# FORMING ATTOSECOND ELECTRON PULSES IN SPACE-CHARGE DOMINATED REGIME

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### Abstract

Short pulses of electrons of femtosecond and attosecond duration are necessary for numerous applications: studying fast processes in physics, chemistry, biology and medicine. They can be used after acceleration to 1–10 MeV, for generation of ultrashort pulses of VUV and X-ray coherent radiation in periodic fields or as a relativistic mirror. Final bunching stage is inevitably space-charge dominated. Two models are studied: a spherical bunch and a thin plate bunch. It was shown that bunches of attosecond and subattosecond duration with peak currents of more than 1 MA can be obtained. Possible applications of such electron bunches are reviewed including obtaining attosecond and subattosecond pulses of tunable coherent radiation in UV and X-ray regions. Reflection coefficients of such relativistic mirrors are evaluated.

## CALCULATION OF BUNCHING REGIMES TAKING INTO ACCOUNT SPACE CHARGE FORCES

The dynamics of electron bunches in a regime of dominating space-charge forces is studied more thoroughly and extensively as well as focusing in the lateral direction.

For a laser wavelength of 1  $\mu$ m, the bunch duration upon emission lies in the range 0.4 – 2 fs and the bunch length after acceleration to 10 – 50 keV in the interval 20 – 300 nm. Further shortening of the size and duration of the bunch occurs due to bunching as a result of modulation of the velocity. If the instantaneous spread in velocities is small, the limiting factor for further bunching becomes space-charge force.

Two models permit to obtain analytical solutions of the problem. The first is spherically symmetrical, corresponding to a single-spike cathode and modeling a short bunch of length comparable to the diameter. Uniform compression of a homogeneously charged sphere is secured by linear distribution of radial velocities directed inwardly. In the longitudinal direction, such a distribution is secured by a laser field, and in the transversal should be created in the main by focusing of a quasi-static field in a specially designed geometry of the near-cathode region.

The equation of motion in a system of rest of the sphere center for an electron of mass m and charge e is:

$$m\frac{dv_{\rm i}}{dt} = \frac{4\pi e\rho_0}{3} \times \frac{r_{\rm i0}^3}{r_{\rm i}^2},\tag{1}$$



Figure 1: Dependence of bunch duration on time in the laboratory system;  $\lambda = 10 \ \mu\text{m}$ ,  $E_{qst} = 1 \cdot 10^9 \ \text{V/cm}$ ,  $E_{L0} = 1.6 \cdot 10^9$ ,  $v_{\parallel} = 1.64 \cdot 10^{10} \ \text{cm/s}$ ,  $I_0 = 13 \ \text{mA}$ ,  $I_{max} = 10 \ \text{A}$ , number of electrons with bunch  $N_e = 760$ ,  $d_{drift} = 80 \ \mu\text{m}$ .

where  $\rho_0$  is the initial charge density,  $r_{i0}$  the initial radius at which the electron is located,  $v_i$  its velocity,  $r_i$  the radius. The velocity at the outer boundary of the bunch will be designated as v without index and the initial velocity there as  $v(0) = v_0$ .

The first integral of the equation is:

$$\frac{v}{v_0} = \sqrt{1 - \frac{8I_{r_0}}{3I_0 \beta_0^2 \beta_{\parallel}} \times (\frac{r_0}{r} - 1)},$$
(2)

where  $r_0$  is the initial radius of the sphere, r the current radius,  $I_{r_0}$  the maximal current in the initial movement of the bunch (in the cross-section through the center of the sphere),  $I_0 = mc^3/e \approx 17$  kA,  $\beta_0 = v_0/c$ ,  $\beta_{\parallel} = v_{\parallel}/c$  the velocity of the bunch in the laboratory system in units of the velocity of light c.

Assuming v = 0, we obtain a minimal radius of the sphere and minimal duration:

$$r_{min} = \frac{r_0}{\frac{3I_0\beta_0{}^2\beta_\parallel}{8I_{ro}} + 1}, \qquad \tau_{min} = \frac{r_{min}}{v_\parallel}.$$

The maximal bunch current and the amplitude of linear modulation of longitudinal velocity are:

$$I_{max} = I_{r_0} + \frac{3I_0\beta_0^2\beta_{\parallel}}{8}, \qquad \frac{\delta\beta_{\parallel}}{\beta_{\parallel}} = \sqrt{\frac{8(I_{max} - I_{r0})}{3I_0\beta_{\parallel}^{-3}}}.$$

Figure 1 shows the dependence on time of the bunch duration for flight in the laboratory system.During 1 fs the

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Figure 2: bunch duration on time in the laboratory system;  $\lambda = 1 \ \mu m, v_0 = 2, 5 \cdot 10^9 \text{ cm/s}, v_{\parallel} = 3 \cdot 10^9 \text{ cm/s}, I_0 = 10 \text{ mA}, I_{max} = 1.5 \text{ A}$ , number of electrons with bunch  $N_e = 90$ .



