# COMPARISON OF THE ACTION AND PHASE ANALYSIS ON LHC ORBITS WITH OTHER TECHNIQUES * 

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## Abstract

Recently acquired turn-by-turn data of the LHC is analyzed using the action and phase jump technique. The results of this analysis show a visible variation of the action and phase plots at the interaction regions from which optic error estimations can be done. In this paper error estimations will be presented and comparisons with other existing techniques in the LHC, such as the recently implemented Segment-by-segment technique, will be discussed.

## INTRODUCTION

The action and phase jump technique has been used in the past to successfully estimate normal and skew quadrupole components at RHIC Interaction Regions (IRs) (see [1], [2] , [3]). This has motivated the application of this etechnique on LHC orbits. In the first section of this paper, $\dot{r}$ it is shown how this technique can be adapted to estimate and correct linear errors at the LHC interaction regions. In the second section, action and phase jump analysis of LHC Turn By Turn TBT data is presented and compared to re$\because$ sults of the Segment-by-Segment Technique (SBST) which is regularly used at LHC for optics commissioning.

## CALCULATION OF CORRECTIONS SETTINGS FROM ACTION AND PHASE ANALYSIS

It is shown in [4] that the action and phase jump analysis can be used to calculate the kick $\left(\theta_{x}, \theta_{y}\right)$ that a particle experience due to a magnetic error. If such a magnetic error is composed of an integrated normal quadrupole component $B_{1}$ and an integrated skew quadrupole component $A_{1}$, these values can be determined inverting the equations:

$$
\begin{align*}
& \theta_{x}=A_{1} y\left(s^{*}\right)-B_{1} x\left(s^{*}\right)  \tag{1}\\
& \theta_{y}=A_{1} x\left(s^{*}\right)+B_{1} y\left(s^{*}\right)
\end{align*}
$$

where $x$ and $y$ are the horizontal and vertical position of the beam at the place $s^{*}$ where the error is. This procedure can be applied to all magnets in the interaction regions provided that at least two beam position measurement are available at each side of every magnet. This is not usually the case for any accelerator. In LHC, for example, no more than 6 bpms are available for each interaction region.

[^0]It is possible, however, to consider a group of magnets and estimate an equivalent magnetic error for the whole group. The best choice for a group of magnets is the triplet which is the most common magnet configuration at the interaction regions. For this case, it can be shown that Eq. 1 transforms into:

$$
\begin{align*}
& \theta_{x}^{t}=A_{1}^{t} y\left(s^{*}\right)-B_{1}^{t x} x\left(s^{*}\right) \\
& \theta_{y}^{t}=  \tag{2}\\
& A_{1}^{t} x\left(s^{*}\right)+B_{1}^{t y} y\left(s^{*}\right)
\end{align*}
$$

where $A_{1}^{t}, B_{1}^{t x}$ and $B_{1}^{t y}$ will depend on the individual skew and normal quadrupole errors present in the quadrupoles of the triplet. In practice, it is possible to estimate action and phases before and after a particular triplet and hence $\theta_{x}^{t}, \theta_{y}^{t}$, $A_{1}^{t}, B_{1}^{t x}$ and $B_{1}^{t y}$ can be determined.

The last three quantities can be very useful for local correction at the IRs. Indeed, it can be demonstrated that if $s^{*}$ is chosen at the place where the skew quadrupole corrector of the triplet is located $-A_{1}^{t} / L c$ ( $L c$ is the longitude of the skew quad corrector) is approximately equal to the strength needed in this corrector to eliminate local coupling at the triplet. On the other hand, the effects of normal quadrupole errors in the triplet can be suppressed by changing the strengths of two quadrupoles of the triplet according to the values of $B_{1}^{t x}$ and $B_{1}^{t y}$. Such relationships are given by:

$$
\begin{equation*}
\Delta k\left(Q_{1}\right)=\frac{B_{1}^{t y} \beta_{y}\left(s^{*}\right) \int_{Q_{2}} \beta_{x} d s-B_{1}^{t x} \beta_{x}\left(s^{*}\right) \int_{Q_{2}} \beta_{y} d s}{\int_{Q_{1}} \beta_{x} d s \int_{Q_{2}} \beta_{y} d s-\int_{Q_{2}} \beta_{x} d s \int_{Q_{1}} \beta_{y} d s} \tag{3}
\end{equation*}
$$

$\Delta k\left(Q_{2}\right)=\frac{B_{1}^{t x} \beta_{x}\left(s^{*}\right) \int_{Q_{1}} \beta_{y} d s-B_{1}^{t y} \beta_{y}\left(s^{*}\right) \int_{Q_{1}} \beta_{x} d s}{\int_{Q_{1}} \beta_{x} d s \int_{Q_{2}} \beta_{y} d s-\int_{Q_{2}} \beta_{x} d s \int_{Q_{1}} \beta_{y} d s}$
where $\Delta k\left(Q_{1}\right) \Delta k\left(Q_{2}\right)$ correspond to the values at which $Q_{1}$ and $Q_{2}$ should be changed in order to compensate all possible normal gradient present in the 3 quadrupoles of the triplet. Selection of the 2 quadrupoles out of the three is arbitrary. In principle, any combination is allowed to do the correction but some of them might be determined with better accuracy than others.

