# AN INNOVATIVE LATTICE DESIGN FOR A COMPACT STORAGE RING

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### Abstract

We propose a new concept of lattice design for a compact storage ring. The new lattice can be realized by placing bending magnets, quadrupole magnets, a RF cavity, and other necessary accelerator components on a projected torus knot in the horizontal orbit plane. In a ring with this new scheme, the electron beam may have extremely longer design orbit than that of a conventional ring because a design orbit closes after completing multiple turns.

#### **INTRODUCTION**

The Hiroshima Synchrotron Radiation Center has a plan to construct a third generation compact storage ring, called HiSOR-II, in order to serve brilliant VUV and soft X-ray radiation for synchrotron radiation users in the field of materials science, solid state physics, bio-molecular science, etc. Due to the requirement for the photon energy ranges from user community and the space limitation of construction area on the campus of Hiroshima University, the aimed ring energy should be 0.7 GeV and the ring circumference should be smaller than 50 m.

Figure 1 shows the original HiSOR-II lattice that we have been considering for several years [1]. This 4-cell double-bend lattice was designed based on the design of MAX-III [2].



Figure 1: Original design of conventional HiSOR-II ring.

As shown in the figure, it has eight combined function bending magnets, and accommodate with four 3.4-m straight and four 2.0-m straight sections. The circumference of this ring is about 40 m, and designed natural emittance is as small as 13.6 nm-rad. It is believed that a machine with these specifications can provide one of the World best performances as the VUV radiation source.

However, in general, a small storage ring has some limitations which do not exist in a large or a medium size ring. One of such limitations is that a number of straight sections and a length of each straight section are limited

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due to a small circumference. Also, a small circumference means a short length of closed orbit which lead to a short bunch-to-bunch interval even if the machine is operated in a single-bunch mode. For example, a bunch of electron beam circulates in a 50-m circumference ring at the frequency of 6 MHz, which means the pulse to pulse interval of synchrotron radiation (SR) is only 167 ns. Such a short pulse interval restricts to perform some of advanced SR experiments including ARTOF photoelectron spectroscopy [3].

In this paper, we see how these limitations can be eased by introducing a totally new concept in designing accelerator lattice for a small storage ring.

## **MÖBIUS STRIP AND TORUS KNOT**

It is well known that a Möbius strip is a one-sided nonorientable surface. It can be made from a band or a single strip by connecting both ends, so that one of the ends is half-twisted each other as shown in Figure 2.



Figure 2: An example of Möbius strip.

As one can easily see by drawing a line on the surface of Möbius strip, the line closes after completing two turns  $(4\pi)$  around the ring. Analogically, if the top and bottom surfaces of a tall triangular prism are twisted  $2\pi/3$  angle and then connected, the ridgeline closes after three turns  $(6\pi)$  [4]. These mathematical features are extended, generalized, and categorized as the group of torus knots [5]. Figure 3 shows some examples of torus knots.



Figure 3: Examples of torus knots.

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The mathematical notations of these examples are: from top left to bottom right as (5,2), (7,2), (11,2), (5,3), (7,3), (11,3), respectively. Parameters p and q in the bracket (p,q) stand for the number of crossing of longitude and the number of crossing of meridian of the torus, respectively. Parameters p and q should be co-prime each other.

Geometrically in 3D Euclid space, a torus knot is defined by using the cylindrical coordinate and the parameter t ( $0 \le t \le 1$ ):

$$r = R + a\cos 2\pi p t$$

$$\theta = 2\pi qt$$

$$z = a \sin 2\pi p t$$

where, R is the radius from the center of the hole to the center of the torus tube and a is the radius of the tube, respectively..

### APPLICATIN TO ACCELERATOR LATTICE

In general, the particle beam orbit of a storage ring lies on the horizontal plane. Therefore, in order to generate an accelerator lattice starting from one of the torus knot, it is a good idea to project a torus knot onto a flat plane. Figure 4 shows projected (5,2) and (10,3) torus knots and bending magnet positioning in the corresponding lattices.



Figure 4: Examples of projected torus knots and bending magnet configurations of corresponding lattices.

As it is obvious form this figure, the basic structure of accelerator lattice can be made by placing a bending magnet at each crossing point of projected (5,2) torus knot, and at each outer crossing point of projected (10,3) torus knot. Although particle beam trajectory crosses in every bending magnet, the bending radius of either trajectory is the same. The orbit entered from inboard exit outboard and vice versa.

Whatever choosing any torus knot as the starting point, one can complete a required lattice by adding necessary

components after the position of each bending magnet is determined. As one can recognize by tracing the beam orbit, the unit cell of this type of lattice consists of two bending magnets, i.e. the double-bend type lattice. The numbers of unit cells are five for geometrically 5-fold symmetry lattice, and ten for the 10-fold symmetry lattice, respectively.

Figure 5 shows the (11,3) torus knot lattice and a onecell configuration of magnets. This lattice may be used for the HiSOR-II storage ring [6, 7].



Figure 5: An example of (11,3) torus knot lattice for a compact light source and a unit cell.

This compact 11-fold symmetry light source ring has an outer perimeter of 46 m. The length of electron beam orbit is about 130 m. The length between crossing points of inner straight section is 3.6 m. The length of outer straight section is 1.8 m where a RF cavity and other necessary accelerator components may be installed.

An expected layout of HiSOR-II light source accelerator complex is shown in Figure 6.



Figure 6: Birdseye view of HiSOR-II light source accelerator complex.

As explained for the (10,3) torus knot lattice in this section previously, the (11,3) torus knot lattice also has

eleven double-bend unit cells though the beam passes twice in each bending magnet for completing the closed orbit in the real space.

### FURTHER EXPANSION OF TORUS KNOT LATTICE SCHEME

The scheme of projected torus knot lattice can be extended toward various directions. One of such extensions is to introduce long straight sections by breaking the lattice symmetry as has been done on many conventional storage rings for synchrotron light sources and colliders. An example of extended torus knot lattice with long straight sections is shown in Figure 7.



Figure 7: Schematic view of extended torus knot lattice with long straight sections: modified form (11,3) T-K lattice.

This modification may introduce additional advantages for utilizing straight sections. For example, a long straight section can accommodate with linear acceleration tubes for an Energy Recovery Linac (ERL). Also, very long insertion devices can be installed.

The possible other direction of expansion of T-K lattice concept is to elongate the closed orbit as much as possible by increasing the number of turns, i.e., using a large integer for q. By this modification, the amount of stored charge in the ring can be dramatically increased.

### **DISCUSSION AND SUMMARY**

We found a new concept for designing a lattice of ring accelerator. In a conventional ring accelerator, the circumference of the ring and the length of closed orbit of a charged particle beam is the same. On the other hand, the orbit length is elongated multiple times in the ring having a projected torus knot lattice. This lead to the multiple increment of stored charge when the multi-bunch operation mode is chosen. Also, when the single-bunch mode operation is selected, bunch-to-bunch interval time becomes multiple longer. If this type of lattice is adopted for a light source ring, many insertion devices can be installed for the use of synchrotron radiation and some new experimental techniques become available which could not be used in a small storage ring.

The ring with this new lattice has another advantage when it is used as an optical-cavity type FEL as well. The beam instability may be suppressed because the ratio of spontaneous radiation by the FEL radiation is much larger than that of conventional one-turn ring [8]. Furthermore, the ring FEL with this lattice may be used as a coherent  $\gamma$ -ray source because the FEL radiation and an electron bunch may be able to collide at the position of cross-chamber.

This new scheme may be applicable not only for light source rings but also for compact proton and/or heavy ion synchrotrons of urban use including medical and industrial applications.

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