AUTOCORRELATION FUNCTION AND POWER SPECTRUM OF A TRAIN OF QUASIPERIODIC SEQUENCE OF PULSES

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Abstract

The statistical relationship of the autocorrelation function and spectrum of a train of quasi-periodic sequence of pulses having a time jitter of the repetition rate is obtained.Presented the accordance of autocorrelation function as well as power spectrum of the bounded quasi-periodic sequence of pulses and timing jitter of their repetition rate. The results can be used at the measurements of timing jitter of a train of electron bunches.

PROBLEM STATEMENT

Let's consider a bounded quasi-periodic sequence x*(t)* consisting of (*N+1*) rectangular pulses following each other at intervals $T_0 \pm |\Delta T_i|$ (Fig.1).

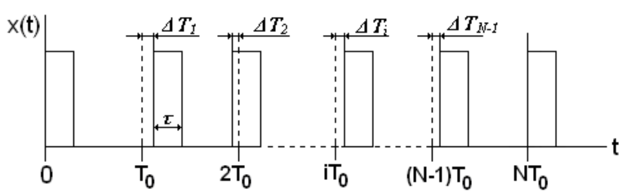

Figure 1: A train of (*N+1)* quasi-periodic sequence of pulses ($\Delta T_i \neq 0$).

At $\Delta T_i = 0$ =0 we will have a bounded strongly periodic sequence of (N+1) pulses. Let's assume that the amplitude and width of pulses as well as the length of the train (ΔT0 = ΔTN = 0) are constant, therefore the energy and power of train will be the same at any ΔT_i. Suppose, in particular, N=30, T0 = 1000 psec , τ = 5 psec.

Let's now consider how a timing jitter affects the autocorrelation function and power spectrum of the train (Figs. 2-4). For that we will specify the random deviation from strict periodicity, using the generators of the discrete random numbers with uniform distribution in intervals:

a) $\Delta T_i = 0 \div |1|$ psec with step 0.1 psec,

b) $\Delta T_i = 0 \div |10|$ psecwithstep 1 psec,

where |A| means a setof discreterandom numbers |-A, -(A-1), ..., -1, 0, 1, ..., (A-1), A|.

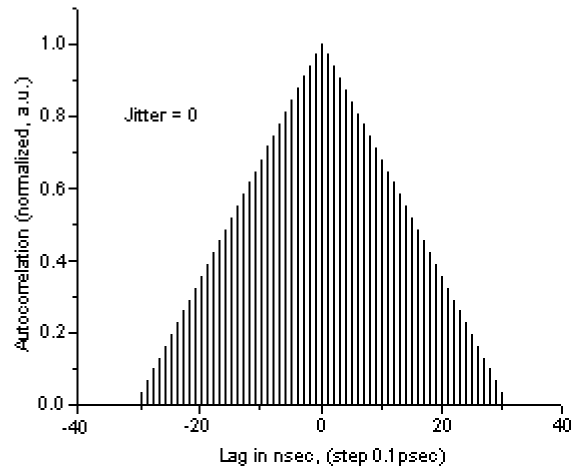

Figure 2a: Autocorrelation function at $\Delta T_i = 0$

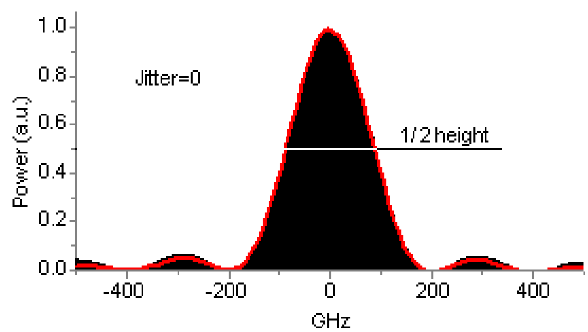

Figure 2b: Power spectrum at $\Delta T_i = 0$

AUTOCORRELATION FUNCTION (AF)

Let’s recall some main properties of an autocorrelation function *(AF).* Autocorrelationof a random processdescribes the correlationbetween the valuesof the processat different points intime.For a discreteprocessof length *n* (X_1, X_2,...,X_n) with knownexpectationand dispersionthe autocorrelationcan be calculated bythe following formula:

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$$\hat{R}(k) = \frac{1}{(n-k)\sigma^2}\sum_{t=1}^{n-k}[X_t - \mu][X_{t+k} - \mu]$$

for any positiveintegers $k$and n.

