AMPLITUDE DEPENDENT ORBIT SHIFT AND ITS EFFECT ON BEAM INJECTION *

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Abstract

The betatron oscillation amplitude dependent orbit shift was measured at the electron storage ring, NewSUBARU. The result agreed with the theoretical calculations. The effect of this shift on the beam injection is discussed using parameters of NewSUBARU and SPring-8 SR. Generally there exists a better side for the injection, the inner side or the outer side of the ring, which depends on the sign of the orbit shift at the injection septum.

INTRODUCTION

Sextupoles in a synchrotron produce an amplitude dependent shift of the oscillation center. However, this shift has importance only in special cases. One is that the shift can be used to detect the beam instability [1]. The other case is that of beam injection, whereby the injected beam has a large oscillation amplitude.

One subject of this report is to give a simple analytical formula for the shift, which has been subsequently confirmed by measurement.

Another aim is to discuss the effect of the orbit shift on beam injection. Generally there exists a better side for the injection, the inner side or the outer side of the ring, depending on the sign of the orbit shift at the injection septum. In case of the NewSUBARU, the beam is injected from the outer side and the shift is positive. The effective thickness of the septum could be reduced with the large oscillation amplitude of the injected beam. On the other hand at SPring-8 SR, the beam is injected from the inner side of the ring with a negative the orbit shift. In either case, the rings are use the optimal side for injection.

AMPLITUDE DEPENDENT ORBIT SHIFT

Analytical Formulae

We define the sextupole component g as

$$d\theta_x = (g/2)x^2 ds \ . \qquad (1)$$

Here θ_x is the horizontal deflection angle. We will also use the normalized sextupole components defined by

$$G_x \equiv g\beta_x^{3/2} \ , \qquad (2a)$$

$$G_y \equiv g\beta_y\beta_x^{1/2} \ . \qquad (2b)$$

The linear betatron oscillation is given by

$$x = \sqrt{\beta_x J_x}\cos\psi_x \qquad (3)$$

Here, J_x is the Courant Snyder Invariant (C.S.I) of the horizontal betatron oscillation, which corresponds to the action. Substituting Eq. (3) into Eq. (1) and we obtain

$$d\theta_x = (g/4)\beta_x J_x (1+\cos 2\psi_x) ds \ . \qquad (4)$$

The deflection by a sextupole has two frequency components, the constant deflection θ_{x0} and the deflection with twice the betatron oscillation frequency θ_{x2}.

The constant deflection produces a shift of the oscillation center given by

$$dx_0 = \left[\frac{\sqrt{\beta_x\beta_{xS}}}{2\sin\pi\nu_x}C(s,s_S)\right]d\theta_{x0} \ , \qquad (5)$$

Here, $C_x(s, s_S)$ is the phase factor defined by

$$C(s,s_S) \equiv \cos(|\psi_x(s) - \psi_x(s_S)| - \pi\nu_x) \ . \qquad (6)$$

The displacement produced by distributing sextupoles is

$$x_0(s) = \frac{\sqrt{\beta_x(s)}}{8\sin\pi\nu_x} J_x \int_0^L G_x(s_S) C(s,s_S) ds_S \ . \qquad (7)$$

The displacement caused by the vertical oscillation with C.S.I of J_y can be calculated by the same manner. Finally, the displacement is given by

$$x_0(s) = \frac{\sqrt{\beta_x(s)}}{8\sin\pi\nu_x}\int_0^L [J_x G_x(s_S) - J_y G_y(s_S)] C(s,s_S) ds \ . \qquad (8)$$

The betatron oscillation also changes the circumference, the averaged path-length for a revolution [2,3]. Then, after a single deflection, the particle would start synchrotron oscillation. The shift of the circumference is given by

$$\Delta L = -2\pi(\xi_x J_x + \xi_y J_y) \ . \qquad (11)$$

Here, ξ_x and ξ_y are the horizontal and the vertical chromaticities. The center of the energy oscillation is given by

$$\delta = (2\pi/\alpha_P L_0)(\xi_x J_x + \xi_y J_y) \ , \qquad (12)$$

where α_p is the momentum compaction factor and L_0 is the circumference. Over a long range, averaged over a synchrotron oscillation period, a shift by the energy displacement should be added to Eq. (8).

Amplitude Dependent Tune Shift

The orbit displacement is one of the origins of the amplitude dependent tune shift. It goes to infinity at integer tune. The other origin is the second harmonic oscillation produced by the second harmonic deflection (θ_{x2}). It goes to infinity for third integer tune also. Am analytical expression for the shift is given elsewhere using an action-angle formula [4]. We do not repeat this calculation, but it is not difficult to derive the shift using an x, y, s coordinate system.

