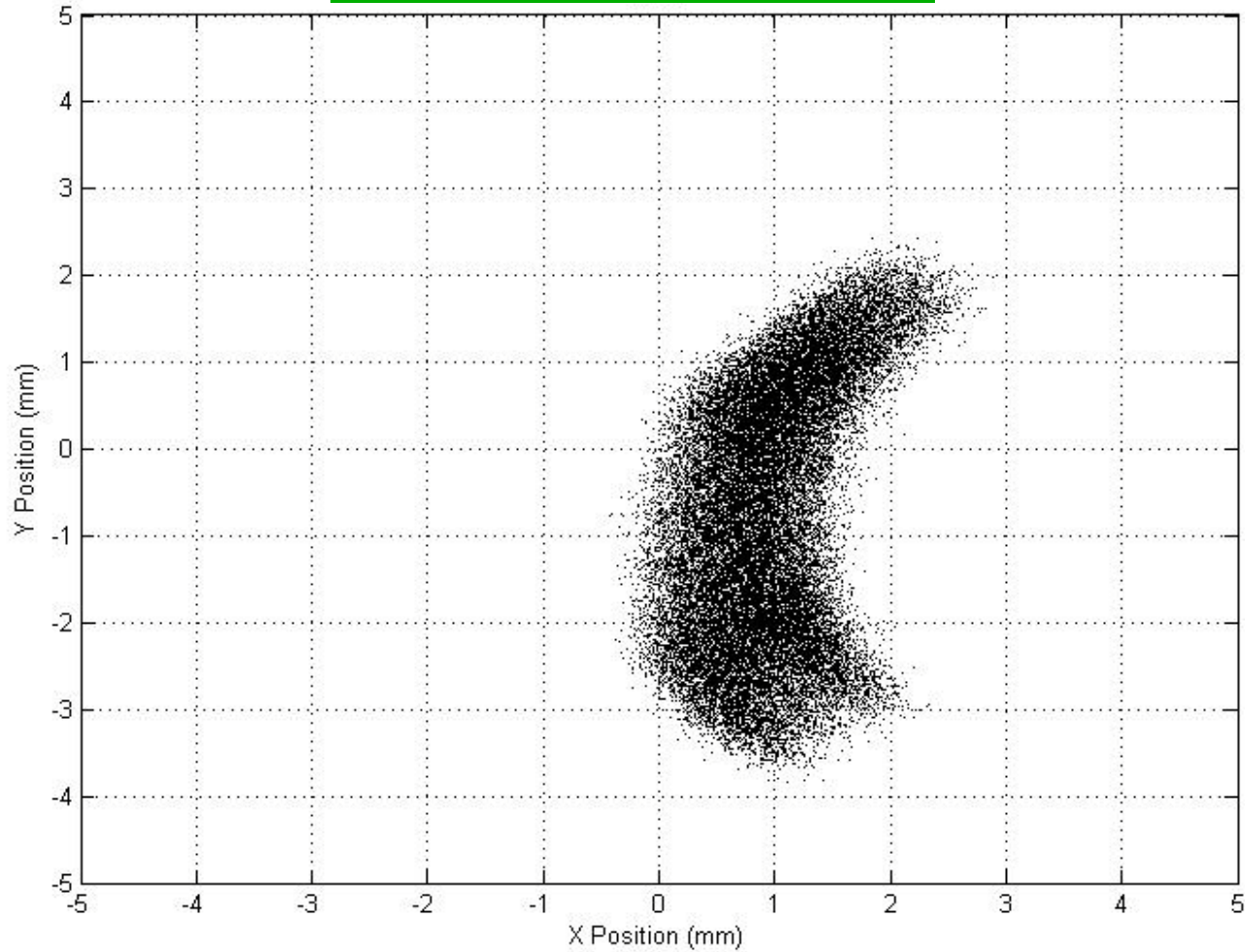
1A18 BPM After Sextupole





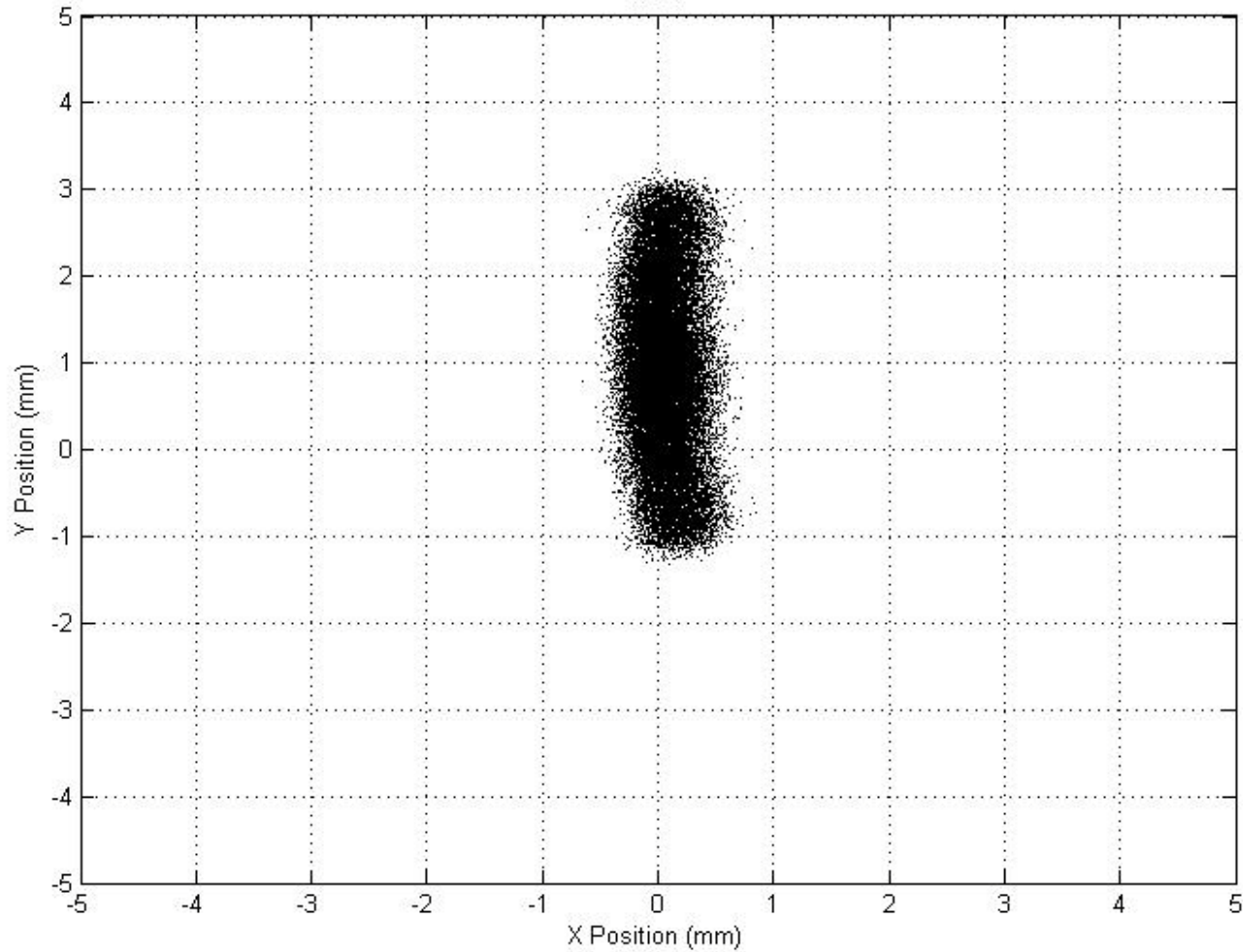
# X-Y Beam Profiles

1A19 BPM After Sextupole



# X-Y Beam Profiles

1A21 BPM After Sextupole

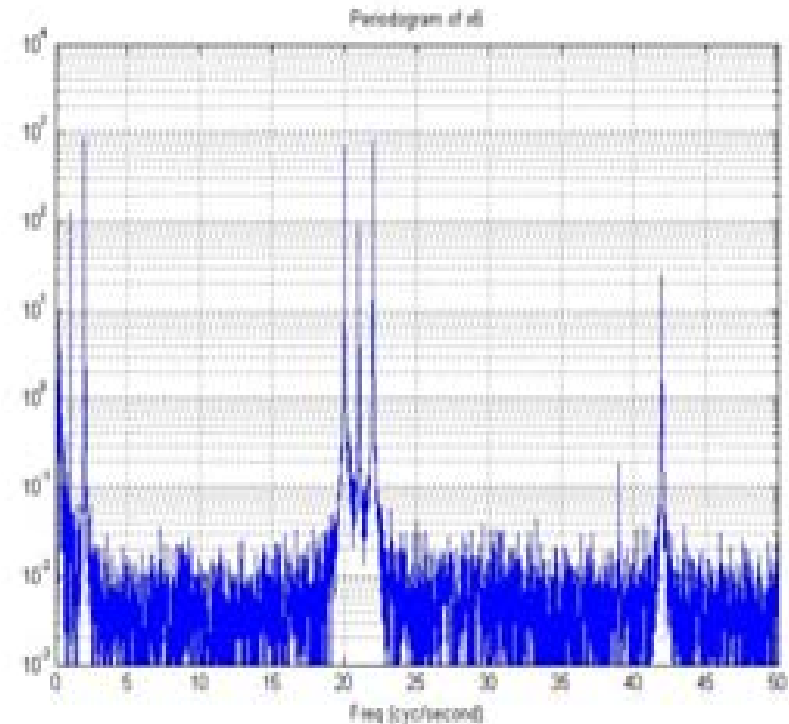
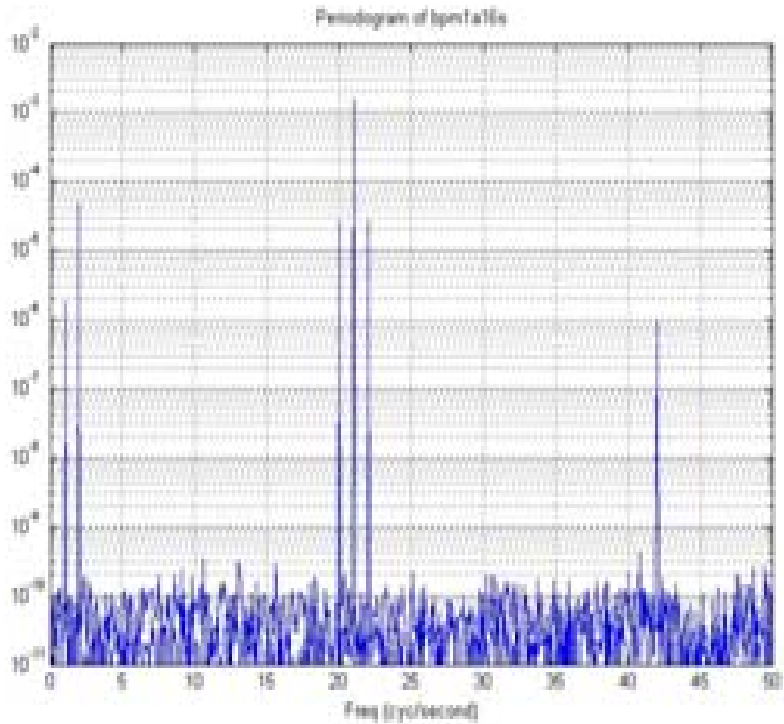




