LOCALIZATION OF TRANSVERSE IMPEDANCE SOURCES IN THE SPS USING HEADTAIL MACROPARTICLE SIMULATIONS

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Abstract

In particle accelerators, the beam coupling impedance is one of the main contributors to instability phenomena that lead to particle losses and beam quality deterioration. For this reason these machines are continuously monitored and the global and local amount of impedance needs to be evaluated. In this work we present our studies on the local transverse impedance detection algorithm. The main assumptions behind the algorithm are described in order to understand limits in reconstructing the impedance location. The phase advance response matrix is analyzed in particular for the nominal gamma transition SPS lattice (Q26), studying the different response from 90, 180, 270 degree phase advance sections. The thin lenses scheme is also implemented and new analytical formulas for phase advance beating were derived. This enabled us to put reconstructing lenses everywhere in the lattice, and to study their positioning scheme. Limits in linear response are analyzed. This relates the upper and lower limits in reconstruction to the phase advance measurement accuracy and the linear response regime limit.

INTRODUCTION

Transverse Mode Coupling Instability driven by machine impedance is one of the major single bunch intensity limitations in the SPS. Possible cures include an increase of the slippage factor η [1] or the identification and elimination of the major sources of impedance of the machine which has been built more than 30 years ago. The transverse effective impedance can be inferred by measuring the tune shift as a function of the bunch intensity. This was theoretically studied for instance in [2] where the following formula relating the coherent tune shift of mode 0 and the generalized (i.e. dipolar + quadrupolar) impedance is given:

$$\Delta Q_0^{x,y} = -\frac{eI_b}{4\sqrt{\pi}\gamma m_0 \omega_0{}^2 Q_0^{x,y}\sigma} Im(Z_{eff}^{x,y}), \quad (1)$$

where $I_b = N_b e/T_0$ is the beam current, N_b the single bunch population, e the proton charge, m_0 the proton mass, T_0 the revolution period, ω_0 the revolution frequency, $Q_0^{x,y}$ the betatron tune, $\omega_\beta^{x,y} = Q_0^{x,y}\omega_0$ the betatron frequency respectively for x and y planes, σ the rms bunch length for a Gaussian bunch, γ the relativistic gamma and $Im(Z_{eff}^{x,y})$ the imaginary part of the effective impedance for both planes.

The impedance obtained in this way is a global parameter. In order to have a local description of the impedance distribution, the overall effect of the tune shift is considered as the summation of smaller tune shift provoked by single impedance sources:

$$\Delta Q_0^{x,y} = \sum_{j=1}^{N_{imp}} \Delta Q_j^{x,y}. \quad (2)$$

Similar tune shifts can be reproduced by thin quadrupole errors. For small errors Δk_j a beta-beating wave provokes a tune shift given by:

$$\Delta Q_j^{x,y} = \frac{1}{4\pi}\beta_j^{x,y}\Delta k_j^{x,y}, \quad (3)$$

where $\beta_j^{x,y}$ is the beta function at the kick location.

In the limit in which the impedance detuning can be associated with a thin quadrupole detuning, Eqs. (1), (2) and (3) can be chained obtaining:

$$\frac{1}{4\pi}\sum_{j=1}^{N_{imp}} \beta_j^{x,y}\Delta k_j^{x,y} = \sum_{j=1}^{N_{imp}} \frac{-eI_b}{4\sqrt{\pi}\gamma m_0\omega_0{}^2 Q_0^{x,y}\sigma} Im(Z_{eff}^{x,y})_j. \quad (4)$$

This moves the problem to finding the errors Δk_j along the studied accelerator. As the tune shift with intensity is the parameter to get the global impedance, the change of the phase advance between two positions as a function of the bunch intensity is the observable to get the local impedance [3].