Figure 3: Dependence of the duration of a flat bunch on time in the laboratory system:  $\lambda = 1 \ \mu m$ .

bunch has duration of 12 - 14 as, having passed through a distance of 160 nm.

Figure 2 shows analogous curve for  $\lambda = 1 \ \mu$ m. The minimal duration of the bunch in this case is less than for  $\lambda = 10 \ \mu$ m. It should be noted that during compression of a spherical bunch the field rises sharply  $\propto 1/r^2$ , which decreases the "life" time of a short bunch. The situation is hardly improved by decreasing the current: one or two electrons cannot be left in the bunch.

The use of multi-spike and blade cathodes opens up great possibilities since current in a bunch can be increased  $10^3 - 10^4$  times. In this case, a one-dimensional, flat model can be used to calculate bunching. Another important advantage of this scheme is that electric field of a flat bunch does not depend on its thickness and the "life" time of the bunch is significantly greater than for a spherical bunch. Just as in the spherical case, the field increases linearly from the middle of the bunch to its edge. Layers do not mix and conserve the charge within the layer. The movements of the boundaries of layers are similar and are described by similar equations:

$$m\frac{dv_i}{dt} = 2\pi e q_0 \frac{x_{i0}}{x_0},\tag{3}$$

where  $v_i$  is the velocity of the boundary of a layer with initial coordinate  $x_{i0}$ ,  $q_0$  the initial charge density,  $2x_0$  the initial thickness of the flat bunch, m and e the mass and charge of the electron.

An equation with constant right term permits direct integration:

$$v_{i} = \frac{2\pi x_{i0}}{S\beta_{\parallel}} \cdot \frac{I}{I_{0}} \cdot c^{2}t + v_{i0}, \qquad (4)$$

$$x_{i} = \frac{\pi x_{i0}}{S\beta_{\parallel}} \cdot \frac{I}{I_{0}} \cdot c^{2}t^{2} + v_{i0}t + x_{i0},$$
 (5)

where I is the current in a bunch of area S and thickness  $2x_0$ ,  $\beta_{\parallel} = v_{\parallel}/c$ . It can be seen that both  $v_i$  and  $x_i$  are proportional to  $x_{i0}$  if  $v_{i0} = 0$ . Upon expansion of the bunch,  $v_{i0}$ ; upon compression, the velocity directed toward the center of the bunch, linearly increases in absolute value from zero at the center to  $v_0$  at the edge.

The current of the bunch with a multi-spike cathode is  $I = I_1 N_s \approx \pi \rho_c^2 j N_s$ , where j – density of emission current at a spike and  $N_s$  the overall number of spikes. For a blade cathode, the current I is approximately  $I \approx \pi \rho_c l j N_{bl}$ , where l is blade length and  $N_{bl}$  number of blades. Upon compression, bunch current  $I_b = I x_{i0}/x_i \equiv I x_0/x$ , where x is half of running bunch length.

The time of bunching to the minimal length of the bunch (minimal thickness) and its size are:

$$_{min} = \frac{I_0 S \beta_0 \beta_{\parallel}}{2\pi I x_0 c}, \qquad 2x_{min} = 2x_0 - \frac{I_0 S \beta_0^2 \beta_{\parallel}}{\pi I x_0}.$$

The maximum current, minimum time and relative lengthening of the bunch in the presence of instantaneous spread of velocities are:

$$I_{max} = I \frac{x_0}{x_{min}}, \qquad \tau_{min} = \frac{2x_{min}}{v_{\parallel}}, \qquad \frac{\delta \tau_{min}}{\tau_{min}} = \frac{\delta v_0}{v_0}.$$

The minimal duration of the bunch (and its maximum "lifetime") are attained when the initial speed of compression is precisely adjusted. In this case, the minimal duration can be limited by the instantaneous spread in velocities. The bunch current could exceed by hundreds or thousands of times the initial one. The "lifetime" of the maximum current is the most when the velocity is precisely adjusted; for lower velocity it gradually decreases and for velocities greater than optimal sharply drops. This is due to the fact that forces of space charge do not stop the electrons and the two halves of the bunch, moving toward each other, pass through each other (Fig. 3). The current can exceed  $10^6 - 10^7$  A. Each bunch contains  $4 \cdot 10^4$  electrons.