# Simulation and Measurements

Elegant Simulation

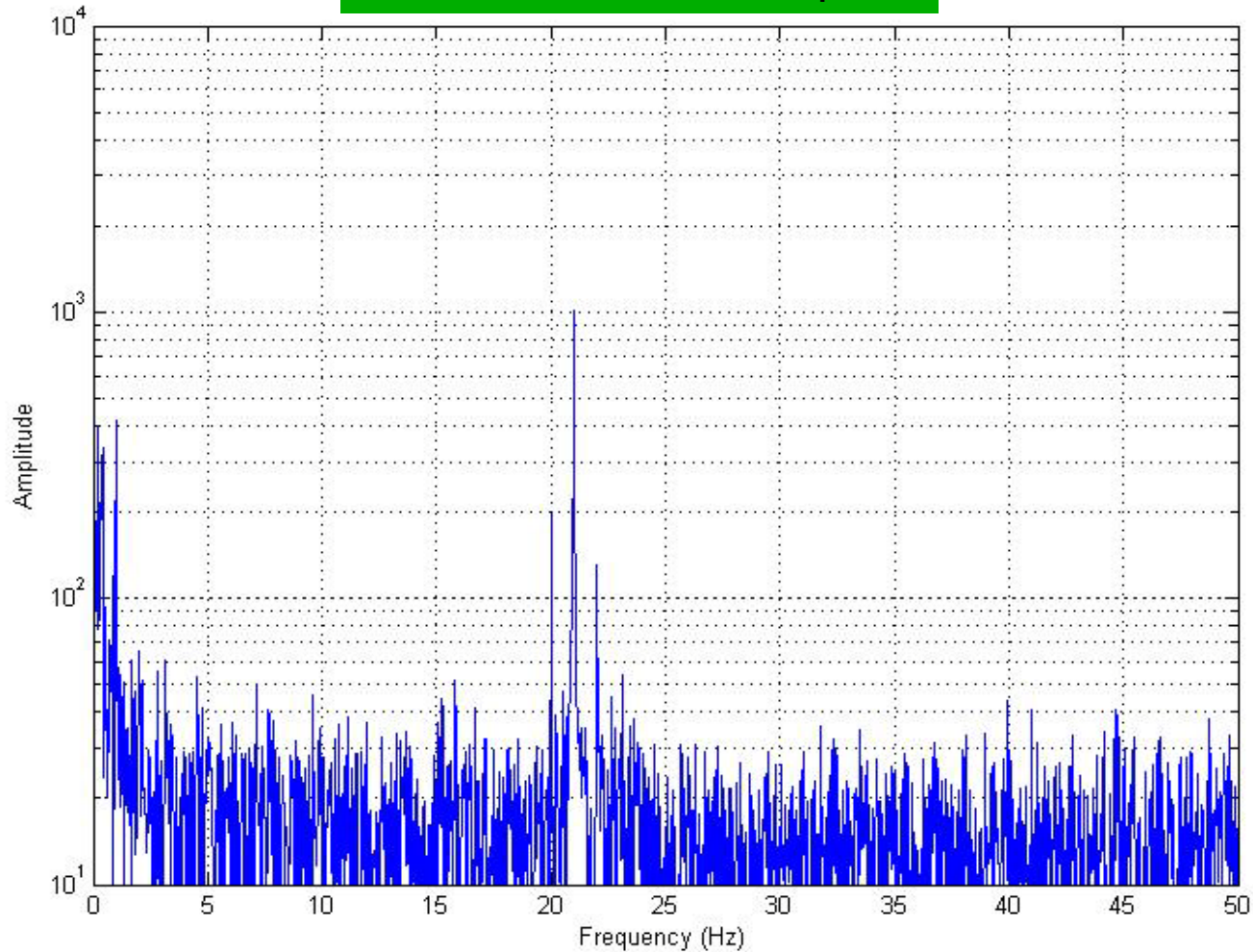
Beam Measurement



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

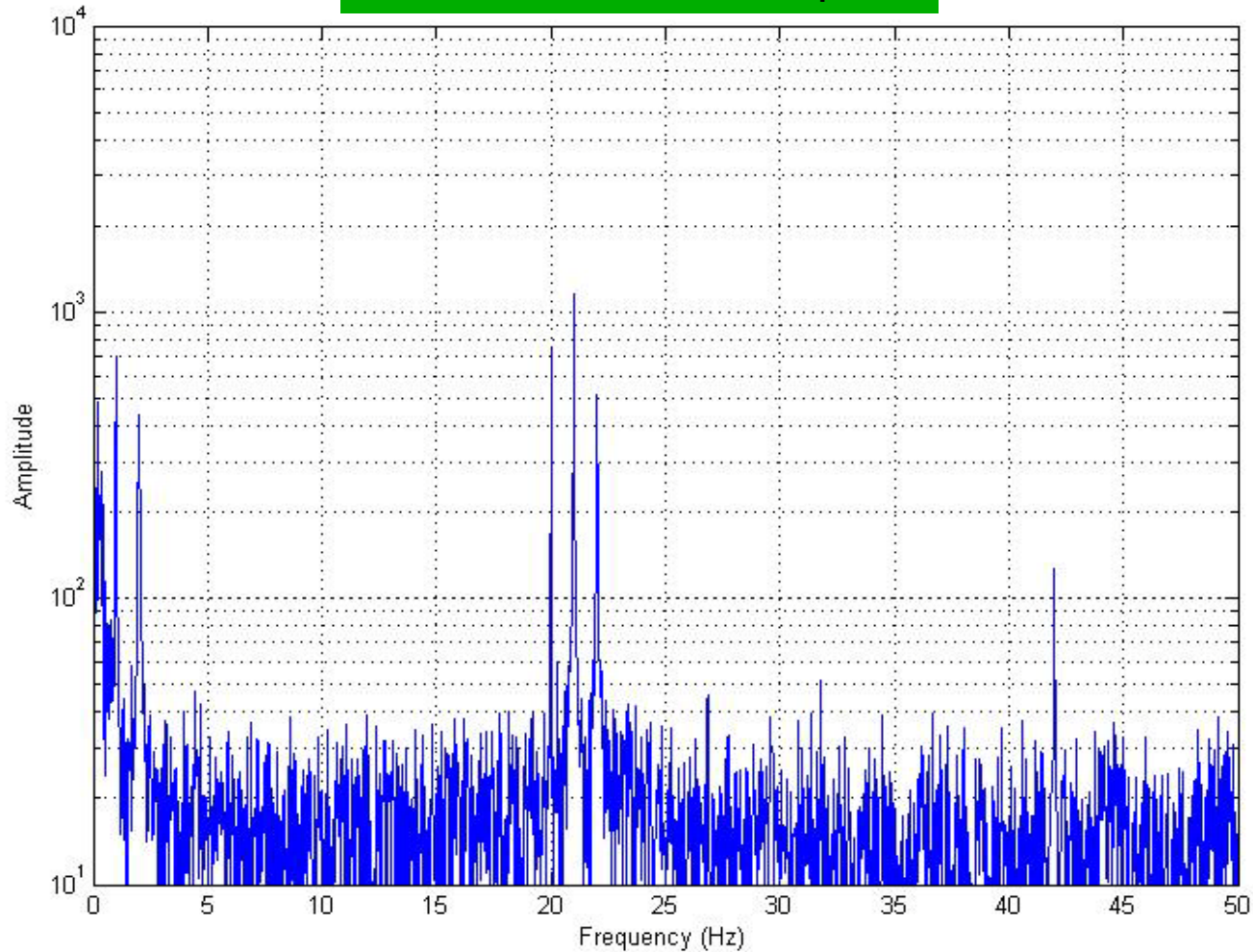
0 G/cm



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

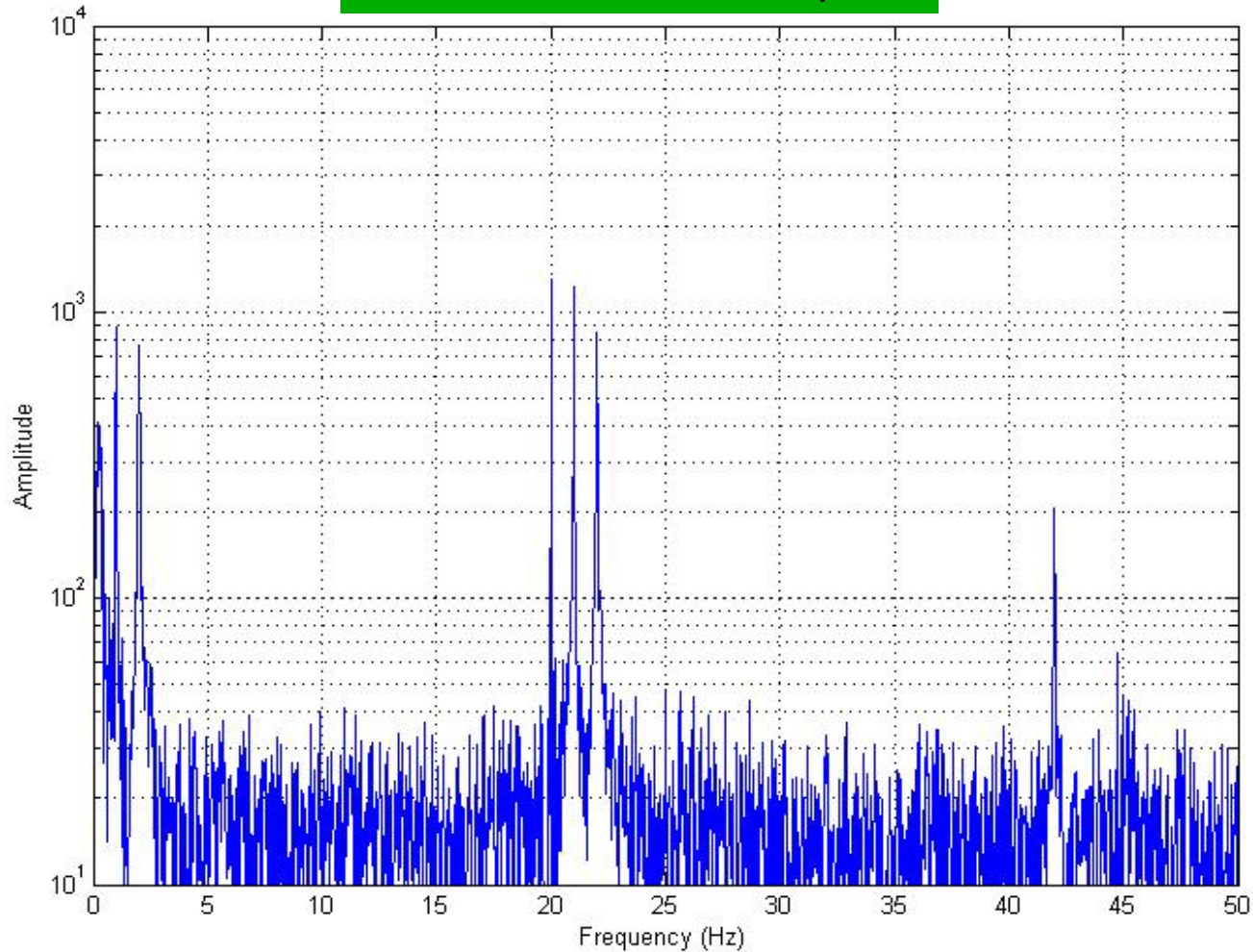
100 G/cm



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

200 G/cm

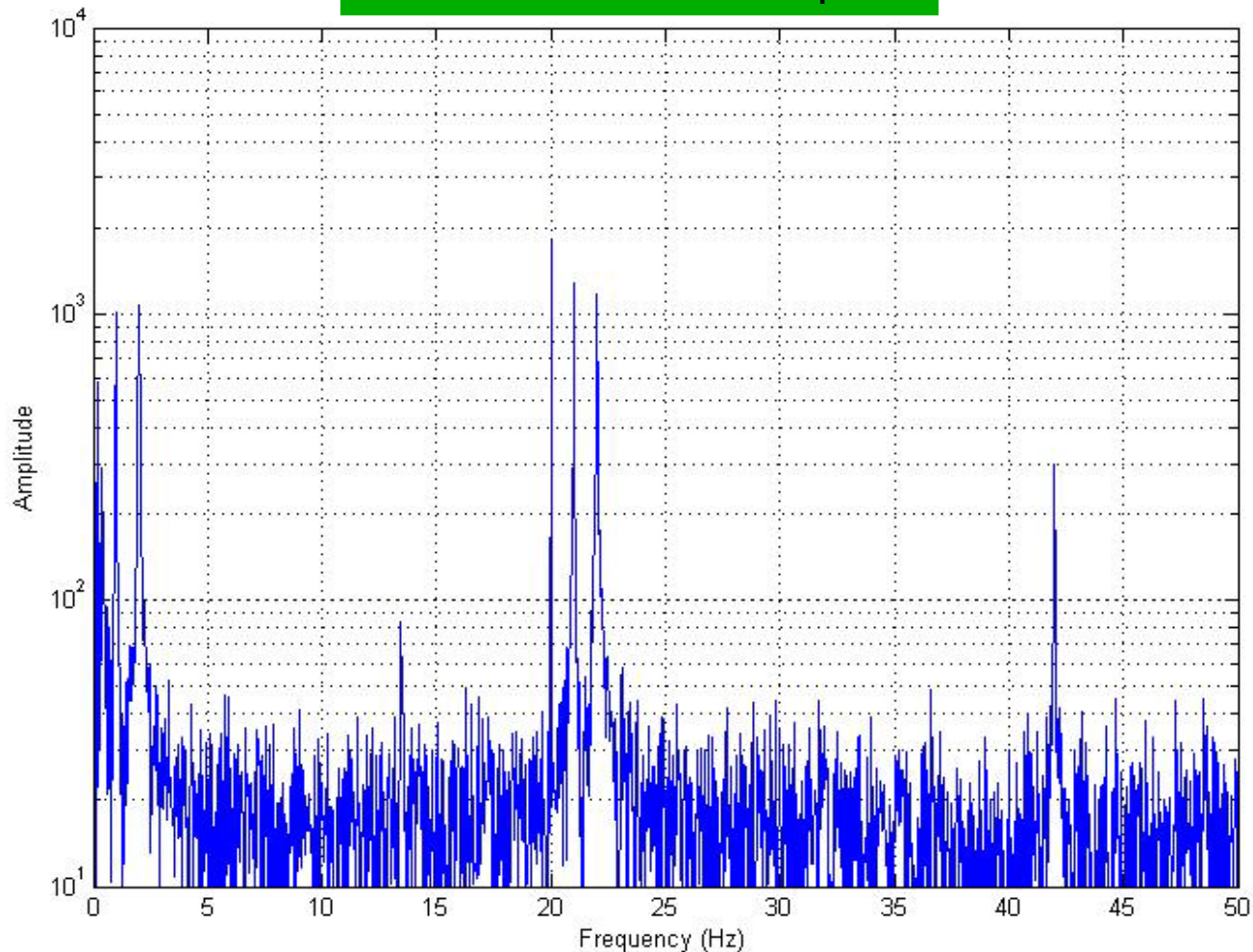




# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