METHOD

Here we briefly describe the method [3] we use to obtain the $\Delta k_j^{x,y}$ errors along the SPS accelerator. For small quadrupole errors a phase-beating wave is excited as well as a beta-beating one. The latter can be calculated analytically starting from the beta-beating formula:

$$\frac{\Delta\beta(s)}{\beta_0(s)} = \frac{-\beta_0(s_j)}{2\sin(2\pi Q_0)}\cos(2|\varphi(s)-\varphi(s_j)| - 2\pi Q_0)\Delta k_j, \quad (5)$$

where $\beta(s)$ and $\varphi(s)$ are the perturbed beta and phase function at the "s" location in the ring, "*0*" subscript refers to the unperturbed quantities, "*j*" to the position of the focusing error Δk_j [4]. The phase beating is given by:

$$\varphi(s) = \int_0^s \frac{1}{\beta(\tau)}d\tau + \varphi(0).$$

Setting the initial phase $\varphi(0) = 0$ and expanding the β function to the first order we get:

$$\varphi(s) = \int_0^s \frac{1}{\beta_0(\tau)\left(1+\frac{\Delta\beta(\tau)}{\beta_0(\tau)}\right)}d\tau = \int_0^s \frac{1}{\beta_0(\tau)}d\tau - \int_0^s \frac{\Delta\beta(\tau)}{\beta_0^2(\tau)}d\tau \quad (6)$$

The second term in Eq. (6) can be integrated to get the phase-beating along the entire accelerator. More than phase-beating, we are interested in the phase advance beating between Beam Position Monitors (BPMs). This is given by the following formula [5]:

$$\Delta\varphi(s_{j12}) = \frac{\beta_j \Delta k_j}{2s(2\pi Q_0)} \begin{pmatrix} c(2\varphi_j - \varphi_2)s(\varphi_2 - 2\pi Q_0) + \\ -c(2\varphi_j - \varphi_1)s(\varphi_1 - 2\pi Q_0) \end{pmatrix},$$

$$\Delta\varphi(s_{1j2}) = \frac{\beta_j \Delta k_j}{2s(2\pi Q_0)} \begin{pmatrix} c(2\varphi_j - \varphi_2)s(\varphi_2 - 2\pi Q_0) + \\ -s(\varphi_1)c(\varphi_1 - 2\varphi_j + 2\pi Q_0) \end{pmatrix} + \frac{\beta_j \Delta k_j}{2} \quad (7)$$

$$\Delta\varphi(s_{12j}) = \frac{\beta_j \Delta k_j}{2s(2\pi Q_0)} \begin{pmatrix} s(\varphi_2)c(\varphi_2 - 2\varphi_j + 2\pi Q_0) + \\ -s(\varphi_1)c(\varphi_1 - 2\varphi_j + 2\pi Q_0) \end{pmatrix},$$

where the index "*j*" refers to the position s_j of the quadrupole error, "1" and "2" to the generic BPMs position $s_1 < s_2$, "*s*()" and "*c*()" stand for "*sin*()" and "*cos*()". The function is divided in three expressions depending on the relative position of the kick, BPM1 and BPM2 (expression "s_{j12}" stands for "$s_j < s_1 < s_2$").

Given a number N_{BPM} of observation points, using Eq. (7), we can construct a response matrix exciting a quadrupole error arbitrarily in the lattice and collecting the phase advance of the $N_{BPM} - 1$ pairs of BPMs. Choosing N_Q locations for quadrupoles we construct $\Delta\varphi_i = R_{i,j}\Delta k_j$ where "$R_{i,j}$" are elements of the response matrix "*R*" (for vertical or horizontal plane) with $i \in (1..N_{BPM} - 1)$ and $j \in (1..N_Q - 1)$. This is in general a rectangular matrix that can be inverted in a least square sense to get, from the measured phase advances between BPMs, the values of Δk_j along the ring.

Since, from Eq. (4), we see that the impedance is acting as a quadrupole whose strength is linearly dependent on the bunch population N_b, the effective imaginary impedance $Im(Z^{x,y}_{eff})$ can be computed at each location from the slope $\Delta k / \Delta N_b$.

HEADTAIL SIMULATIONS

Simulation Scheme

Following the studies initiated in [6] we set up a simulation scheme in order to study the reliability of this method. As shown in Fig. (1) the SPS accelerator was simulated via the HEADTAIL tracking code, extended by D. Quatraro in order to include a twiss lattice description for the tracking between BPMs and impedance interaction points [7]. The beam is tracked for different intensities and the BPMs data are collected in both planes and analysed in order to get the phase advance beating with intensity. The "R" matrix is constructed from Eq. (7) using the MAD-X [8] SPS 2007 twiss lattice output and is then pseudo-inverted.

