# AMPLITUDE DEPENDENT BETATRON OSCILLATION CENTER SHIFT BY NON-LINEARITY AND BEAM INSTABILITY INTERLOCK 

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## Abstract

As a result of the even symmetry of the Sextupole field, it creates the horizontal shift of the averaged position of horizontal and vertical betatron oscillation and the amount of the shift depends on its oscillation amplitude. This shift can be observed with usual slow orbit beam position monitor.
At the SPring-8 storage ring, this shift is used to detect the excitation of the betatron oscillation for the interlock system for the protection of the vacuum components from strong radiation of insertion devices.

## AMPLITUDE DEPENDENT BETATRON OSCILLATION CENTER SHIFT

The transverse beam instability drives a horizontal or vertical betatron oscillation, and if this occurs in light sources, the strong synchrotron radiation from insertion devices also oscillates as the beam and cause heat damages on beam pipe components if the radiation continuously hit them. The even symmetry of the sextupole field produces the horizontal shift of the time averaged horizontal position if the horizontal and vertical betatron oscillation exists. Here we call it an amplitude dependent betatron oscillation center shift (ADCS). The ADCS on the sextupole strength can be derived by a canonical perturbation theory as the first order effect by sextupole field [1] as

$$
\begin{align*}
& \overline{x(s)}=\frac{\beta_{x}^{1 / 2}(s)}{2 \sin \pi v_{x}}  \tag{1}\\
& \times \int_{s}^{s+C} \frac{1}{2} g_{1}(s)\left[J_{x} \beta_{x}\left(s^{\prime}\right)-J_{y} \beta_{y}\left(s^{\prime}\right)\right] \beta_{x}^{1 / 2}\left(s^{\prime}\right) \cos \left(\Phi_{x}\left(s^{\prime}, s\right)\right) d s^{\prime}
\end{align*}
$$

where the symbols with overline are the time averaged values, $J_{z}$ and $\phi_{z}(z=x, y)$ are the action and the phase, respectively, and related to the position and beta function $\beta_{z}(s)$ as

$$
\begin{equation*}
z=\sqrt{2 J_{z} \beta_{z}(s)} \cos \phi_{z}, \tag{2}
\end{equation*}
$$

$$
\phi_{z}(s)=\int^{s} \frac{d s^{\prime}}{\beta_{z}\left(s^{\prime}\right)},
$$

and $\Phi_{x}\left(s^{\prime}, s\right)=\phi_{x}\left(s^{\prime}\right)-\phi_{x}(s)-\pi v_{x}$.
The sextupole strength is expressed as

$$
\begin{equation*}
g_{1}(s)=\frac{B^{\prime \prime}}{B \rho} \tag{3}
\end{equation*}
$$

for the magnetic field:

$$
\begin{equation*}
B_{y}(x, y)=\frac{1}{2} B^{\prime \prime}\left(x^{2}-y^{2}\right) . \tag{4}
\end{equation*}
$$

The Eq. (1) has the form of the closed orbit distortion produced by the time average kick in the sextupole filed:

$$
\begin{equation*}
\frac{\overline{B_{y}(x, y, s)}}{B \rho}=\frac{1}{2} \frac{B^{\prime \prime}}{B \underline{\rho}}\left(\overline{x^{2}}-\overline{y^{2}}\right)=\frac{1}{2} g_{1}(s)\left[J_{x} \beta_{x}(s)-J_{y} \beta_{y}(s)\right] \tag{5}
\end{equation*}
$$

where we used $z^{2}=J_{z} \beta_{z}(s)$. This process is intuitively understandable with Fig. 1.

We divide Eq. (1) to the contribution by the horizontal and the vertical oscillation as

respectively. In later section, we sometimes use $x_{\text {ave }}, \sigma_{x}{ }^{2}$, and $\sigma_{y}{ }^{2}$ to represent $\overline{x(s)}, \overline{x(s)^{2}}$, and $\overline{y(s)^{2}}$, respectively.


Figure 1: The kick by sextupole field on the electrons with the finite amplitude betatron oscillation.

The ADCS at the beam position monitors (BPMs) for the SPring-8 storage ring (total 288 BPMs, 6 BPM per cell) is calculated with Eq. (6) and (7) as shown in Fig. 2.


Figure 2: The ADCS at BPMs for horizontal ( $x_{\text {ave }} / J_{\mathrm{x}}$ ) and vertical ( $x_{\text {ave }} / J_{\mathrm{y}}$ ) oscillations.

## TRACKING SIMULATION

The tracking simulation was performed to confirm the analytical treatment. In the simulation, a kicker is implemented to the simulation code CETRA [2] to excite 05 Beam Dynamics and Electromagnetic Fields
the beam. However, the resonant frequency of the betatron oscillation shifts as amplitude increase by amplitude dependent tune shift (the second order perturbation by sextupole field), the resonant frequency was searched by changing the excitation frequency of the kicker as the kicker strength changed.

## EXPERIMANT

The ADCS in the SPring-8 storage ring was observed by exciting the horizontal and vertical betatron oscillations. The oscillations were excited by the power amplifiers and kickers for the bunch-by-bunch feedback system, by sending the excitation signal to the input of the feedback. The excitation signal was produced by a tracking generator of a spectrum analyzer with the betatron frequency: $\sim 30 \mathrm{kHz}$ for horizontal and $\sim 70 \mathrm{kHz}$ for vertical. The signal from a beam position monitor was fed to the spectrum analyzer to find the resonant betatron frequency. Because of the amplitude dependent tune shift, the excitation frequency was adjusted as increase of the excitation power.

The amplitude of the excited betatron oscillation was measured by the single-pass beam position monitor. Then, $J_{x}$ and $J_{y}$ were calculated as $\overline{z^{2}}=J_{z} \beta_{z}(s)$.