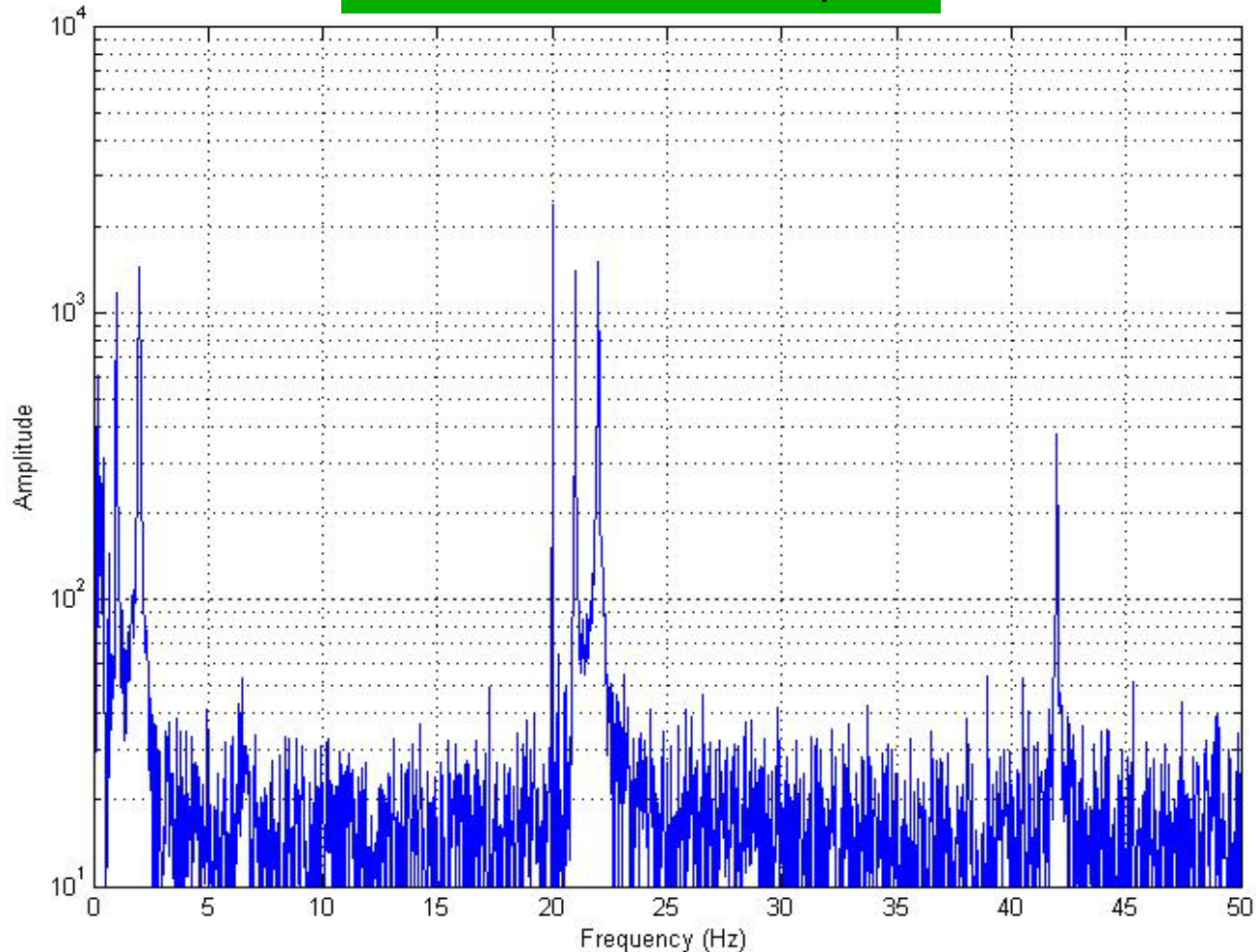
300 G/cm



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

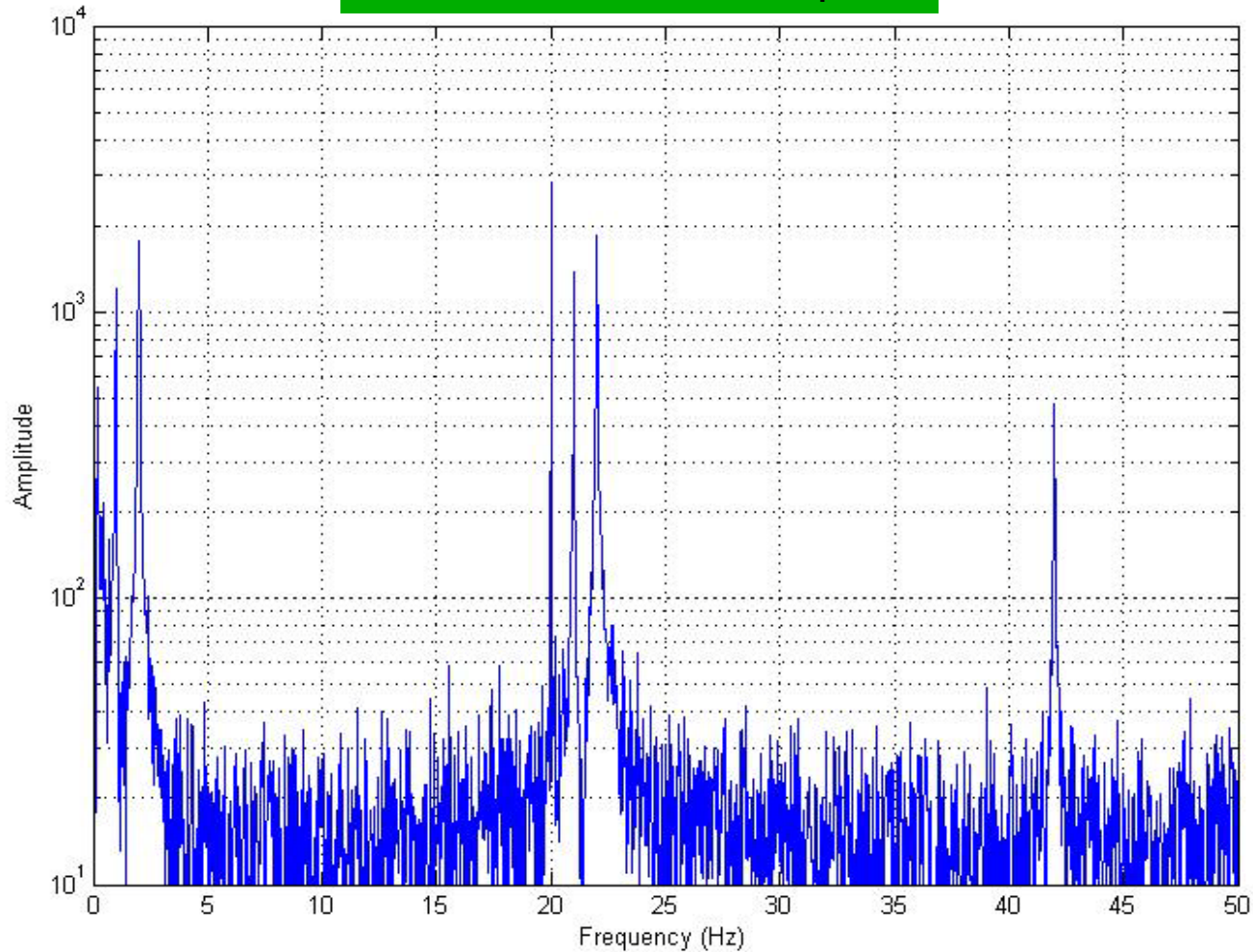
400 G/cm



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

500 G/cm

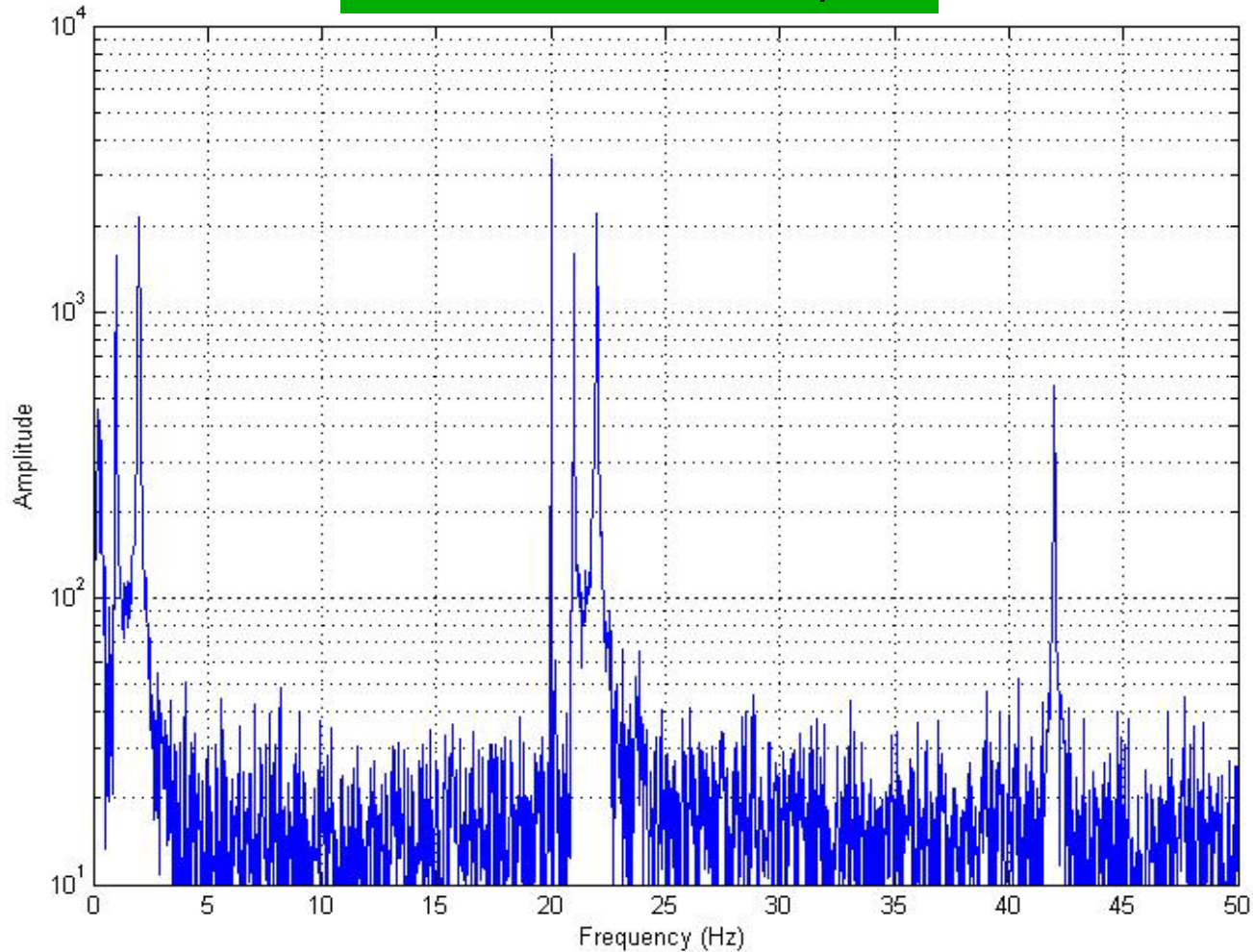




# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

600 G/cm

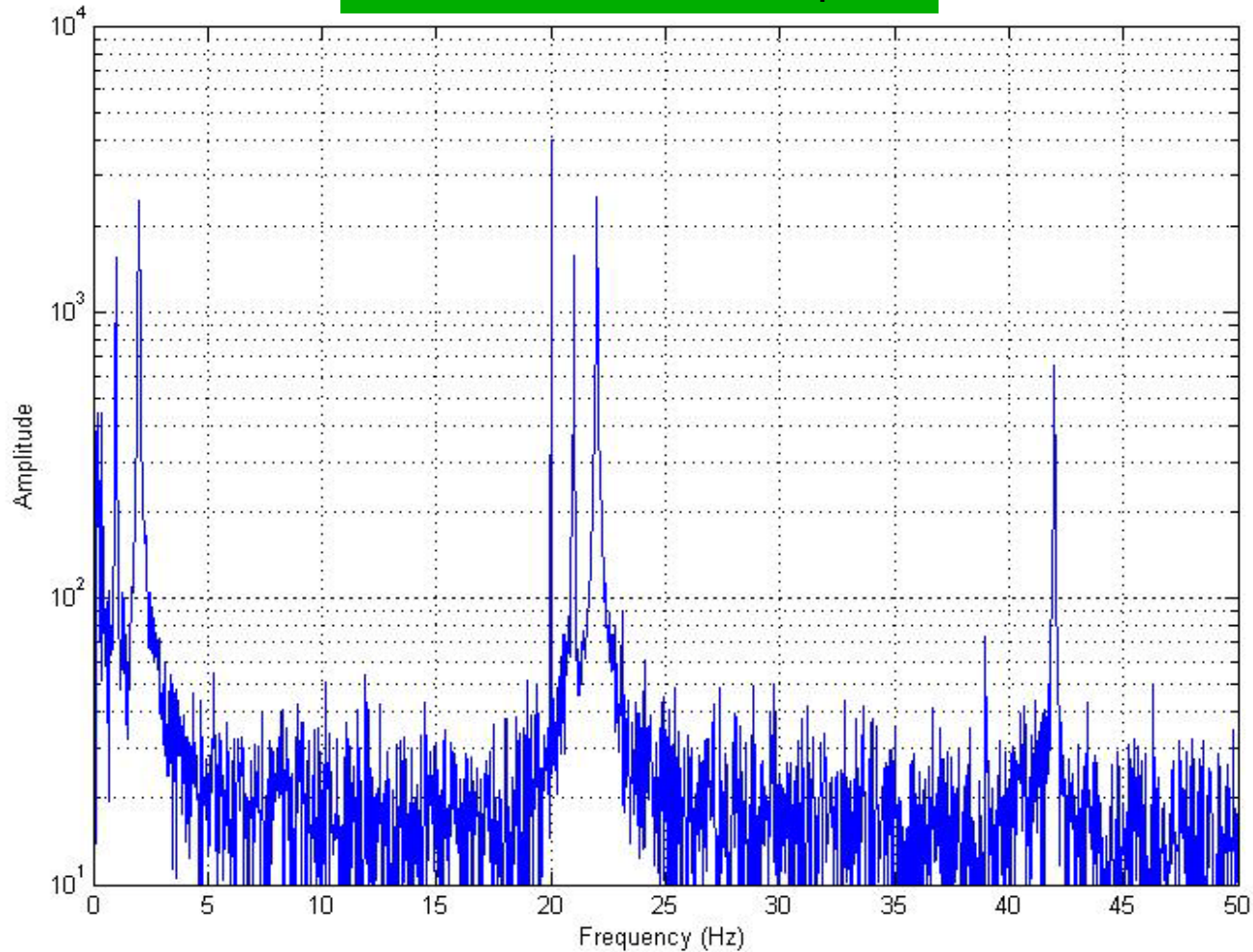




# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

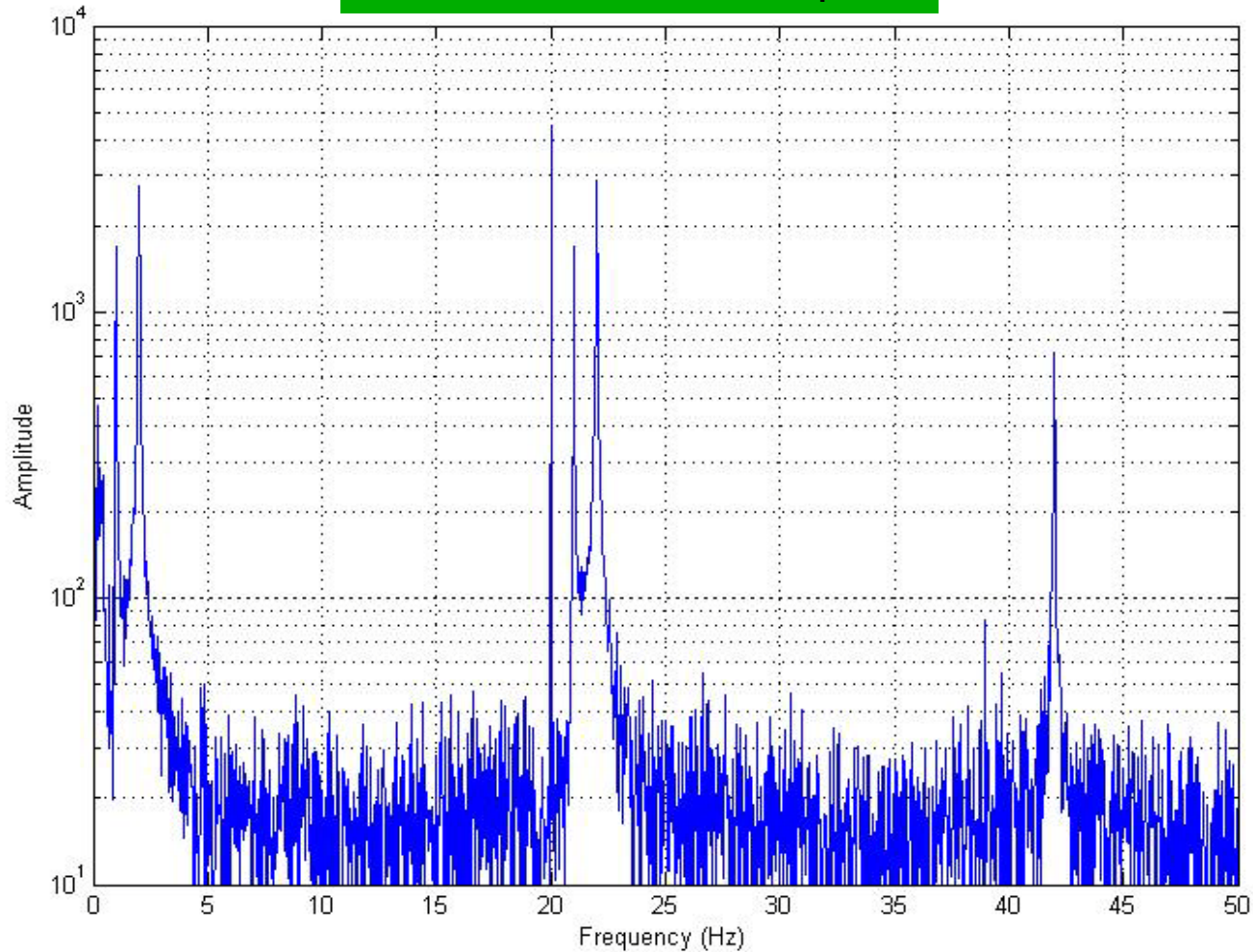
