# TOUSCHEK LIFETIME AND MOMENTUM ACCEPTANCE MEASUREMENTS FOR ESRF

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#### Abstract

The Touschek lifetime of a synchrotron results from electrons scattering off one another within the bunch and subsequently being lost. We have measured the Touschek lifetime for the major operating modes of the ESRF as a function of RF voltages. This includes multibunch and few bunch filling patterns with correspondingly different chromaticity values. Through calibration of the RF voltage and measurement of the other beam parameters such as bunch length and vertical emittance, we may understand the momentum acceptance in the regime where this is determined by non-linear dynamics effects.

#### **OVERVIEW**

The electron beam lifetime is one of the most important parameters for a synchrotron radiation facility, being directly proportional to the corresponding lifetime of radiation intensity. It is defined as the relative loss rate of the current and is readily measurable by finding the time dependence of the total stored current. The loss of electrons is due to two different effects, and the lifetime may correspondingly be split into two terms:

$$\frac{1}{\tau} = \frac{1}{I}\frac{dI}{dt} = \frac{1}{\tau_V} + \frac{1}{\tau_T} \tag{1}$$

The first represents the vacuum lifetime and the second represents the Touschek lifetime which typically dominates the lifetime at the ESRF, due to the ultra high vacuum conditions.

Consider figure 1 which shows a plot of Touschek lifetime vs. RF voltage. In region I, the lifetime depends on the RF acceptance, which is determined by the RF voltage. We plot the RF acceptance vs. RF voltage for the ESRF in Figure 3. In region II, the RF acceptance is large enough that electrons are lost due to exceeding the transverse dynamic aperture during their post-scattering trajectories. There is also an intermediate region where both effects come into play and may be used to estimate the dynamic momentum acceptance of the lattice.

The Touschek lifetime  $\tau_T$  is calculated as [1]

$$\tau_T^{-1} = \frac{r_e^2 I_b}{8\pi e \gamma^3 \sigma_s} \oint_C \frac{F\left(\left[\frac{\delta_{\rm acc}(s)}{\gamma \sigma_{x'}(s)}\right]^2\right)}{\sigma_x(s)\sigma_{x'}(s)\sigma_z(s)\delta_{\rm acc}^2} \,\mathrm{d}s,\qquad(2)$$

Here,  $r_e$  denotes the classical electron radius,  $\sigma_s$  the bunch length,  $I_b$  is the bunch current.  $\sigma_x(s)$  and  $\sigma_z(s)$  the rms horizontal and vertical beam sizes. c is the vacuum velocity of light, e the electron charge, and  $\gamma$  the relativistic Lorentz







Figure 1: Lifetime vs. RF voltage curve. In region I, the momentum acceptance is set by the RF acceptance. In region II, the momentum acceptance is the dynamic acceptance, set by the stability region, and dependant on sextupole settings. The region in between is a transition where both effects play a role in stability.

factor of the beam. The function F(x) is defined as

$$F(x) = \int_0^1 \left(\frac{1}{u} - \frac{1}{2}\ln\frac{1}{u} - 1\right) \cdot \exp\left(-\frac{x}{u}\right) \, \mathrm{d}u.$$
 (3)

From this formula, one finds that for a constant momentum acceptance, the Touschek lifetime scales as

$$\tau_T \sim \frac{\sigma_s \sqrt{\epsilon_z} \delta^a_{acc}}{I_b} \tag{4}$$

For the ESRF lattice, one finds that *a* is close to 3. To be more precise, the momentum dependence will have an *s* dependence, i.e. it changes around the ring. How then is one to interpret the value of momentum acceptance one finds from a curve such as Figure 1? A careful analysis would require a combination of particle tracking to find the expected momentum acceptance, together with detailed curve fitting.<sup>1</sup> The s dependence of the momentum acceptance typically roughly follows the dispersion invariant  $\mathcal{H}_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta {\eta_x'}^2$  constructed from the Courant Snyder paramaters and the dispersion. We plot  $\mathcal{H}_x$  for ESRF in figure 2. Figure 2 also contains a plot of the local Touschek scattering rate. This is the scattering rate (the integrand of Eq. (2)) assuming a constant momentum acceptance plotted for one super period. One finds

<sup>&</sup>lt;sup>1</sup>See [2] and [3] for examples of such momentum aperture model based analysis.

that for ESRF, it is sharply peaked in the center of the lowbeta straight section.<sup>2</sup> Hence, the integral determining the overall Touschek scattering rate will get most of the contribution from this region. We may thus take the derived momentum acceptance to be roughly the value in this region.



Figure 2: Local Touschek scattering rate, and  $\mathcal{H}_x$  function for one superperiod of ESRF. Approximate locations of dipoles are indicated.



Figure 3: Momentum acceptance vs. RF voltage.

#### VACUUM LIFETIME

We measure the vacuum lifetime by blowing up the beam vertically (using a white noise shaker) and measuring the lifetime for each emittance value. At large emittances the Touschek lifetime is large (Eq. 4) and one expects the lifetime to be determined by the vacuum lifetime as given by Eq. 1. Figure 4 shows the results of such a measurement, in which a vacuum lifetime of 387 hrs was determined. The

vacuum lifetime depends on the total current. So in order that it is invariant with the measurements, we keep the total current fixed for the measurements.



Figure 4: Vacuum lifetime measurement yielding a value of 387 hours.

