# **EMITTANCE MINIMIZATION BY COURANT-SNYDER PARAMETER** SCAN IN MERGER SECTION AT THE COMPACT ENERGY RECOVERY LINEAR ACCELERATOR

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#### Abstract

The project of compact-Energy Recovery Linac(c-ERL) at Photon Factory in KEK is a test facility for the 5 GeV ERL, which is one of the candidates of next generation light source. It consists of injector system, merger section, main SRF section, return arc, long straight section and beam dump. The injector system produces beams with a low-energy of 5 MeV and low-emittance less than 1 mmmrad. It causes the large emittance growth by space charge force in merger section, which consists of two rectangular type dipole magnets and one sector type magnet. Dispersion also causes the displacement of bunch sllice on horizontal plane. The displacement of bunch slice is laid on the kick angle induced by space charge force. Also, each slice has the orientation of the phase ellipse on horizontal phase space. Therefore, the emittance growth due to the displacement of bunch sllice induced by space charge force in the horizontal phase space can be minimized by matching the displacement to the orientation of the phase ellipse at the exit of merger. We present the results of the emittance minimization performed by mathcing of the angle of the phase ellipse by scan of CS (Courant-Snyder) parameter.

### **INTRODUCTION**

Many users require a fourth generation light source that can produce high charge, short pulse, low transverse emittance and a high peak current electron beams[1]. The Energy Recovery Linac is one of the candidates for the fourth generation light sources that can meet these requirements. The compact-ERL at KEK, in the final stage, will provide a beam energy of around 125 MeV and a bunch charge of 77 pC, which is a prototype for the future 5 GeV ERL at KEK. The layout of the compact-ERL is shown in Fig 1. The c-ERL consists of an injector system, a merger section, a superconducting RF (SRF) section, two return loops and two straight sections[2]. In the early comissioning phase, the injector produces electron beams with a bunch charge of 77 pC, beam energy of 5 MeV and bunch length of 0.6 mm rms. The beam energy is increased by 30 MeV with two 9 cell SRF cavities. Since the beam energy in c-ERL is a low, we need to consider the several effects, e.g., the space charge effect, the coherent synchrotron radiation (CSR) effect, the wake function, ion effects and beam break up[3].

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The energy spread induced in an achromatic cell results in the growth of projection emittance at the exit of the achromatic cell. It is known that this emittance growth can be compensated by setting the cell-to-cell betatron phase advance at an appropriate value[4].



Figure 1: Layout of a compact-ERL.

## ANALYTICAL CALCULATION TO MINIMIZE THE EMITTANCE GROWTH

A merger section with 3-dipole was adopted for the flexible beam transport of the high energy circulating beam. The layout of the 3-dipoles merger is shown in Fig. 2.



Figure 2: Layout of a merger section.

As shown in Fig. 2, the bending magnet at the center of the merger is sector type and bending magnets at the entrance and exit of the merger are rectangular type. The center bending magnet have an edge angle to achieve zero dispersion at the exit of the merger section. Firstly, the analytical calculation of the emittance growth in the merger section was performed by using the first-order theory[5]. The longitudinal space charge force (LSCF) is the main source of the emittance growth in the low-energy beam. In the analytical calculation, we assume that the longitudinal and transverse bunch lengths and sizes are not largely changed in the merger section. The energy spread of the beam due to the SC force was induced in the upstream part of the merger section. The slice of the beam in horizon-

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tal phase space has a different position in the downstream part of the merger section, due to the energy spread. The projected emittance grew due to the SC effect, shown in Fig. 3.



Figure 3: Growth of projected emittance due to the SC effect in merger section. (a) Maximum case of projected emittance growth (b) Mimimum case of projected emittance growth

The emittance growth due to the displacement of the bunch slices in phase space can be minimized by matching the displacement to the orientation of the phase ellipse at the exit of the merger. The first-order theory was used to calculate the space charge kick angle, because the displacement of the bunch slice is laid on the  $\zeta_x x' - \zeta'_x x = 0$ , where  $\zeta$  is the space charge dispersion,  $\zeta'$  is its derivative, x is the horizontal position and x' is its derivative. Therefore, the angle of the displacements due to the SC effect is given by

$$\phi_{\zeta} = \tan^{-1}(\zeta_x'/\zeta_x) \tag{1}$$

The analytical calculation of the space charge dispersion requires the space charge kick angle. The result of the space charge dispersion in the merger section is shown in Fig. 4.



Figure 4: Space charge dispersion at the merger section.