700 G/cm



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

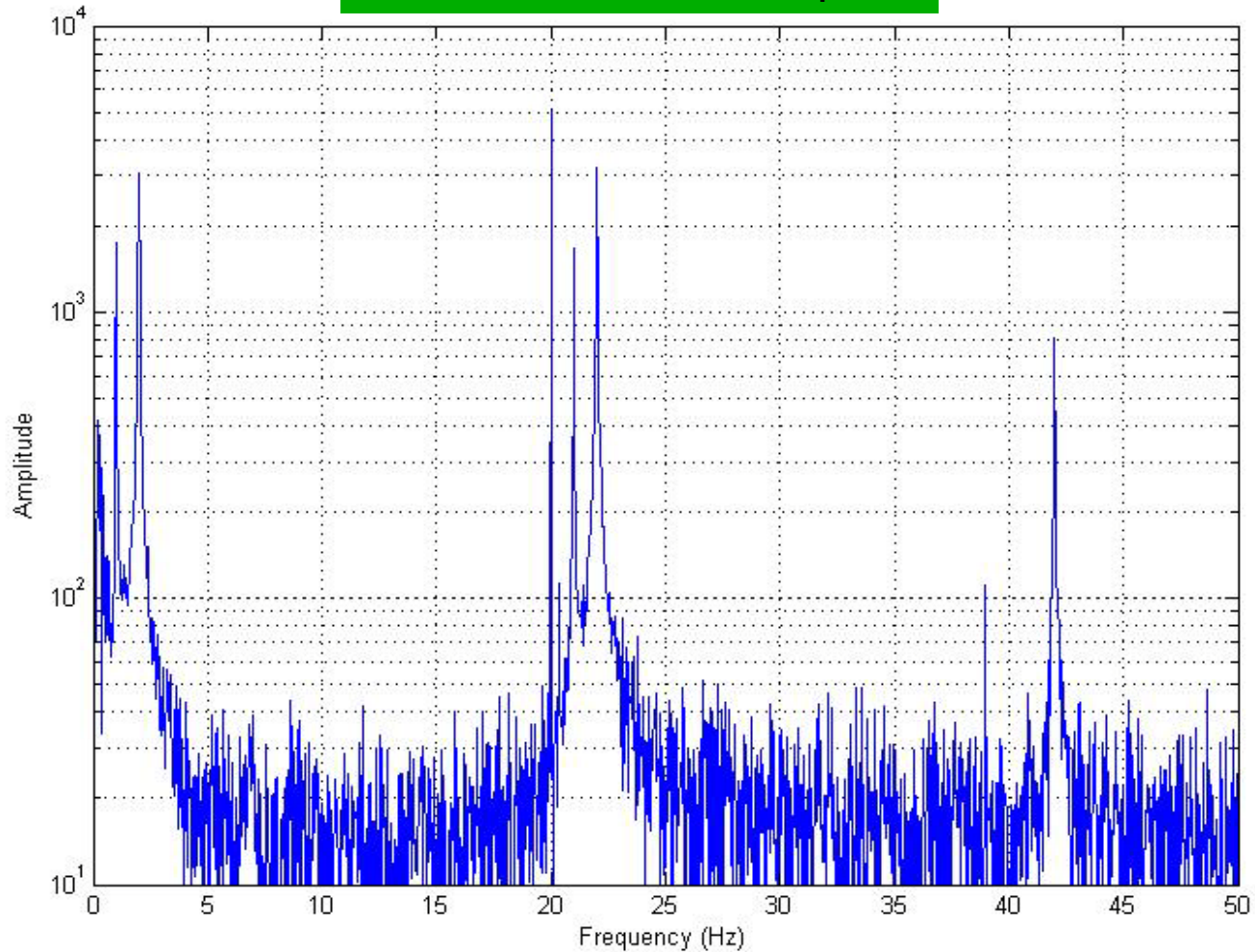
800 G/cm



# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

900 G/cm

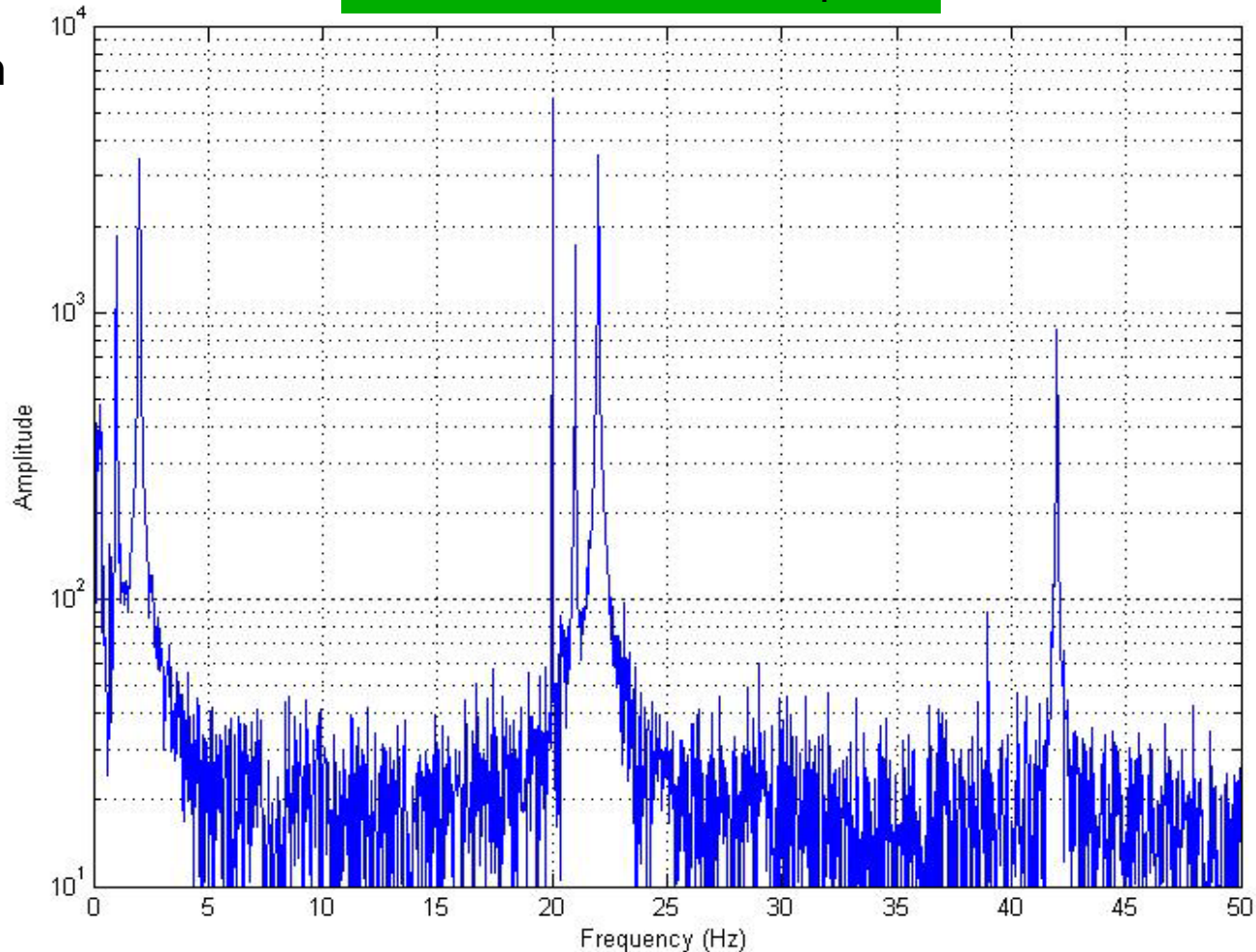




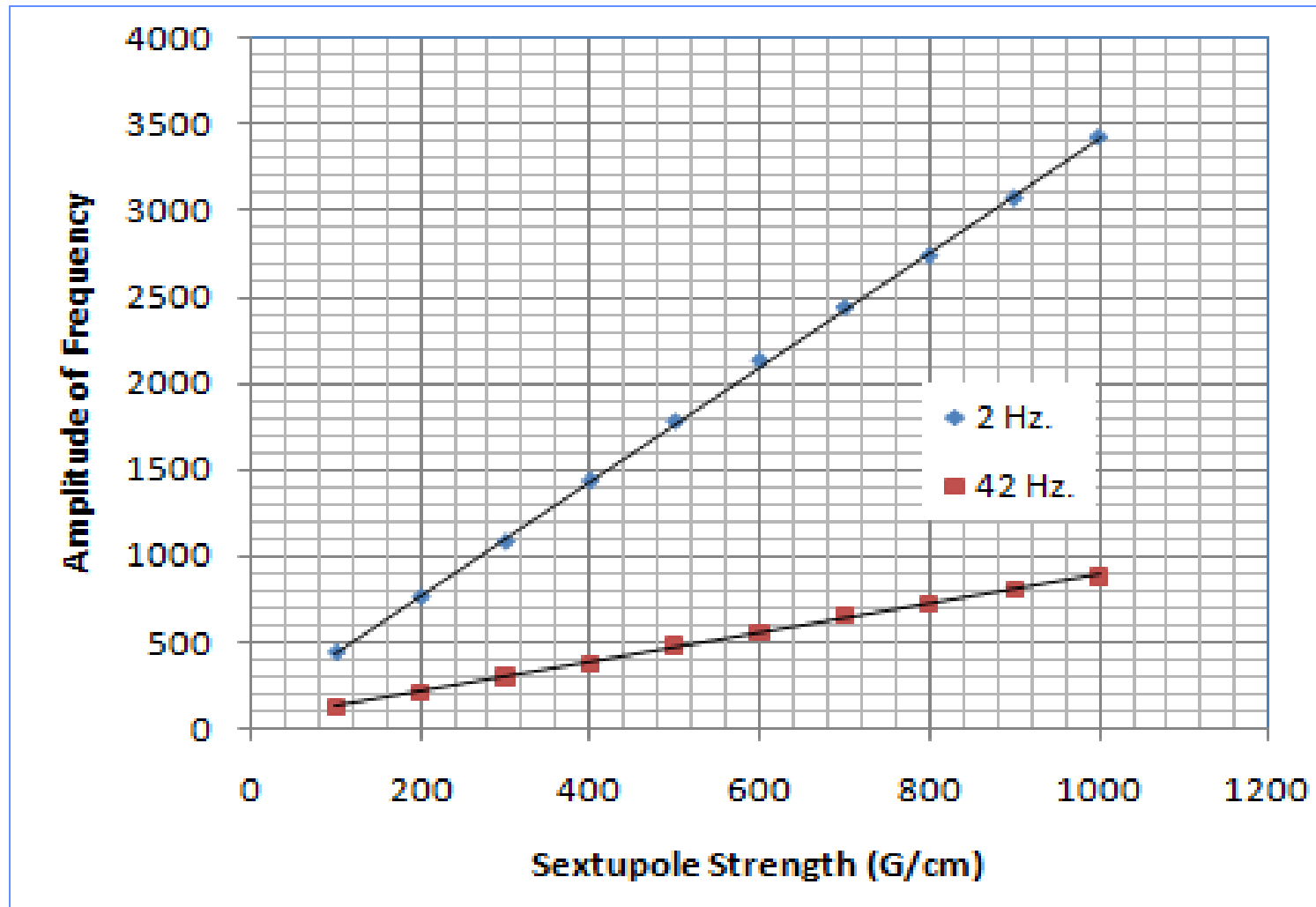
# Spectrum vs. Sextupole Strength

1A16 BPM After Sextupole

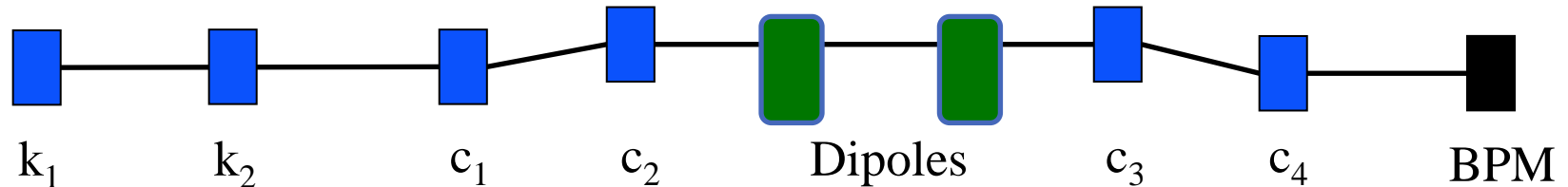
1000 G/cm



# Amplitudes vs. Sextupole Strength

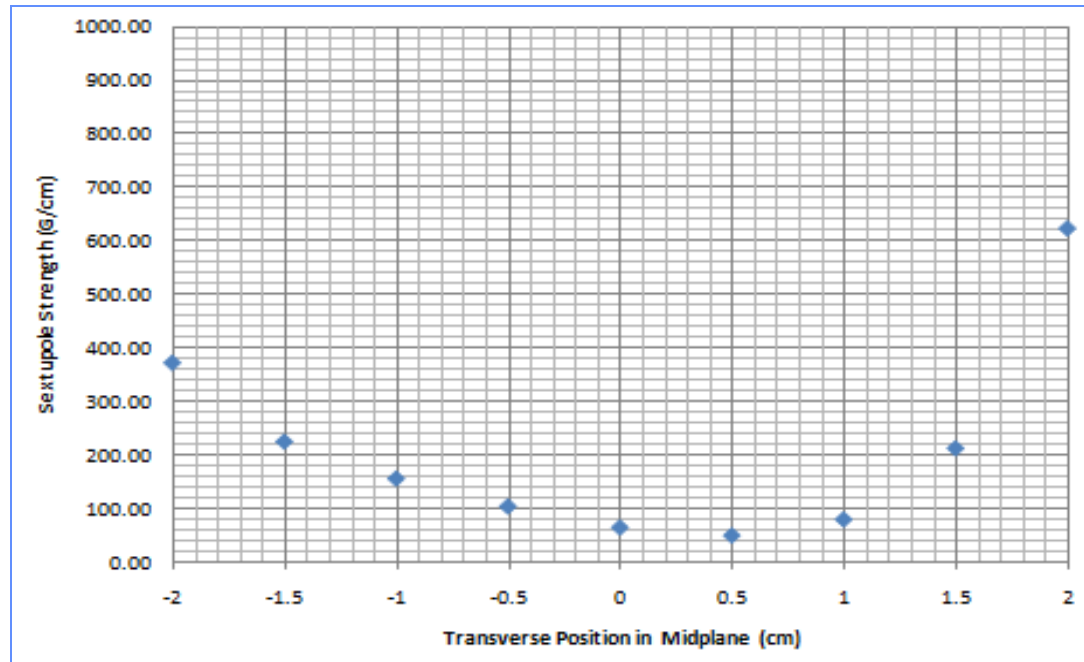


