# SOME PRELIMINARY EXPERIMENTS USING LIBERA BPMs IN BEPCII* 

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## Abstract

There are total 16 LIBERA BPMs in BEPCII, which is a double ring e+e- collider. The turn-by-turn BPMs serve as only tune measurement system in most cases during normal operation. We tried to do some more machine study using them: the local coupling parameter at the BPM, the resonance driving term. We also compare the difference from the different exciting method: single time kick with injection kicker or continuous sinusoidal kick with feedback system.

## INTRODUCTION

In the storage rings of BEPCII, a total of 16 units of LIBERA Electron BPM are installed. The LIBERA BPM gave us great help to store beam in the very first beginning of BEPCII. The LIBERA also help us reduce the residual orbit oscillation during injection. The TBT(turn-by-turn) mode has already become a powerful tool for testing machine ${ }_{2}$ characteristics such as damping time, tune measurements and other beam behavior.

We would present some preliminary experiments result on beam dynamics study using LIBERA: 1) The local cou© pling parameters measurement using Bagley \& Rubin's [1] and Ohnish's [2] method respectively. The result obtained by different methods is compared. The beam is excited by injection kicker or continuous sinusoidal kick with transverse feedback system. 2) It is tested if the resonance driving terms could be measured in our machine.

## LOCAL COUPLING PARAMETERS

## Local Coupling Representation

We follow the parametrization of coupling in [3]. The one-turn transfer matrix of 4D phase space vector $\mathbf{x}=$ $\left(x, x^{\prime}, y, y^{\prime}\right)$ is represented by $\mathbf{T}$, written in terms of $2 \times 2$ submatrices,

$$
\mathbf{T}=\left(\begin{array}{cc}
\mathbf{M} & \mathbf{m}  \tag{1}\\
\mathbf{n} & \mathbf{N}
\end{array}\right)=\mathbf{V U V} \mathbf{V}^{-1}
$$

where

$$
\mathbf{U}=\left(\begin{array}{cc}
\mathbf{A} & \mathbf{0}  \tag{2}\\
\mathbf{0} & \mathbf{B}
\end{array}\right) ; \mathbf{V}=\left(\begin{array}{cc}
\gamma \mathbf{I} & \mathbf{C} \\
-\mathbf{C}^{+} & \gamma \mathbf{I}
\end{array}\right) ; \gamma^{2}+|\mathbf{C}|=1
$$

[^0]A is a $2 \times 2$ normal mode transfer matrix, which can be represented by Courant-Snyder parameters,

$$
\mathbf{A}=\left(\begin{array}{cc}
\cos 2 \pi \nu_{A}+\alpha_{A} \sin 2 \pi \nu_{A} & \beta_{A} \sin 2 \pi \nu_{A}  \tag{3}\\
-\gamma_{A} \sin 2 \pi \nu_{A} & \cos 2 \pi \nu_{a}-\alpha_{A} \sin 2 \pi \nu_{A}
\end{array}\right)
$$

It is similar for $\mathbf{B}$. The symplectic conjugate is

$$
\mathbf{C}^{+}=\left(\begin{array}{cc}
C_{22} & -C_{12}  \tag{4}\\
-C_{21} & C_{11}
\end{array}\right)
$$

In a weak coupling machine, mode $A / B$ is similar to horizontal/vertical oscillation respectively. $\nu_{A} / \nu_{B}$ is the socalled horizontal/vertical tune. The one turn coupled transfer matrix can be rewritten in the following form

$$
\begin{equation*}
\mathbf{T}=\mathbf{G}^{-1} \overline{\mathbf{V}} \overline{\mathbf{U}} \overline{\mathbf{V}}^{-1} \mathbf{G} \tag{5}
\end{equation*}
$$