In the analytical calculation, by using the first-order theory, the transfer matrix for each element is derived by Green's function method[6]. At the exit of the merger section, the space charge dispersion,  $\zeta_x$ , and derivative of the space charge dispersion,  $\zeta_{x'}$ , are -0.163 m<sup>2</sup> and 0.319 m, respectively. From the Eq. 1, the space charge kick angle,  $\phi_s$ , in horizontal phase space was calculated to be -1.09 rad which is based on the first-order theory. Using the above calculation parameters, the LSCF wake potential was given by  $\Delta W_s$ =5.62 [keV/m]. Also,  $\kappa_s$  is the normalized space charge wake potential in the bending path and it is dependent on the space charge wake potential

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W[eV/m], and the reference energy  $E_0[\text{eV}]$  at  $s = s_0$ , which is the entrance of the bending magnet. It is given by  $\Delta \kappa_s = \Delta W_s / E_0 = 1.23 \times 10^{-3} [1/m]$ . When the merger section is optimized by matching the envelope between the LSCF-induced dispersion function and the betatron function at the exit of the merger section, all the bunch slices align along the orientation of the phase ellipse and have a distribution of the displacement ( $\Delta \kappa_s \zeta$ ,  $\Delta \kappa_s \zeta'$ )=(-0.200 mm, 0.392 mrad). Based on the first-order theory, the transverse emittance growth in the merger section is given by

$$\varepsilon^2 = (\varepsilon_0 \beta_x + D^2)(\varepsilon_0 \gamma_x + D'^2) - (\varepsilon_0 \alpha_x - DD')^2 \quad (2)$$

,where  $\varepsilon_0$  and  $\varepsilon$  are the initial and final emittance as un-normalized values, respectively, and  $(D,D')=(\Delta\kappa_s\zeta,\Delta\kappa_s\zeta')$  is the rms spread of the bunch slice displacement in (x,x') phase space. From Eq. 2, the result of analytical calculation of the emittance growth in the merger section is shown in Fig. 5.



Figure 5: Analytical calculation results of emittance as function of twiss parameter at the entrance of merger

## NUMERICAL CALCULATION BY USING GPT CODE INCLUDED 3D SC EFFECT

The numerical calculation is done by using the code of General Particle Tracer (GPT), which includes the calculation of 3-dimensional SC force with actual electric and magnetic fields [7]. In the particle tracking simulation by using GPT code, the initial normalized transverse emittance is 0.1 mm·mrad, the bunch length is 3 ps (rms), the beam energy is 5 MeV and the the bunch included the particle distribution of 10000 macroparticles. The bunch distribuition was assumed to be beer-can shape, which has same vertical and horizontal emittances. The transverse emittance growth was scanned by the initial and final CS parameter at the entrance and exit of merger section. In the tracking simulation, the  $\beta_{xi}$  and  $\alpha_{xi}$  were varied from 0.5 m to 10 m by 0.5 m step and from -4 to 4 by 0.2 step, respectively. Fig. 6 show transeverse emittance as a function of CS parameter at the entrance of the merger section.

As shown by Fig. 6, amount of the growth of the transverse emittance due to the SC effect is around 1.09 mm-

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Figure 6: Transverse emittance at the exit of the merger as function of the initial  $\alpha_{xi}$  and  $\beta_{xi}$ .

mrad. Also, the growth of the transverse emittance has minmum value at the minus value of the  $\alpha_{xi}$  with small value of  $\beta_{xi}$ . The initial CS parameter is  $\alpha_x = -1.6$ ,  $\beta_x$ = 5.5 m when growth of the emittance is minimized. The growth of the transverse emittance as function of the orientation of the phase ellipse in the (x, x') phase space, which calculated by using the results of the numerical calculation as shown by Fig. 6, is shown in Fig. 7.



Figure 7: Transverse emittance at the exit of merger section as a function the orientation of the phase ellipse.

From the numerical calculation result, the emittance growth induced by the LSCF in the merger section was compensated by changing the orientation angle of the 🖞 ellipse in horizontal phase space. We found a minigrowth of 1.09 mm mrad in the merger section when the vertical CS parameter was fixed to  $\beta_y = 9$  m, with  $\alpha_y = 0$ . Also, the effect of coupled motion of the beam was investigated to minimize the emittance growth in the merger section. The vertical CS parameter was changed to investigate the effect of coupled motion. In the calculation, the horizontal CS parameter was fixed to  $\beta_x = 5.5$  m with  $\alpha_x = -1.6$  and shows a minimum emittance growth. The result is shown in Fig. 8.

#### **SUMMARY**

The study for the compensation of emittance growth on the low energy around 5 MeV in the merger section was



Figure 8: The change of transverse emittance at the exit of merger section as function the orientation of the phase ellipse due to the change of vertical CS parameters.

performed to minimize the growth of the emittance in a beam with 77 pC of bunch charge, a bunch length of 3 ps and a bunch energy of 5 MeV. Based on the first-order theory, the emittance growth due to the displacement of bunch slices in phase space was minimized by matching the orientation of the phase ellipse to the kick angle induced by the SC force at the exit of the merger. As shown in Fig. 8, the minimum horizontal emittance growth at the exit of the merger becomes 0.773 mm·mrad at the  $\beta_y = 0.5$  m and  $\alpha_{u}$ =-6. From this result, the orientation of the phase ellipse was calculated to -0.541 rad. It is shows that the smaller angle of the phase ellipse which is calculated by the analytical model, -1.09 rad, gives the smaller growth of the emittance. To achieve the small angle of the phase ellipse, which is around -1.09 rad, it needs the small  $\beta_{u}$  with large  $\alpha_{y}$ . It causes the growth of the vertical betatron function after passing the merger section. Therefore, the optimumu of  $\beta_x$ ,  $\alpha_x$ ,  $\beta_y$  and  $\alpha_y$  were determined to 5.5 m, -1.6, 0.5 and -6 m, respectively.

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