# Dipole Measurements



- Local orbit bump to offset the beam within the dipoles
- Measurements made at 1 mm steps in transverse direction from the reference orbit out to 1 cm

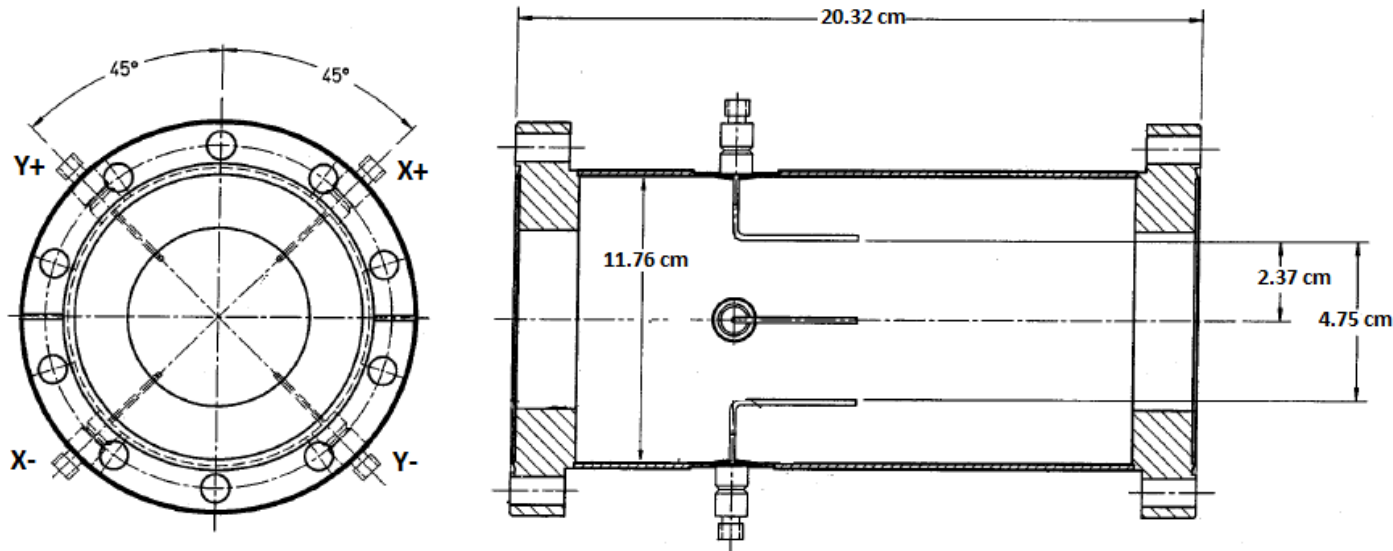
# ARC1 Dipole Hall Probe Data



- Dipoles also measured at the Magnet Measurement Facility with a Hall Probe stepper
  - 21 tracks through the dipoles at 0.5 mm spacing
  - Measurements made every 2 mm along the tracks to create a grid of points