The impedance (or wake functions in time domain) can be arbitrarily placed along the accelerator model: we chose wakes coming from kickers since these are the most important contributors to the SPS impedance budget (~500kΩ/m for each kicker).

RESPONSE MATRIX

In our simulations we used the SPS lattice from 2007. The tune is Q_x=26.13 and Q_y=26.18, the phase advance between BPMs is nearly 90° both in vertical and horizontal plane, with some sections of 180° and 270° as shown in Fig. 2 for the vertical plane.

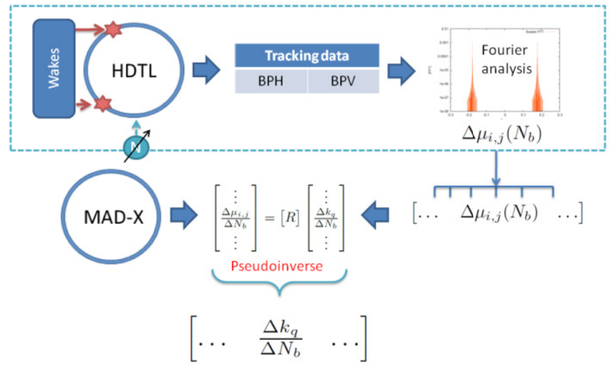

Figure 1: Scheme of the simulation set up using the HEADTAIL code.

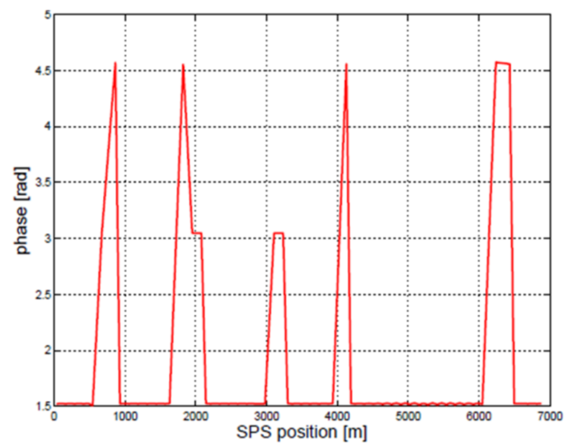

Figure 2: Phase advance in radiants between SPS vertical BPMs.

The phase advance beating has a periodicity of twice the tune as shown in Eq. (7), i.e. corresponding to 132m space period. We chose to cover this periodicity with 8 lenses, spaced by 16.5m, leading to a total number of lenses of 412 to cover the whole length of the SPS accelerator (6911m). Since we have 93 BPMs for the vertical plane and 105 for the horizontal, the response matrix "*R*" has a size of 412x93 in the vertical plane and 412x105 in the horizontal. Especially in the 270° sections, we expect that reconstructing lenses spaced by 132m will have the same response seen by BPMs, leading to degeneration in reconstruction. A useful way to visualize this degeneracy is shown in Fig. 3 where the "*R*" response matrix for the vertical plane is drawn as a contour plot: each horizontal line represents the response seen by each pair of BPMs to the corresponding lens excitation. The peak (red spot) appears when a kick is given between the BPMs pair. Two peaks vertically aligned, as in the 270° section at 4000m position (see zoom on Fig. 3) means that two different kick positions produce the same peak signal between the correspondent BPM pair. This means we cannot reconstruct impedances in these locations since in the reconstruction we do the inverse procedure: from the BPM pair phase signal, to the impedance (kick) location.

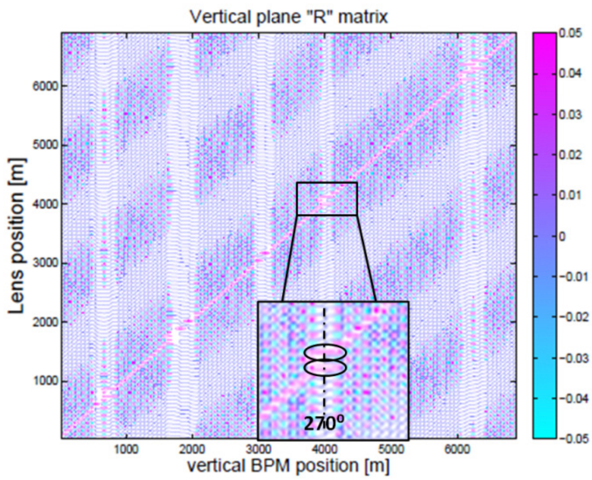

Figure 3: Vertical plane response matrix color coded with maximum response.