$\mathbf{G}$ is the Courant-Snyder transformation matrix,

$$
\begin{gather*}
\mathbf{G}=\left(\begin{array}{cc}
\mathbf{G}_{A} & \mathbf{0} \\
\mathbf{0} & \mathbf{G}_{B}
\end{array}\right) ; \mathbf{G}_{u}=\left(\begin{array}{cc}
1 / \sqrt{\beta_{u}} & 0 \\
\alpha_{u} / \sqrt{\beta_{u}} & \sqrt{\beta_{u}}
\end{array}\right)  \tag{6}\\
\begin{aligned}
& \overline{\mathbf{V}}=\mathbf{G V G}^{-\mathbf{1}}=\left(\begin{array}{cc}
\gamma \mathbf{I} & \mathbf{G}_{\mathbf{A}} \mathbf{C G}_{\mathbf{B}}^{-\mathbf{1}} \\
-\mathbf{G}_{\mathbf{B}} \mathbf{C}^{+} \mathbf{G}_{\mathbf{A}}^{-\mathbf{1}} & \gamma \mathbf{I}
\end{array}\right) \\
&=\left(\begin{array}{cc}
\gamma \mathbf{I} & \overline{\mathbf{C}} \\
-\overline{\mathbf{C}}^{+} & \gamma \mathbf{I}
\end{array}\right) \\
& \overline{\mathbf{U}}=\mathbf{G U G G}^{-\mathbf{1}}=\left(\begin{array}{cc}
\mathbf{R}\left(2 \pi \nu_{A}\right) & \mathbf{0} \\
0 & \mathbf{R}\left(2 \pi \nu_{B}\right)
\end{array}\right)
\end{aligned}
\end{gather*}
$$

where $\mathbf{R}$ is the rotation matrix.

## Measurement Method

The $x / y$ turn-by-turn oscilation at BPM could be recorded using LIBERA. If only the A mode is exicited, we could get [1]

$$
\begin{equation*}
\bar{C}_{12}=\gamma \sqrt{\frac{\beta_{A}}{\beta_{B}}}\left(\frac{y}{x}\right)_{A} \sin \Delta \phi_{A} ; \Delta \phi_{A}=\left(\phi_{y}-\phi_{x}\right)_{A} \tag{9}
\end{equation*}
$$

where $(y / x)_{A}$ is the ratio of the y amplitude to the x amplitude for the A mode and $\left(\phi_{y}-\phi_{x}\right)_{A}$ is the phase difference between the two motions for the A mode.

If neighbour bpms are equipped with LIBERA, we could construct the turn-by-turn coordinate in 4D phase space at some observation point using the theory model parameters and least square method [4]. When only the horizontal 06 Beam Instrumentation and Feedback
mode is excited, we could obtain all of the 4 local coupling parameters with [2]

$$
\left(\begin{array}{cc}
-C_{22} & C_{21}  \tag{10}\\
C_{12} & -C_{11}
\end{array}\right)=\left(\begin{array}{cc}
C_{x}^{A} & C_{x^{\prime}}^{A} \\
S_{x}^{A} & S_{x^{\prime}}^{A}
\end{array}\right)^{-1}\left(\begin{array}{cc}
C_{y}^{A} & C_{y^{\prime}}^{A} \\
S_{y}^{A} & S_{y^{\prime}}^{A}
\end{array}\right)
$$

where

$$
\begin{align*}
C_{u}^{A} & =\sum_{n} u(n) * \cos \left(2 \pi \nu_{A} n\right) \\
S_{u}^{A} & =\sum_{n} u(n) * \sin \left(2 \pi \nu_{A} n\right) \tag{11}
\end{align*}
$$

$n$ is the number of turn, and $u$ represents $x, x^{\prime}, y$, or $y^{\prime}$. It could be derived directly that

$$
\begin{equation*}
\bar{C}_{12}=C_{12} / \sqrt{\beta_{A} \beta_{B}} \tag{12}
\end{equation*}
$$

we could compare the result obtained by the two methods.

## Simulation

We use MADX to track a single particle where the strengh of skew quadrupole is non-zero, and use Ohnishi's method to obtain the coupling parameters. The result is shown in Fig. 1, where the model parameter calculated with MADX is also shown.


Figure 1: The local coupling parameters obtained by analyzing tracking data. Ohnishi's method is used. The result concides well with that calculated by MADX.

