HIGH ORDER NON-LINEAR MOTION IN ELECTROSTATIC RINGS

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Abstract

The advantages of an electrostatic storage ring as compared to a magnetic ring are obvious from the point of view to search for the proton electric dipole moment (pEDM). However the magnetic and electrostatic fields have the different nature and, consequently, different features. In particular, particles moving in electrostatic field, can change their own kinetic energy as electrical field coincides with the direction of motion, which is not so for the magnetic field, where the force is always perpendicular to the direction of motion. The electrostatic rings found many applications in the atomic physics and partly the beam dynamics has been already investigated. However in EDM ring some additional specific features are added, which are considered in this paper.

INTRODUCTION

The possible experiment to search for the electric dipole moment (EDM) using an electrostatic storage ring is widely discussed now [1]. It may be a ring either with only electrostatic elements, such as in the case of proton EDM search, or with electrostatic and magnetic elements for experiment with deuterons.

Electrostatic elements in accelerators have been used in the past. Particularly in BNL laboratory the AGS electrostatic analog was first assembled [2]. Much later, great interest has arisen for electrostatic storage rings in atomic physics, biology and chemistry [3]. In paper [4] the beam dynamics in an electrostatic ring has been studied on the example of the ring ELISA to increase the life time, which was mainly determined by the dynamic aperture. The most successful experiment to search for the EDM could be based on measurement of the spin precession dependence on external field strength. In this regard, the new requirements for the ring lattice appeared to weaken the decoherent effects in polarized beam. Since we are interested in the features of storage ring introduced namely by the electrostatic elements, we consider a purely electrostatic ring, where $B=0,\ E\neq 0$ and the spin precession frequency ω_G relative to the momentum direction is:

$$\vec{\omega}_G = -\frac{e}{m_0\gamma c}\left[\left(\frac{1}{\gamma^2-1}-G\right)\cdot\vec{\beta}\times\vec{E}\right] \qquad (1)$$

Where G is the anomalous magnetic moment. The advantages of purely electrostatic rings are especially evident in the so-called magic rings, when

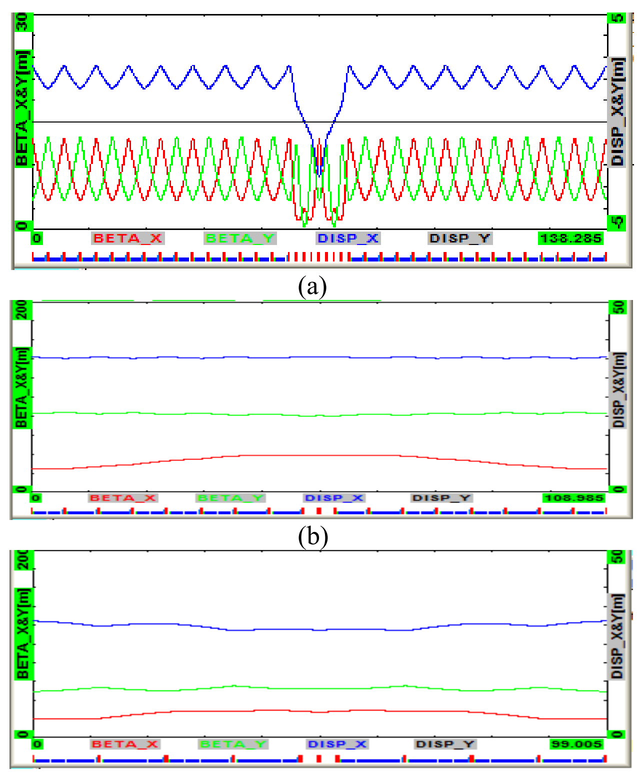
$$G-1/(\gamma_{mag}^2-1)=0 \qquad (2)$$

and the spin oriented in the longitudinal direction rotates in the horizontal plane with the same frequency as the momentum vector, resulting in $\omega_G=0$ [5].

In the paper we study the beam dynamics only, and everything about the spin will be considered in paper [6] of this proceedings. But all properties of the lattice considered here must be subjected to the magic condition (2). All numerical simulations performed in the simulation programs OptiM [7] and COSY Infinity [8].

LATTICE

We have considered three purely electrostatic lattices (see fig.1) based on electrostatic elements only. The rings structure are designed for accumulation of polarized protons with magic energy for proton 248 MeV. They all have two arcs and differ in the number of periods and total radial tunes.



(a)
(b)
(c)

Figure1: Lattices with tunes: $\nu_x/\nu_y = 7.9/7.8;\ 1.12/0.42;\ 1.32/0.6$

In the first structure (a) we have 32 cells, in the second (b) 16 cells and in the third (c) 8cells per arc. Each cell is a FODO structure having either two, or four, or eight electrostatic deflectors and two electrostatic quadrupoles, focusing and defocusing (see fig. 2).