  - Multipole fields calculated along curved trajectories



# Beam Position Monitor Nonlinearity



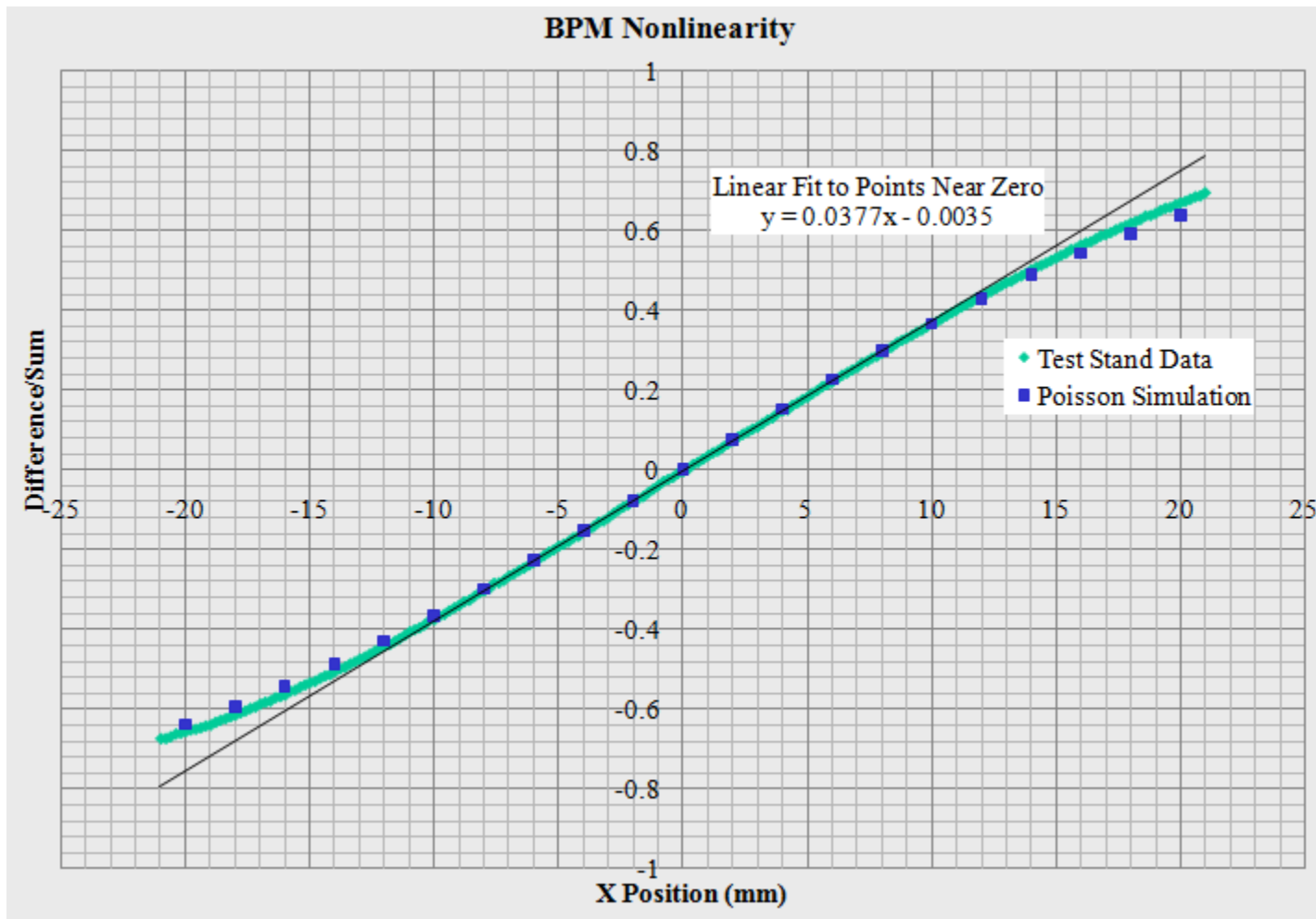
$$X_{rot} = k_x \frac{(X^+ - X_{off}^+) - \alpha_x (X^- - X_{off}^-)}{(X^+ - X_{off}^+) + \alpha_x (X^- - X_{off}^-)}$$

$$Y_{rot} = k_y \frac{(Y^+ - Y_{off}^+) - \alpha_y (Y^- - Y_{off}^-)}{(Y^+ - Y_{off}^+) + \alpha_y (Y^- - Y_{off}^-)}$$

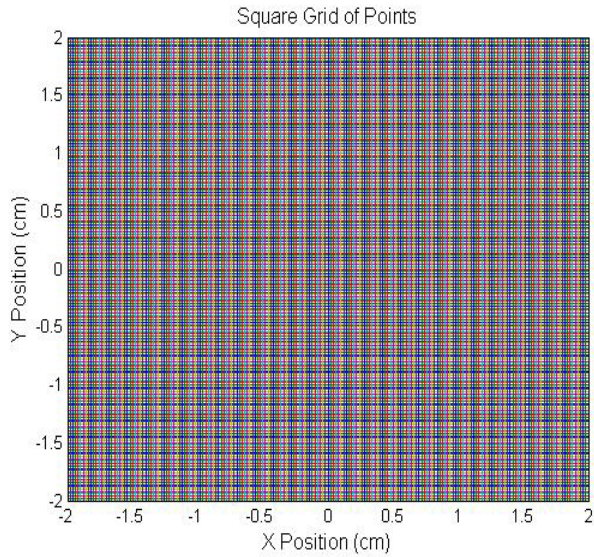
$$X = \frac{1}{\sqrt{2}} (X_{ROT} - Y_{ROT})$$