## Experiment

Only two LIBERAs are used in our experiment. Fig. 2 shows the measurement result with the single time kick. $C_{12}$ obtained by two analysis methods concides well.

The horizontal tune is 0.5374 , and we scan the exciting frequency near the tune. Fig. 3 shows $C_{12}$ versus the exciting frequency. In order to ensure the accuracy of measurement, the deviation of exciting frequency from the tune should be less than 0.0002 . Fig. 4 shows that the measurement agrees well between two exciting methods.

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Figure 2: The four local coupling parameters measured by Ohnishi's method with single time kick. $C_{12}$ concides well with that obtained by Bagley-Rubin's method.


Figure 3: $C_{12}$ versus exciting frequency.

## RESONANCE DRIVING TERMS

## Theory

According to the first order perturbative theory of nonlinear betatron motion, the beam oscillation in the horizon$\mathrm{tal} /$ vertical plane can be written in normalized coordinates, as [5]

$$
\begin{align*}
& \hat{x}-i \hat{p}_{x}=\sqrt{2 A_{x}} e^{i \phi_{x}}- \\
& \quad \sum_{a b c d} 2 i a f_{a b c d}^{(3)}\left(2 A_{x}\right)^{\frac{a+b-1}{2}}\left(2 A_{y}\right)^{\frac{c+d}{2}} e^{i(b-a+1) \phi_{x}} e^{i(d-c) \phi_{y}} \\
& \hat{y}-i \hat{p}_{y}=\sqrt{2 A_{y}} e^{i \phi_{y}}- \\
& \quad \sum_{a b c d} 2 i c f_{a b c d}^{(3)}\left(2 A_{x}\right)^{\frac{a+b}{2}}\left(2 A_{y}\right)^{\frac{c+d-1}{2}} e^{i(b-a) \phi_{x}} e^{i(d-c+1) \phi_{y}} \tag{13}
\end{align*}
$$

Figure 4: Measurement result at R3OBPM05 with two exciting methods.
where $A_{u}\left(u\right.$ is $x$ or $y$ ) is action, $\phi_{u}=m \mu_{u}+\phi_{u, 0}$ is the phase angle of fundamental frequency $\left(\mu_{u}=2 \pi \nu_{u}\right)$, $m$ is the turn number, and $f_{a b c d}^{(3)}$ is the generating function. With the turn-by-turn position data, we could obtain the resonance driving terms value by frequency decomposition using an enhanced FFT algorithm [6]. The calculation method is shown in Table 1, where $X / Y$ represents the am-

Table 1: Calculation of RDT

| Spectral Line | RDT Value |
| :--- | :---: |
| $2 Q_{x}$ | $\frac{X_{2 Q_{x}}}{X_{Q_{x}}^{2}} e^{i\left(\Phi_{2 Q_{x}}-2 \Phi_{Q_{x}}\right)}$ |
| $Q_{x}-Q_{y}$ | $\frac{Y_{Q_{x}}-Q_{y}}{X_{Q_{x}} Y_{Q_{y}}} e^{i\left(\Phi_{Q_{x}-Q_{y}}-\Phi_{Q_{x}}+\Phi_{Q_{y}}\right)}$ |
| $Q_{x}+Q_{y}$ | $\frac{Y_{Q_{x}+Q_{y}}}{X_{Q_{x}} Y_{Q_{y}}} e^{i\left(\Phi_{Q_{x}+Q_{y}}-\Phi_{Q_{x}}-\Phi_{Q_{y}}\right)}$ |
| $2 Q_{y}$ | $\frac{X_{2 Q_{y}}}{Y_{Q_{y}}^{2}} e^{i\left(\Phi_{2 Q_{y}}-2 \Phi_{Q_{y}}\right)}$ |

plitude of the specific frequency in the $x / y$ direction, and $\Phi$ represents the corresponding phase angle. Only the composition value instead of all the generatin function at BPM is obtained. The so-called RDT value $(\mathcal{R})$ in this paper can be represented by the generating functions,