#### **RF VOLTAGE CALIBRATION**

An important aspect of these measurements and their interpretation is to be clear about the value of the RF voltage  $V_{RF}$  for a given readout value. For this purpose, one may measure the incoherent synchrotron frequency  $f_s$  which is related to the RF voltage  $V_{RF}$  by the expression

$$f_s = f_{RF} \sqrt{\frac{\alpha_c}{2\pi h}} \frac{\left(V_{RF}^2 - U_0^2\right)^{1/4}}{\sqrt{E_0}}$$
(5)

with  $f_{RF}$  the RF frequency,  $\alpha_c$  the momentum compaction factor, h the harmonic number,  $U_0$  the energy loss per turn and  $E_0$  the beam energy. We measured  $f_s$  by driving the beam with the longitudinal bunch by bunch feedback system, and locating a side-band of a higher revolution harmonic in order to avoid the reduction in  $f_s$  around the RF frequency due to the Robinson effect. We may assume that the actual RF voltage is given by a scale factor multiplied by the read-out value. i.e.  $V_{RF} = kV_{ro}$ . We have measured the synchrotron frequency vs. readout voltage in order to determine this scale factor.

Fitting the synchrotron tune to the measured values with the assumption of a linear scale factor requires knowledge of the parameter  $\alpha_c E_0^3$ .<sup>3</sup> It was found that the expected values of  $\alpha_c = 1.78 \times 10^{-4}$  and  $E_0 = 6.04 GeV$  did not produce an adequate fit, and required small adjustments of either of the two parameters. Depending on how these two parameters are chosen, one finds a value of k within the range of 0.96 to 0.91. However, some analysis of Touschek lifetime in Region I shows a value closer to 1. Thus, this calibration factor remains a question for further inquiry.

<sup>&</sup>lt;sup>2</sup>See [4] for the similar analysis applied to the NSLS-II.

<sup>&</sup>lt;sup>3</sup>To derive this, one also uses the fact that the energy losses  $U_0$  is proportional to the beam energy to the fourth power,  $E_0^4$ .

## BUNCH LENGTH AND VERTICAL EMITTANCE MEASUREMENTS

In order to fully measure the beam distribution parameters which appear in the Touschek formula, we also need the bunch length and the vertical emittance. The longitudinal bunch distribution has been measured with a streak camera and the vertical emittances have been measured by dipole radiation projection monitors and by pinhole cameras. We use the average value around the ring. See [5] for a recent discussion and analysis of vertical emittance measurement and reduction at ESRF.



Figure 5: Bunchlength vs.  $V_{RF}$  for bunch current I = 1 mA.

### LIFETIME VS. RF VOLTAGE RESULTS

Several measurement shifts were carried out to measure all the parameters for understanding the Touschek lifetime and momentum acceptance for the different filling patterns and machine settings. For the multi-bunch, 16 bunch, and 4 bunch, settings we have derived momentum acceptances from these measurements which are shown in table 2. In these measurements, a current of 2 mA was used with a 32 bunch filling pattern.

We also obtained data at a lower current of 1mA per bunch, and analyzed the region I. All beam parameters were measured at this time. We compare the calculated to measured values for a k value of 1 vs. 0.95. One finds that whereas a value of 1 fits the measured Touschek lifetime, a value of 0.95 is not consistent. The strong deviation results from setting the RF voltage to be close to the value of 4.88 MV where the losses from synchrotron radiation are just compensated for.

### CONCLUSION

We have measured the Touschek lifetime vs. RF voltage for the major operation modes at the ESRF. We find a momentum acceptance of 2.5-2.8% for the multi-bunch mode, 2.4-2.7% for the 16 bunch mode and 2.16-2.2% for the 4 bunch mode. We interpret this to be the momentum acceptance in the low-beta straight section where the majority of the Touschek scattering occurs. We have also measured a

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$V_{RF}$ (MV)	$I_b (mA)$	$\epsilon_z \ (\text{pm})$	$\sigma_s (\mathrm{mm})$	$ au_{meas}$	$\tau_{calc}$	
5.25	0.93	5.6	13.0	1.9 hr	1.6 hr	
5.5	0.95	5.7	11.6	4.15 hr	4.4 hr	

Table 1: Result of Touschek lifetime calculation in region I, using all measured beam parameters. A voltage calibration factor of 1 was used in this case. The horizontal emittance used in the calculation was 4 nm. Using a calibration factor of k = 0.95 drops the computed lifetimes to 0.1 hr for the 5.25 MV case and 1.15 hr for the 5.5 MV case.

sextupole setting	$\delta_{acc}$		
multi-bunch	2.5-2.8%		
16 bunch	2.4-2.7%		
4 bunch	2.16-2.2%		

Table 2: Momentum acceptance results using the following convention: we find  $V_{RF}$  with the maximum lifetime, and then compute the RF acceptance corresponding to this value. The range represents a range of voltage calibration factors between 0.91 and 0.96.

consistent set of parameters in the region in which the momentum acceptance is given by the RF acceptance. Further investigation includes comparison of these results to tracking studies, and resolution of the ambiguity regarding the RF voltage calibration factor.



Figure 6: Lifetime vs. RF acceptance measurements.

### REFERENCES

- H. Bruck, Accelerateurs Circulaires de Particules, Presses Universitaires de France (1966);
- [2] C. Steier et. al., "Measuring and optimizing the momentum aperture in a particle accelerator", Phys. Rev. E (65) 056506
- [3] M. Takao et. al., "Touschek lifetime and momentum aperture of the Spring-8 storage ring", Proceedings, EPAC '00.
- [4] B. Nash, S. Kramer, "Touschek Lifetime Calculations for NSLS-II", Proceedings, PAC '09.
- [5] A. Franchi, et. al., Phys. Rev. ST Accel. Beams 14, 034002 (2011)