$$Y = \frac{1}{\sqrt{2}} (X_{ROT} + Y_{ROT})$$

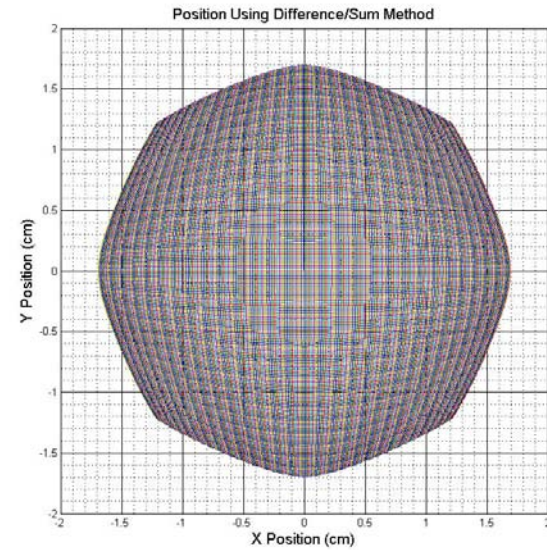
# Beam Position Monitor Nonlinearity



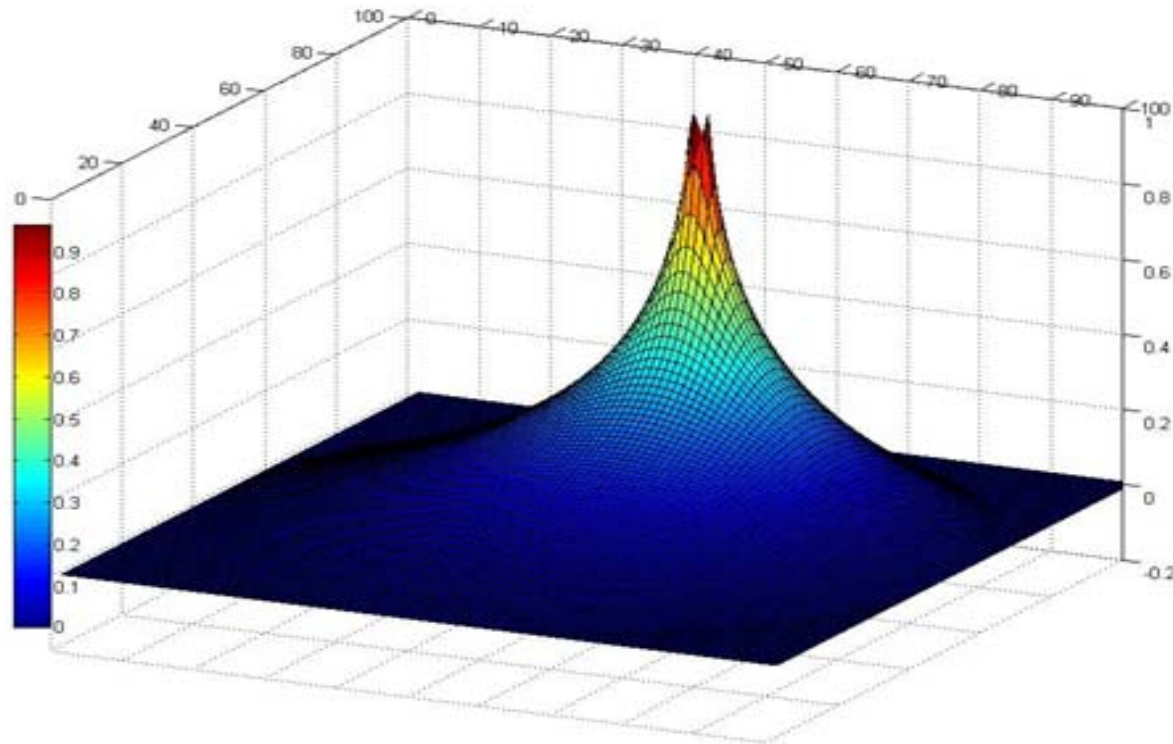
# Beam Position Monitor Nonlinearity



Difference/Sum Method  
 →



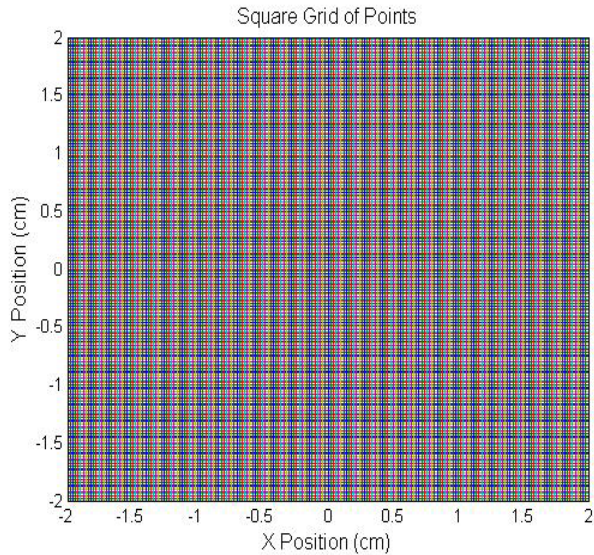
# Beam Position Monitor Nonlinearity



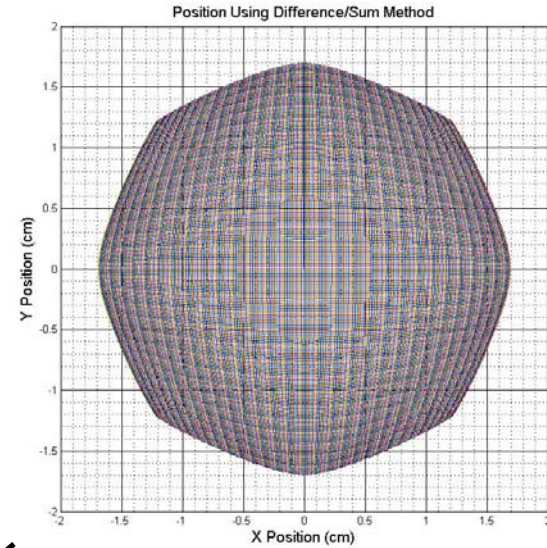
Poisson potential map of a single wire used to calculate coefficients for a two-dimensional polynomial correction of BPM data



# Beam Position Monitor Nonlinearity



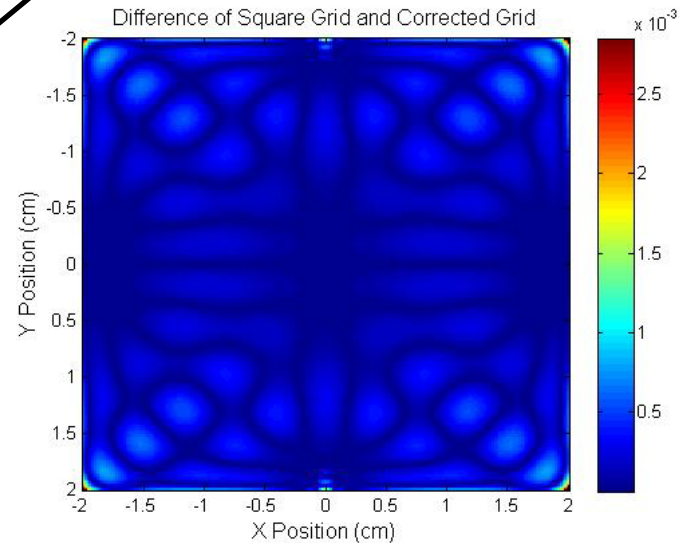
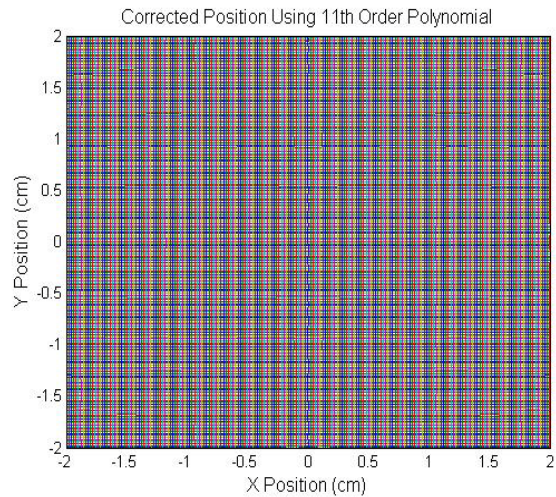
Difference/Sum Method