A shift in equilibrium energy can also lead to a tune shift. The appearance of this effect depends on the how the betatron oscillation starts.

Orbit Shift at NewSUBARU

NewSUBARU is a 1.0 – 1.5 GeV electron storage ring at the SPring-8 site. Table 1 shows the basic parameters of the ring. The ring is a race-track type and has four-fold symmetry. Figure 1 shows the calculated amplitude dependent orbit shift in 1/4 of NewSUBARU.

Table 1: Basic parameters of NewSUBARU

Betatron tune: ν_x / ν_y	6.30 / 2.23
Chromaticity: ξ_x / ξ_y	3.4 / 5.8
Momentum compaction factor: α_p	0.00136
Horizontal emittance	50 π nm
Septum wall from the beam center: x_{MAX}	+21 mm
Septum thickness: Δx_{SEP}	3 mm

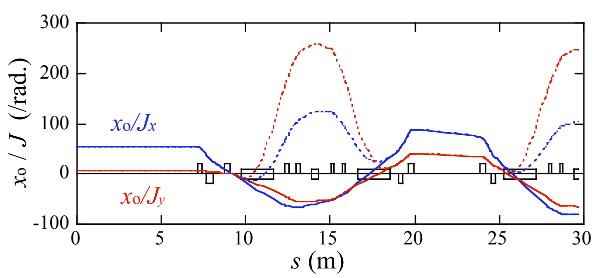

Figure 1: Amplitude dependent orbit shift in 1/4 of NewSUBARU storage ring. The broken lines are the shifts of the synchrotron oscillation center.

We measured the horizontal orbit shift of the horizontal betatron oscillation. The stationary stored electron beam was deflected by an injection kicker. The turn-by-turn beam position for about 40 turns after the deflection was recorded at 18 beam position monitors (BPM) in the ring. 0.4% of the stored beam was lost by a deflection corresponds to J_x≈5µm rad. The orbit was measured for the deflection of 80%, 50%, 30%, and 10% of that. The measured period (16µs) was much shorter than the synchrotron oscillation period (190µs), as well as that given by Eq. (8).

Figure 2 shows the result. It agrees well with the calculations based on Eq. (8).

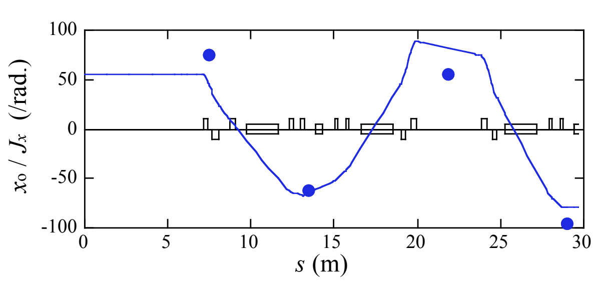

Figure 2: Amplitude dependent orbit shift. The circles are the measured shift and the solid line is the calculated shift using the analytical formula.

The same measurement gave the amplitude dependent tune shift. However, it provided limited useful information due to large measurement inaccuracy.

EFFECT ON BEAM INJECTION

Basic Effect on the Beam Injection

The effect of the amplitude dependent orbit shift on the beam injection is qualitatively understood as the following. We start with the maximum displacement of the circulating beam and the injection point of the injected beam, written as

$$x_{MAX} = \sqrt{J_{xMAX}\beta_x} + (\partial x_0/\partial J_x)J_{xMAX} + x_{BUMP} \quad (13)$$

and

$$x_{INJ} = \sqrt{J_{xINJ}\beta_x} + (\partial x_0/\partial J_x)J_{xINJ} + x_{BUMP} \quad (14)$$

respectively, with

$$x_{INJ} = x_{MAX} + \Delta x_{SEP}. \quad (15)$$

Here, J_{xMAX} and J_{xINJ} are the maximum J_x of the circulating beam and J_x of the injected beam, respectively, and x_{BUMP} is the injection bump height and Δx_{SEP} is the thickness of the injection septum. In this case J_{xINJ} is calculated to be

$$J_{xINJ} = J_{xMAX} + \frac{\Delta x_{SEP}}{\dfrac{\sqrt{\beta_x}}{\sqrt{J_{xINJ}} + \sqrt{J_{xMAX}}} + \dfrac{\partial x_0}{\partial J_x}} \quad (16)$$

This means that the effect of the finite septum thickness depends on $\partial x_0/\partial J_x$.

Calculation at NewSUBARU

In this subsection, we will see the effect based on the results of a tracking calculation using the parameters of NewSUBARU. The linear dispersion is zero at the injection location, and the energy spread is ignored for the simplification of the calculation. The higher order multi-pole field is also ignored, giving an idealized set of parameters are not always those used in the real injection.