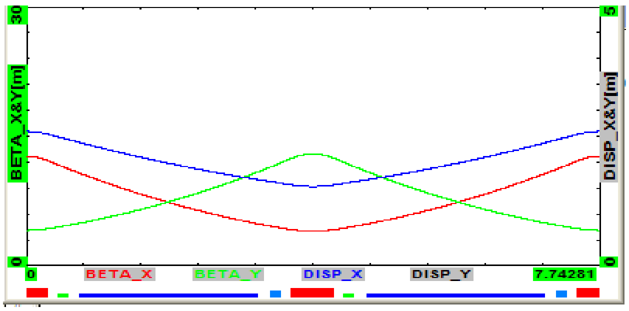


Figure 2: One FODO cell of electrostatic lattice

Around each quadrupole, sextupole and BPM are placed. The deflectors in all options are the same and have the field strength of 170 MV/m. Thus, the total length of all deflectors is constant and equals to ~155 m.

We analyzed all three structures with different shape of deflector plate in terms of the requirements for maximum spin coherence time [6].

OWN NONLINEARITY OF DEFLECTOR

Since deflectors provide a main influence on the spin-orbital motion, we just focused on the beam dynamics study in the deflector. We consider two types of deflectors with spherical and cylindrical shapes of the electrodes.

Motion Equation in Deflector

In paper [4] the equations of motion up to third order of non-linearity in approach of not truncated spheres in spherical deflector and cylinders in cylindrical deflector with radius R$_1$ and R$_2$ and potential on spheres and cylinders $\pm\varphi_0$ have been obtained for spherical:

$$\begin{aligned} &x'' + \frac{1}{R_{eq}^2}x - \frac{1}{R_{eq}^3}x^2 - \frac{3}{2\cdot R_{eq}^3}y^2 = 0 \\ &y'' + \frac{1}{R_{eq}^2}y - \frac{3}{R_{eq}^3}xy = 0 \end{aligned}, \qquad (3)$$

with equilibrium radius $R_{eq} = M^2 \cdot \dfrac{1}{2em\varphi_0} \cdot \dfrac{R_2 - R_1}{R_1 R_2}$ and for cylindrical:

$$\begin{aligned} &x'' + \frac{2}{R_{eq}^2}x - \frac{1}{R_{eq}^3}x^2 = 0 \\ &y'' = 0 \end{aligned}, \qquad (4)$$

with equilibrium radius $R_{eq} = M\sqrt{[\ln(R_2/R_1)]/(2em\varphi_0)}$, where the angular moment $mr^2\dot{\vartheta} = p_{\vartheta}\cdot r = M = const$ in absence of fringe fields due to the axial symmetry.

From (3) you can see the spherical deflector focuses the particle in horizontal and vertical planes with the same force and in the cylindrical deflector in the horizontal plane the focusing term is twice as much as in the spherical deflector, but it is no focusing in the vertical plane. Both deflectors have an own sextupolar component.

Dispersion Equation

Let us suppose that the particle has a non-zero energy deviation. Then the new equilibrium radius $\widetilde{R}_{eq}$ will be in an accordance with expression $\dfrac{1}{\widetilde{R}_{eq}} = \dfrac{1}{R_{eq}}\cdot\left(1 - 2\dfrac{\Delta v_{\vartheta}}{v_{\vartheta}}\right)$, and the full system of the equations in linear approach is

$$\begin{aligned} &x'' + \frac{1}{R_{eq}^2}x = \frac{2}{R_{eq}}\frac{\Delta v_{\vartheta}}{v_{\vartheta}} \\ &y'' + \frac{1}{R_{eq}^2}y = 0 \end{aligned} \qquad (5)$$

The solution of this system can be represented in the matrix form:

$$\begin{aligned} \begin{bmatrix} x \\ x' \end{bmatrix} = &\begin{bmatrix} \cos\phi & R_{eq}\sin\phi \\ -\dfrac{1}{R_{eq}}\sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ x'_0 \end{bmatrix} + \\ &+ 2\frac{\Delta v_{\vartheta}}{v_{\vartheta}} \begin{bmatrix} R_{eq}(1-\cos\phi) \\ \sin\phi \end{bmatrix} \end{aligned}, \qquad (6)$$

Where $\phi = s/R_{eq}$, x, x'_0, y_0, y'_0 are the initial meaning of the coordinates.

LONGITUDINAL-TRANSVERSE COUPLING

The spherical and cylindrical deflectors have the central field symmetry, namely $E \propto \dfrac{\alpha}{r^n}$. In particular, the case of the spherical deflector, when $n = 2$, accords to the Keppler problem and it was investigated by Landau in [9]. The case $n = 1$ accords to the cylindrical deflectors. It is obviously that for both of them the full energy conservation is fulfilled:

$$W(r, \dot{r}, \dot{\vartheta}) = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\vartheta}^2\right) + \varphi(r), \qquad (7)$$

If not take into account the fringe fields, the angular momentum $M = mv_{\vartheta}r$ has to be conserved.