The ADCS was measured by the slow beam position measurement system. The results of the measurement of ADCS by the horizontal oscillation are shown in Fig. 3 and Fig. 4, and agree well with the analytical result by Eq. (6) and the tracking simulation.


Figure 3: Measured ADCS (circle) for the horizontal oscillation with the action $J_{x}=3.0 \times 10^{-7}$. Theoretical values (triangle) are also shown.


Figure 4: ADCS at the third BPMs in each cells to show the agreement; observed values (circle) and theoretical values (triangles). The points with dotted circles are for Figure 5.


Figure 5: Dependence of $\overline{x(s)}$ ( $x_{\text {ave }}$ ) on horizontal action $J_{x}$, at BPMs previously shown at Fig. 4. White marks (Measured \#) are measured values, solid marks are tracking simulation result (TRACK \#), and lines are analytical result (TH \#) .

For vertical oscillation, the beta function and the strength of the kicker are much smaller than for horizontal, therefore, the excitation amplitude is one order smaller than the horizontal. The result is shown in Fig. 6, however, no good agreement and larger amplitude oscillation should be necessary. Later we compare the tracking result and the theoretical values for vertical oscillation to confirm the model.
Also the ADCS observed with the vertical instability is shown in Fig. 7. The corresponding $J_{y}=1.3 \mathrm{e}-8$. Still the agreement is poor.


Figure 6: ADCS for vertical oscillation for two different excitation ( $J_{y}=1.1 \mathrm{e}-8$ and $0.53 \mathrm{e}-8$ ), and the theoretical values.


Figure 7: ADCS observed at vertical instability. Measured (circles with line), theory (solid triangle), inverted value of theory (open triangle).

## INTERLOCK OF INSERTION DEVICES

In SPring-8, the interlock system for the beam direction/position in insertion devices (IDs) is implemented to prevent vacuum components from the radiation of IDs by Hara and Seike. For the interlock, the dedicated RF beam position monitors are installed at the both ends of IDs to observe the position shift of the electron beam from the nominal position, and if the large shift is detected, then the beam is aborted in 1 ms by turning off the RF power of the acceleration system. The window of the shift is $+/-0.5 \mathrm{~mm}$ for horizontal and $+/-$ 0.25 mm for vertical.

However, in case that the betatron oscillation is excited by the instability, but the oscillation amplitude was saturated by some reason like Landau damping by amplitude dependent tune shift, then the radiation of the IDs spreads out continuously and may produce the damage on beam pipe components.

The ID beam direction/position interlock system is basically designed for the slow motion of the beam orbit less then kHz , and is not fast for the detection betatron oscillation. However, the large amplitude betatron oscillation produces large ADCS, which is detectable even by such slow system, and the beam is aborted as the same sequence as slow orbit shift. Now we use this system for the instability interlock for the protection of the components.
For the first version of the interlock for such cases, we build the power detection system of a BPM electrode difference signal with 180 deg. hybrid, and if the betatron oscillation occurs, the power of the difference signal increases and the instability can be detected. However, the SPring-8 ring has many filling patterns with various bunch current and the power varies much as the filling patterns. The ADCS method is free from such trouble and the instability interlock was switched to the ADCS method in 2007.

## ADCS AT INSERTION DEVICES

The ADCS at straight sections for insertion devices are calculated with Eq.(6) and (7) and are shown in Fig. 7, in which the values are normalized for the all the values are evaluated in mm with the beta functions $\beta_{x}=$ 23 m and $\beta_{y}=6.5 \mathrm{~m}$ at the BPMs for interlock at insertion devices. The theoretical values for vertical oscillation are confirmed by the tracking simulation as in Fig. 8.

From the Fig. 7, the largest absolute value is 0.07 for horizontal, and 0.1 for vertical, at multiple IDs. If the horizontal oscillation amplitude exceeds 4 mm , then, $\sigma_{x}^{2}=8 \mathrm{~mm}^{2}$ and $x_{\text {ave }}=0.07 \times 8 \sim 0.5 \mathrm{~mm}$ : the window of the interlock system. For vertical, the amplitude exceeds 2 mm at ID13, then $\sigma_{y}^{2}=2 \mathrm{~mm}^{2}$ and $\left|x_{\text {ave }}\right|=$ $0.12 \times 2=0.24 \mathrm{~mm} \sim 0.25 \mathrm{~mm}$ : the vertical window of interlock system. These interlock amplitude, 4 mm for horizontal and 2 mm for vertical, are rather higher than the slow interlock window, $+/-0.5 \mathrm{~mm}$ for horizontal and $+/-0.25 \mathrm{~mm}$ for vertical, but the irradiation duty is low
compared with slow orbit shift because the radiation axis is oscillating. Also the minimum half gap of insertion devices is 2.5 mm ; larger than the oscillation amplitude.


Figure 7: Theoretical value of ADCS at the center of IDs. The ADCS is normalized as all values are evaluated with mm in this figure for easy estimation of the excited betatron amplitude.


Figure 8: Comparison of ADCS by the tracking simulation result (points) and by theory (lines) fat several IDs shown in Fig. 7 .

## SUMMARY

The amplitude dependent oscillation center shift was analyzed theoretically and experimentally for the SPring8 storage ring. The both result agree well for horizontal oscillation, however, further study is necessary for vertical by increasing vertical oscillation amplitude by strong kicker [3]. This effect adds the capability of the instability interlock to the beam axis interlock system of the insertion devices using the SPring- 8 with slow BPM system. The effect of ADCS on the injection for the New SUBARU ring is also discussed in Ref. [4].

## REFERENCES

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05 Beam Dynamics and Electromagnetic Fields