The determination of $B_{1}^{t x}$ and $B_{1}^{t y}$ must be done with at least two different orbits (Notice that there are 3 unknown

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Figure 1: LHC orbit with a maximum value at the right triplet of IR5 in the horizontal plane and a minimum value at the same place in the vertical plane.This is one of the 4 kinds of orbits used for error estimation at LHC IRs.
variables in the Eq. 2 unlike Eq. 1 which has 2 unknown variables). Several simulations have shown that there are 4 kinds of orbits that allows the precise determination of $A_{1}^{t}$, $B_{1}^{t x}$ and $B_{1}^{t y}$. One of these orbits is shown in Fig. 1 where it is possible to see that the orbit has a maximum excursion of the horizontal position of the beam at the right triplet of IR5 while the vertical position is minimum at the same place. The other kinds of orbits correspond to the combinations: maximum in the horizontal plane while the vertical is maximum, minimum in the horizontal plane while the vertical is maximum and minimum in the horizontal plane while the vertical is minimum.

## ANALYSIS OF EXPERIMENTAL ORBITS AND COMPARISONS

Since 2009, several beam related experiments and optical corrections have been done in the LHC using the SBST (see [5] and [6]). In particular, during the 2010 LHC run the beam was squeezed to 2 m in all IPs to be able to measure errors at the IRs. One of the corrections that might be compared with the results of action and phase analysis is the one done at IR5. During this experiment, correction was performed by changing the strength of two IR quadrupoles MQXB.B2R5 and MQXB.B2L5. In this case, the two triplets of IR5 can be defined as the group of magnets and the equations of the previous sections can be used to estimate the values proposed for correction in MQXB.B2R5 and MQXB.B2L5.
Initially the idea was to select from the turn by turn data single orbits to estimate $A_{1}^{t}, B_{1}^{t x}$ and $B_{1}^{t y}$. However, due 05 Beam Dynamics and Electromagnetic Fields


Figure 2: The solid line corresponds to the phase analysis of the average of all turns shown in Fig. 3 while the dotted line corresponds to the phase analysis of a single turn.


Figure 3: Orbits with a maximum value at the right triplet of IR5 are selected from a single file of TBT data. In this case, 117 orbits were found out of the 2000 turns.
to the wide band of the LHC BPMs, these orbits exhibited a high level of noise in the action and phase plots (see dotted line of Fig. 2). An average orbit can help to reduce the noise but it cannot include all turns because it will converge to the unperturbed small closed orbit of the accelerator. Instead, turns in phase can be selected from the TBT data (see Fig. 3) to build an average orbit. The phase plot of this new average orbit (solid line of Fig. 2) has much lower noise


Figure 4: The equivalent kick that the orbit experience due to errors in IR5 is plotted against the orbit position. The slope of the linear regression (dotted line) gives directly the values of $B_{1}^{t x}$ and $B_{1}^{t y}$.
than the phase plot of a single turn orbit.
Now, orbits with the specific conditions required to estimate $A_{1}^{t}, B_{1}^{t x}$ and $B_{1}^{t y}$ (see Fig. 1) are less frequent due to the double requirement of having maximums or minimums at both planes simultaneously and hence much lower number of turns are found from each TBT data file. Therefore, there are more noise in the resultant average orbits but it is still possible to estimate the errors with the equations presented in the previous section.

The experiment under analysis has 3 different sets of TBT data. From each set of data two errors values for each of the 2 quadrupoles can be calculated. As consequence there will be six different error estimations for each of the 2 quadrupoles. The second row of table 1 reports the average values obtained in this way. The uncertainties correspond to one standard deviation of the mean.

Table 1: Estimated Corrections $\Delta k$

| Analysis | MQXB.B2R5 <br> $* 10^{-5} \mathrm{~m}^{-2}$ | MQXB.B2L5 <br> $* 10^{-5} \mathrm{~m}^{-2}$ |
| :--- | :---: | :---: |
| SBST | 1.3 | 1 |
| 4 kinds of orbits | $3.1 \pm 0.3$ | $0.64 \pm 0.2$ |
| Approximation | $3.2 \pm 0.1$ | $0.55 \pm 0.1$ |

A second approach to estimate the corrections can be used assuming that the effect of coupling in Eq. 2 is negligible. This seems to be the case for the average orbit obtained from the hundred of turns shown in Fig. 3 since the average vertical orbit is very small when compared with the
horizontal one. Under this approximation Eq. 2 becomes:

$$
\begin{equation*}
\theta_{x}^{t}=-B_{1}^{t x} x\left(s^{*}\right), \theta_{y}^{t}=B_{1}^{t y} y\left(s^{*}\right) \tag{4}
\end{equation*}
$$

and hence $B_{1}^{t x}$ and $B_{1}^{t y}$ can be found with one single orbit or one average orbit. This allows to use average orbits made out hundred of turns. In practice, the average of trajectories with a maximum at IR5 and trajectories with a minimum at IR5 are obtained for each set of TBT data. Therefore, for the experiment under analysis, there will be six average orbits which lead to the six points seen in Fig. 4. The slope of these plots give the approximate values of $B_{1}^{t x}$ and $B_{1}^{t y}$ which lead to the values of $\Delta k(Q 1)$ and $\Delta k(Q 2)$ written in the third row of Table 1. These values are very consistent with the values obtained before and the uncertainty has dropped significantly thank to the bigger number of turns used to build the average orbits.

Comparing Table 1 with the values obtained using the SBST, there is still notable differences that cannot be explained by the uncertainties alone. Searching for possible systematic deviations, several simulations have been done using MADX but to this date the errors introduced in the mad files agree well with the values obtained with software for action and phase analysis (with and without noise).

## CONCLUSIONS

The action and phase jump method have been applied to estimate linear corrections at one LHC IR using turn by turn data. Two different approaches have been used, one of them considering coupling and the other without coupling but with much better statistics. The two approaches agree well and the suppression of noise in the second case is excellent. The comparison with the SBST might point to a systematic problem which needs further investigation.

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