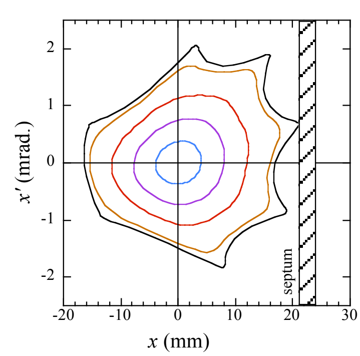

Figure 3: Phase space contour at the injection point of NewSUBARU. The shaded area is the injection septum.

Figure 3 shows the phase space contour (Poincaré map) of the circulating beam at the location of the injection point. Figure 4(a) shows the beam at the instance of the injection. The injection bump height was 18.2 mm, with which only electrons displaced more than 4σ are scraped by the septum. The injected electron moves along the edge of the other side of the septum. The trajectories of the circulating beam and the injected beam after the injection are shown in Figure 4(b). By contrast Figures 5 (c) and (d) show the results when the septum is set at the inner side of the ring. Contrary to our expectation, J_{xINJ} was almost the same for both cases.

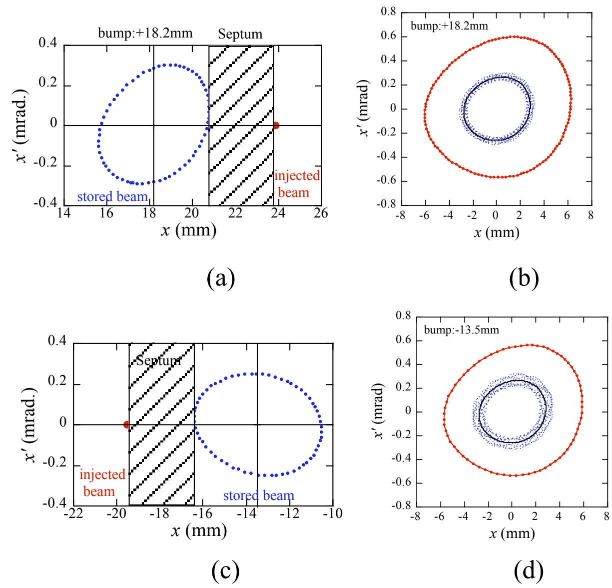

Figure 4: Phase space contour at the injection point. (a) and (c) show those at the initial turn at the injection and (b) and (d) show those after the injection. The upper and lower figures are for injection from the outer side and inner side of the ring, respectively.

Figure 5 gives explanation for this unexpected result. Eq. (13) is not a good approximation and x_{MAX} less depends on J_x. However, when we change ν_x from 6.302 to 6.207, we see a difference as plotted in Figure 5.

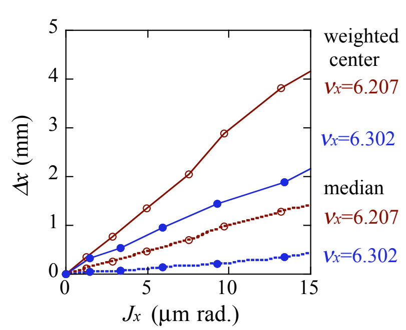

Figure 5: J_x dependence of the weighted center and the median of the stored beam at NewSUBARU for ν_x =6.302 and ν_x =6.207.

Calculation at SPring-8

In SPring-8 SR, unlike NewSUBARU, the orbit shift is negative and the injection septum is set at the inner side of the ring.

Figures 6 and 7 show the phase space contour and the orbit shift, respectively, illustrating that the injection from the inner side is better for SPring-8 SR.

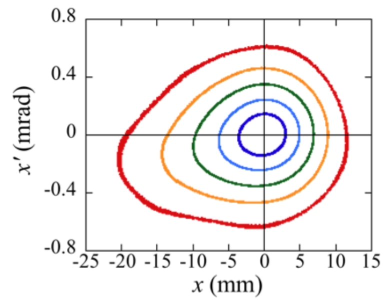

Figure 6: Phase space contour at the injection point of SPring-8 SR.

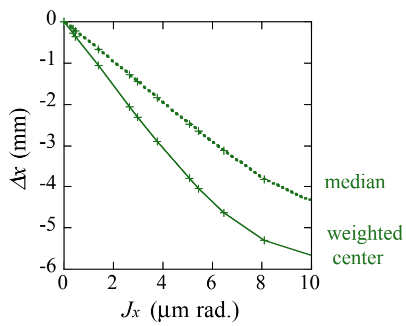

Figure 7: J_x dependence of the weighted center and the median of the stored beam at SPring-8 SR.

SUMMARY

We discussed the effect of the amplitude dependent shift of oscillation center by sextupoles. In many cases the injection from the side of the orbit shift would have an advantage but not always.

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