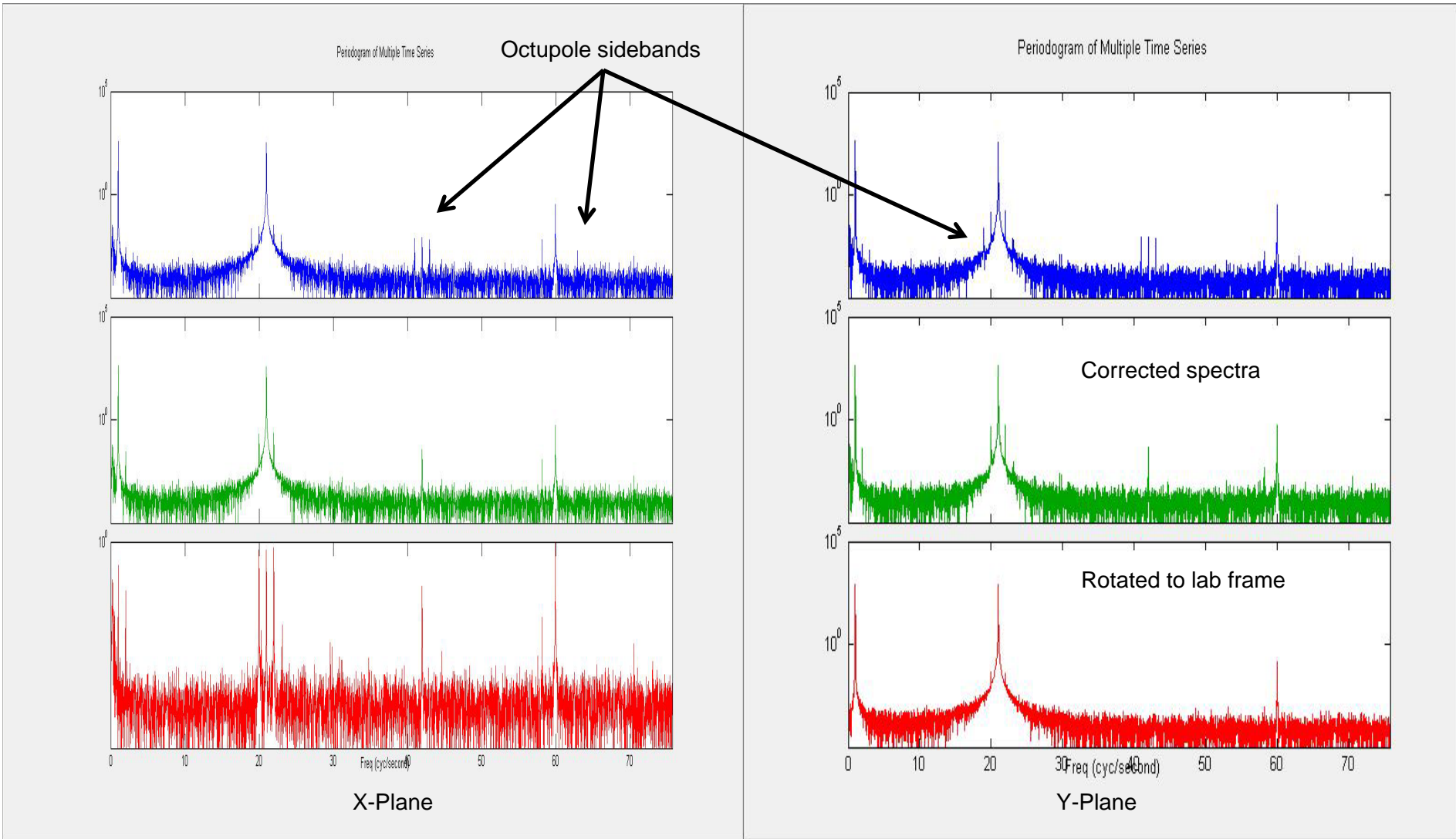
11th Order Polynomial



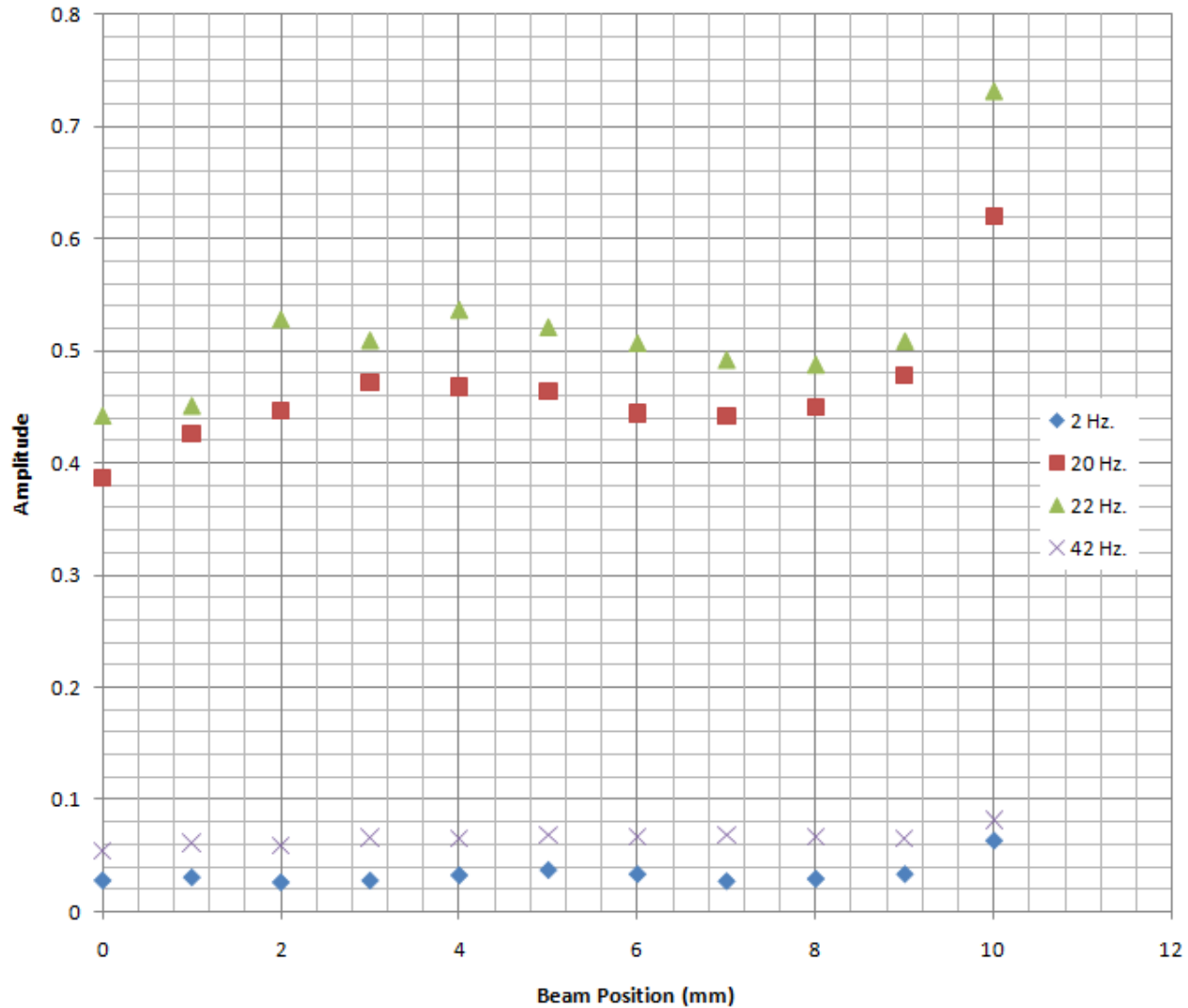
Correction to 100 microns



# Spectra Correction



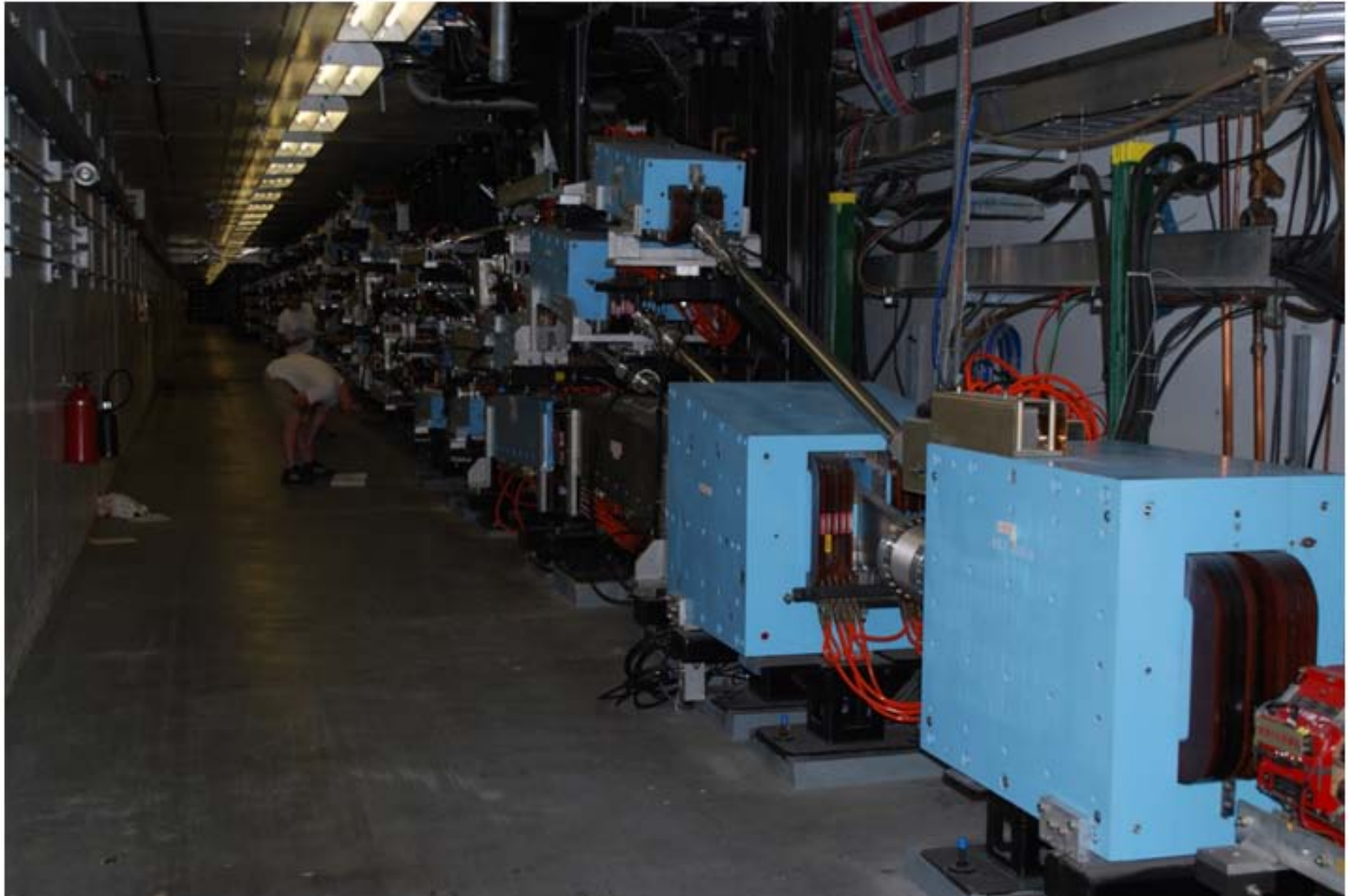
# Dipole Measurements



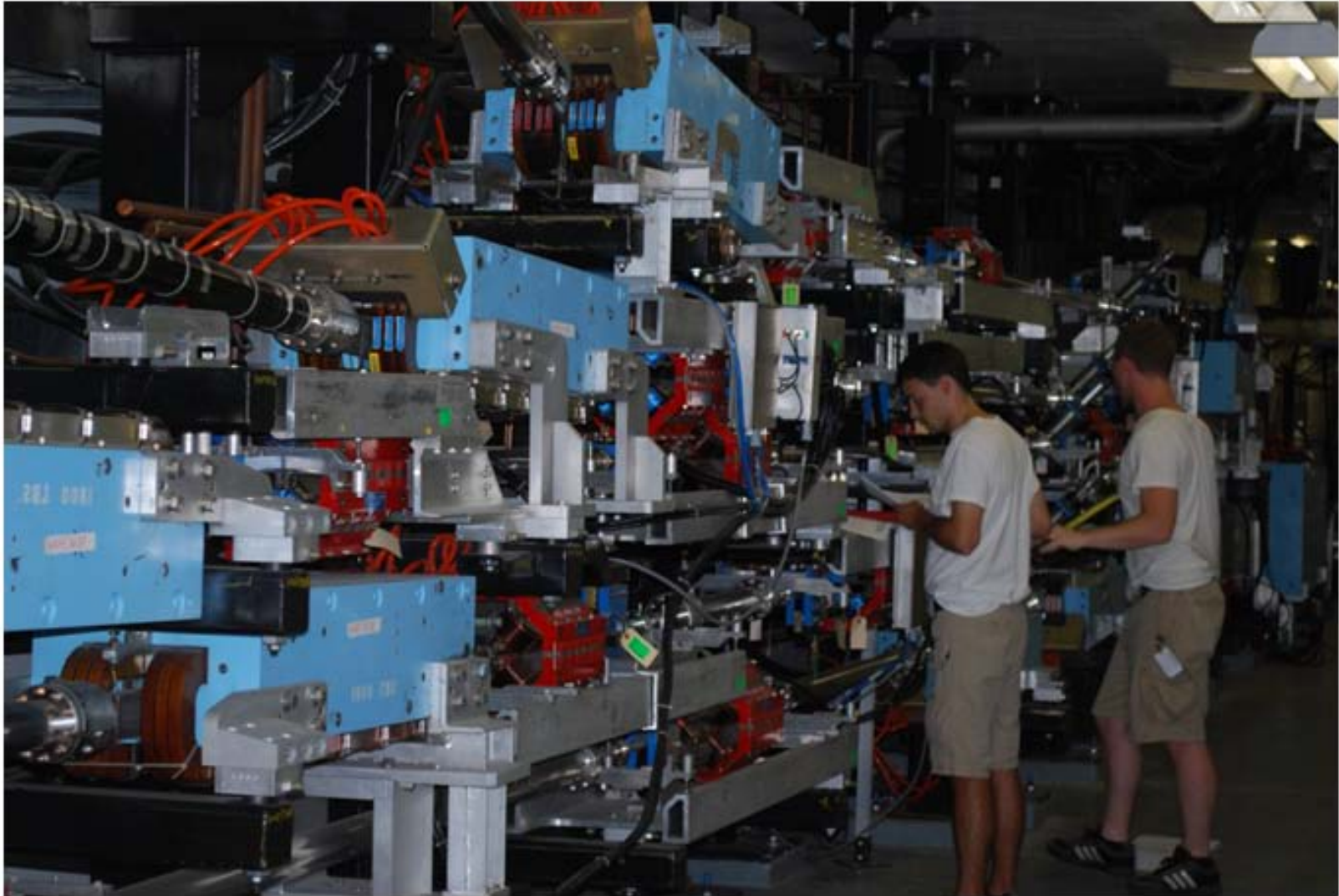




# Recombiner Beamline



# Recombiner Beamline





# Summary

- Dual AC dipole magnets were used to measure nonlinear fields
- Sextupole measurements were made to develop a calibration standard
- Measurements of dipole nonlinearity versus position within the dipole were performed
- Will be ready to characterize new beamlines as the 12 GeV machine is commissioned

Thanks!