Inversion problem

Once the simulated data from the HEADTAIL code is obtained and the response matrix for each plane is calculated correspondingly to the reconstruction lens position scheme, we invert the equation $\Delta\varphi = R\,\Delta k$ in a least-square sense and scale the errors Δk to get the local imaginary impedances. We artificially normalized the response matrix over the beta function from the lattice: in this way all lenses have the same weight in the least square procedure.

We report an example of reconstruction in Fig. 4. Here we tracked single bunch beams of different intensities interacting with kicker impedances represented by black bars along the circumference. We reconstructed impedance locations using a thin lens scheme 16.5m spaced. It can be observed that the impedance can be localized near the black bars, but doubled peaks appear. These are spaced by 132m as the space periodicity of phase advance beating.

We also calculated the expected impedance value looking to the tune shift with intensity (machine total budget) and found almost 9.2 MΩ/m. Summing the local impedances we get the same amount. These values are almost 5% greater than the one predicted by Sacherer's theory that represents itself an approximation of the real coherent tune shift.

CONCLUSION

The impedance localization algorithm was presented, implemented and tested using HEADTAIL simulations.

We found limits in accuracy due to the periodicity of phase advance beating. A solution to this problem could be to increase the phase sampling between BPMs, i.e. using lower tunes like the ones proposed for the low gamma transition SPS lattice (20.13, 20.18) [1] or even lower. Alternatively we could install, if feasible, new BPMs in the long phase advance sections.

Another limit will probably be linked to the accuracy in phase measurement we can achieve with real data. This will be investigated during machine development time and adding noise to the simulations.

Alternative algorithms for reconstruction are also available: we could correct the closed orbit using beta bumps along the ring using few quadrupoles in place of all of them as in the current method [9].

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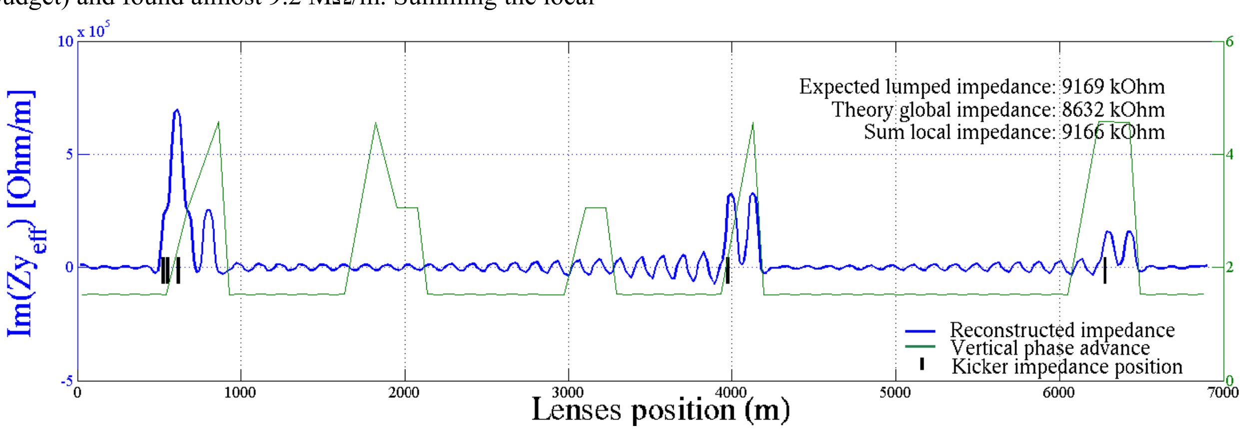

Figure 4: Vertical reconstruction of kicker impedances. Black lines are where the 19 kicker impedances are placed, green line is the vertical phase advance between consecutive BPMs in SPS, blue line is the reconstructed position of impedances.