## **POSSIBLE APPLICATIONS**

A sequence of attosecond/femtosecond electron bunches with the possibility of regulating the spatial interval between them may be useful for various applications. First,

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for the excitation of levels in quantum systems and studying the kinetics of quantum processes (tunneling, ionization and others). Second, such bunches can be used in diffractometry with attosecond and femtosecond timeresolution of ultra-fast phase transitions, expanding and destroying solids and so on. Third, it may be possible to obtain coherent electromagnetic radiation in the VUV and Xray range with smooth tuning of the radiation frequency. A pair of pulses can be obtained with an adjustable interval between them for a pump-probe method.

Subsequent acceleration of electron bunches to weakly relativistic energies (1–10 Mev) broadens the range in the direction of short wavelengths. For this, one can use one of several recently proposed schemes of laser acceleration [7–11] with the required high tempo of acceleration of 1 GeV/cm. For a short bunch duration of < 1 as and a small velocity spread in the bunch of (0.01–0.1%), it is possible to generate radiation wavelength of a fraction of a nanometer.

The number of electrons in a bunch for 10  $\mu$ m and 450 kA current from 30 blades is  $N_e = 3 \cdot 10^{10}$  with cathode area of 0.1 mm<sup>2</sup>. Volume density in a bunch of 0.1-nm thickness is  $n_v = 3 \cdot 10^{21}$  cm<sup>-3</sup>. The densities could be increased to  $n_s = 3 \cdot 10^{16}$  cm<sup>-2</sup> and  $n_v = 3 \cdot 10^{24}$  cm<sup>-3</sup> if the total current were ten times larger and transverse-focusing bunch area diminished 100 times.

As examples, two schemes of generation can be mentioned. The first is a radiator used as an undulator of a periodic structure of fields formed by laser radiation. Due to the shortening of the undulator period to  $10^3$  times less than those used now, large fields of  $\approx 10^9$  V/cm (it is possible to use schemes with a resonator) are required.

The second scheme requires the electron beam of greater precision: using the sequence of short bunches as a multilayer resonance mirror. To secure small spread of phase in the reflected radiation, it is necessary that the bunch length be less than  $\lambda_r/10$ , which secures addition of reflected waves from various bunches with small phase shift ( $\lambda_r$  is the wavelength of reflected radiation).

For counter-propagating electromagnetic radiation of wavelength  $\lambda_i$ , reflected radiation has a wavelength

$$\lambda_r = \frac{\lambda_i}{4\gamma^2},$$

where  $\gamma = \mathcal{E}/mc^2$  is the relativistic factor of electrons.

The condition for the reflected waves from various bunches being cophasal is:

$$d' = n\lambda'_r,\tag{6}$$

where d' is the distance between bunches in the co-moving system of coordinates,  $\lambda'_r = \lambda/(2\gamma)$  the wavelength of incident radiation in the same system, n an integer. If the incident radiation has a broad spectrum, the mirror cuts off from it a line satisfying condition (6). For a large number of bunches and narrow band of incident radiation, fine tuning of resonance is required.

Reflection coefficient can be evaluated for the following reasons. The penetration depth of an electromagnetic wave in a metal (here, copper) is 20-30 nm for 0.05-10  $\mu$ m wavelengths. Reflection is in fact the radiation of coherently oscillating electrons forced by the wave electric field. The reflection coefficient is near 100% under normal incidence and  $\lambda = 1 - 10 \ \mu m$ . Such perfect reflection is ensured by electrons in a layer of 26 nm, i.e.,  $n_s =$  $2 \cdot 10^{17}$  cm<sup>-2</sup>. Coherent reflection of n equidistant bunches with  $n_{s1}$  will be the same (100%) as from one bunch with  $n_s = n_{s1}n = 2 \cdot 10^{17} \text{ cm}^{-2}$ . Otherwise, the part of reflected photons is  $k_{ph} = (n_{s1}n/2 \cdot 10^{17})^2$ . This coefficient coincides with the power reflection coefficient in the comoving coordinate system. In the laboratory system, the energy of photons is  $4\gamma^2$  times more. Thus, the power reflection coefficient is  $k_{power} = 4\gamma^2 (n_{s1}n/2 \cdot 10^{17})^2$ . Power reflection of 100% can be attained with  $\gamma = 40$  and number of bunches n = 100 of density  $n_{s1} = 10^{13}$  cm<sup>-2</sup>.

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