In case of the fringe field we have with good approximation very short “entrance” and “exit” regions, where the angular momentum increases or decreases, but in mainly we have the region where M is constant. Due to this fact the total energy can be represented as the function of the coordinates $r, \dot{r}$:

$$W(r, \dot{r}) = \frac{m}{2}\dot{r}^2 + \frac{M^2}{2mr^2} + \varphi(r) \qquad (8)$$

The last expression shows to us that the radial motion could be considered as one dimensional motion in the field with the effective potential consisting of the centrifugal potential and the electrical potential:

$$\varphi_{eff}(r)=\frac{M^2}{2mr^2}+\varphi(r) \qquad (9)$$

Differentiating (9), we are getting the same expressions for the equilibrium radius, what we have got in (3) and (4), namely, for the spherical deflector:

$$R_{eq}\propto M^2 \text{, or } \Delta R_{eq}/R_{eq}=2\cdot\Delta M/M_0 \qquad (10)$$

and for the cylindrical deflector:

$$R_{eq}\propto M \text{, or } \Delta R_{eq}/R_{eq}=\Delta M/M_0 \qquad (11)$$

Thus, if the matrix of the deflector is described by the equations (6), then

$$\frac{\Delta v}{v}=\frac{(\cos\phi-1)}{R_{eq}}\cdot x_0+\sin\phi\cdot x_0'+\frac{\Delta v_0}{v} \qquad (12)$$

-Hereinafter we will omit the index ϑ of v_ϑ.

Joining (6) with (12) we have

$$\begin{bmatrix} x \\ x' \\ \frac{\Delta v}{v} \end{bmatrix}=\begin{bmatrix} \cos\phi & R_{eq}\sin\phi & 2R_{eq}(1-\cos\phi) \\ -\frac{1}{R_{eq}}\sin\phi & \cos\phi & 2\sin\phi \\ \frac{\cos\phi-1}{R_{eq}} & \sin\phi & 1 \end{bmatrix}\cdot\begin{bmatrix} x_0 \\ x_0' \\ \frac{\Delta v_0}{v} \end{bmatrix} \qquad (13)$$

Where $\phi=s/R_{eq}$ for the spherical deflector and $\phi=\sqrt{2}s/R_{eq}$ for the cylindrical deflector. Thus, we can say that a particle with initial deviation from the equilibrium radius unequal to zero is oscillating around a new equilibrium energy level.

Now let us suppose that the matrix of the transport channel between two deflectors is M_{tr}. Then at the entrance of the next deflector:

$$\left\{x,x',\frac{\Delta v}{v}\right\}=M_{tr}\cdot M_{def}\cdot\left\{x_o,x_0',\frac{\Delta v_0}{v}\right\} \qquad (14)$$

and after n turns:

$$\left\{x,x',\frac{\Delta v}{v}\right\}_n=\left(M_{tr}\cdot M_{Def}\cdot M_{tr}\right)^n\cdot\left\{x_o,x_0',\frac{\Delta v_0}{v}\right\}, \qquad (15)$$

Where M_{Def} is the matrix of the deflector (13).

Since the equilibrium energy level in each deflector depends on the entrance coordinates to the deflector, it is obvious that the instantaneous energy level will vary according to the phase advance of focusing channel of the ring. And if the number of transverse oscillations in the ring is much greater than unit, we can talk about some average energy level with respect to which all particles oscillate. It is a completely different situation when the number of oscillations per revolution approaches to unit. Then the energy level changes within a time comparable with the time of the longitudinal oscillations, and the particles oscillating with respect to time-varying energy levels have phase trajectories of Lissajous curves.

Actually, the oscillation of the equilibrium energy level means the transformation of potential energy into kinetic and vice versa. The process of the energy conversion each into another has the oscillating character, since the tune of the ring is not integer figure. However, the conversion process is suppressed by the fringe field by some factor k. This coefficient k depends on the initial co-ordinate and decrease with the growth of the last one. If now we substitute k into (12), we get:

$$\frac{\Delta v}{v_\vartheta}=\frac{1}{k}\left[\frac{(\cos\phi-1)}{R_{eq}}\cdot x_0+\sin\phi\cdot x_0'\right]+\frac{\Delta v_0}{v} \qquad (16)$$

This mechanism of the equilibrium energy change explains the increase of the spin aberrations in the magic rings.

Each particle oscillating about its own non-magic energy level violates condition (2) in different degrees, which leads to the spin aberrations.

CONCLUSION

In this paper we studied the effects of own nonlinear deflector field on the beam dynamics in electrostatic rings. The main difference of electrostatic rings is that kinetic energy alternately transforms into potential and vice versa. This leads to change in time of the energy level which causes the spin aberrations.

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