$$
\begin{align*}
\mathcal{R}_{2 Q_{x}} & =\left[\left|f_{1200}\right| e^{i\left(\phi_{1200}-\frac{\pi}{2}\right)}+3\left|f_{3000}\right| e^{i\left(\frac{\pi}{2}-\phi_{3000}\right)}\right] \frac{4}{\sqrt{\beta_{x}}} \\
\mathcal{R}_{2 Q_{y}} & =\left[\left|f_{1020}\right| e^{i\left(\frac{\pi}{2}-\phi_{1020}\right)}+\left|f_{1002}\right| e^{i\left(\phi_{1002}-\frac{\pi}{2}\right)}\right] \frac{4 \sqrt{\beta_{x}}}{\beta_{y}} \\
\mathcal{R}_{Q_{x}-Q_{y}} & =\left[\left|f_{1011}\right| e^{i\left(\frac{\pi}{2}-\phi_{1011}\right)}+2\left|f_{0120}\right| e^{i\left(\phi_{0120}-\frac{\pi}{2}\right.}\right] \frac{4}{\sqrt{\beta_{x}}} \\
\mathcal{R}_{Q_{x}+Q_{y}} & =\left[2\left|f_{1020}\right| e^{i\left(\frac{\pi}{2}-\phi_{1020}\right)}+\left|f_{0111}\right| e^{i\left(\phi_{0111}-\frac{\pi}{2}\right)}\right] \frac{4}{\sqrt{\beta_{x}}} \tag{14}
\end{align*}
$$

## Simulation

We use MADX to track a single particle, and use the turn-by-turn data to analyze the RDT value. The result is shown in Fig. 5, where the model value calculated with generating function is also listed and they concides well.


Figure 5: RDT value analyzed using tracking data and caluclated using generating function.

## Experiment

The beam is excited using the feedback kicker in the horizontal/vertical direction with the correspoding tune frequency. We failed to measure the RDT value in our experiment. It seems that the result does not repeat in mulitple measurements. The big error comes from a few facts. The horizontal beam size in our machine is in the order of 1 mm , and the vertical is in the order of 0.1 mm , which is much larger that that in a modern synchrotron light source, and it is harder to measure the small amplitude oscillation of high order spectral. The generating function along the ring is not large enough to measure the RDT value. The resolution of the LIBERA is in the order of $10 \mu \mathrm{~m}$. We calculate the amplitude of the fudamental frequency when the correponding high order harmnoic's amplitude is $20 \mu \mathrm{~m}$. The result is shown in Table 2. It is unfortunate that the mea-

Table 2: Minimum Required Oscilation Amplitude [mm]

| Spectral Line | R3OBPM05 | R3OBPM06 |
| :--- | :---: | :---: |
| $2 Q_{x}$ | $X_{Q_{X}}=1.6$ | $X_{Q_{x}}=1.7$ |
| $Q_{x}-Q_{y}$ | $\sqrt{X_{Q_{x}} Y_{Q_{y}}}=1.3$ | $\sqrt{X_{Q_{x}} Y_{Q_{y}}}=5.0$ |
| $Q_{x}+Q_{y}$ | $\sqrt{X_{Q_{x}} Y_{Q_{y}}}=1.4$ | $\sqrt{X_{Q_{x}} Y_{Q_{y}}}=1.5$ |
| $2 Q_{y}$ | $Y_{Q_{y}}=4.2$ | $Y_{Q_{y}}=1.4$ |

sured oscillation amplitude is smaller since exiting strength is not strong enough in our experiment. Since the beam lifetime become very bad when we try to increase the exciting strength, it seems that the coefficient of LIBERA is not well calibrated or the orbit of beam is outside the linear region of BPM.

## SUMMARY

We check the feasibility of local coupling parameters measurement using LIBERA in our machine. We also check the coincidence between different beam exciting methods. The deviation of exciting frequency from the transverse tune should be less than 0.0002 .

It is hard to measure the RDT value using LIBERA in our machine. The error comes from the big beam size, small generating function and the error of the diagnostics systerm.

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