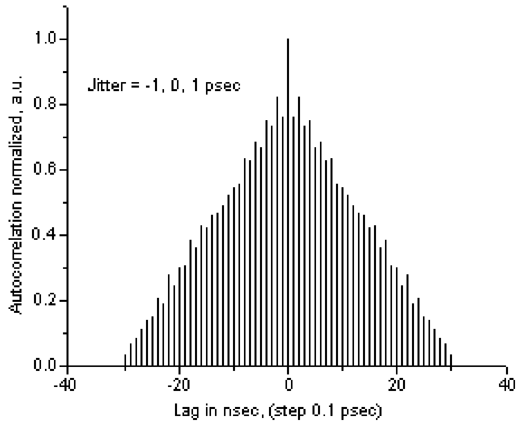

Figure: 3a. Autocorrelation function at $\Delta T_i = |1|\ psec$

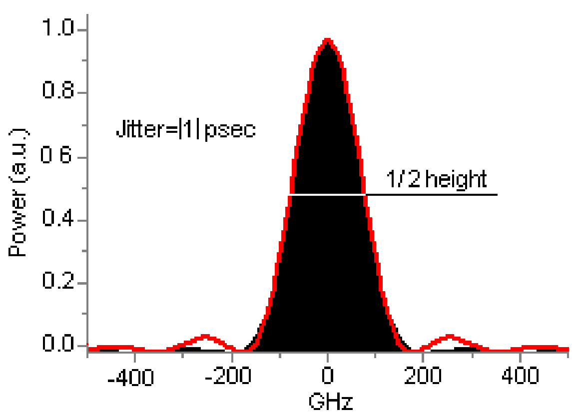

Figure 3b: Power spectrum at $\Delta T_i = |1|\ psec$

The fundamental property of autocorrelation function is its symmetry; the value of *AF* at 0 is proportional to the energy of the signal. Autocorrelation function reaches its maximum at 0 and $|R(\tau)| \leq R(0)$ The Wiener-Khinchin theorem relates the autocorrelation function to the power spectral density via the Fourier transform:

$$R(\tau) = \int_{-\infty}^{\infty} S(f)\exp(i2\pi f\tau)\,df;$$

$$S(f) = \int_{-\infty}^{\infty} R(\tau)\exp(-i2\pi f\tau)\,d\tau$$

Since the energy of the train at any ΔT_i is the same, the ratio of the ordinates' sum (or area) at $\Delta T_i \neq 0$ to ordinates' sum (or area) at $\Delta T_i = 0$in the range of correlation will uniquely determine the ΔT_i

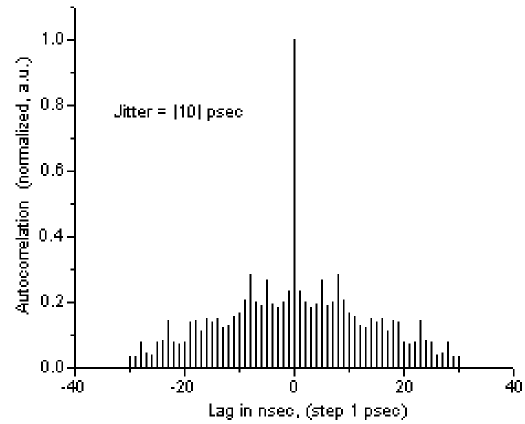

Figure 4a: Autocorrelation function at $\Delta T_i = |10|\ psec$

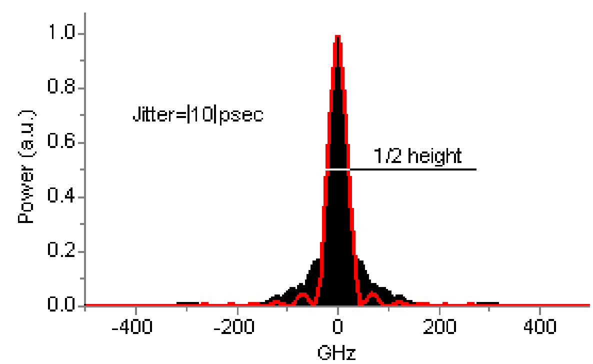

Figure 4b: Power spectrum at $\Delta T_i = |10|\ psec$

POWER SPECTRUM (PS)

Figs. 2b-4b shows that the timing jitter leads to a redistribution of energy over the spectrum. Thus, the ratio of energy in the main maximum to the total energy across the spectrum, as well as the cut-off frequency at half-height contains information about the jitter.

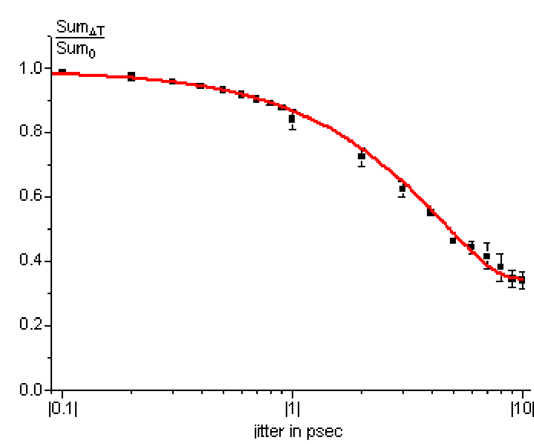

Figure 5: The ratio of ordinate's sum (area) at $\Delta T_i \neq 0$ and $\Delta T_i = 0$

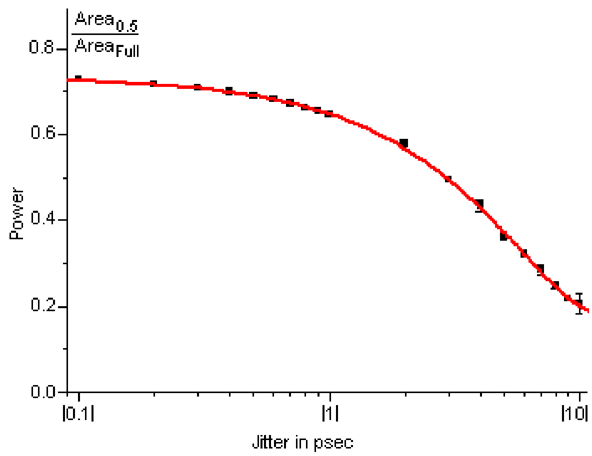

Figure 6: The ratio of energy in the main maximum at half-height / total energy across the spectrum

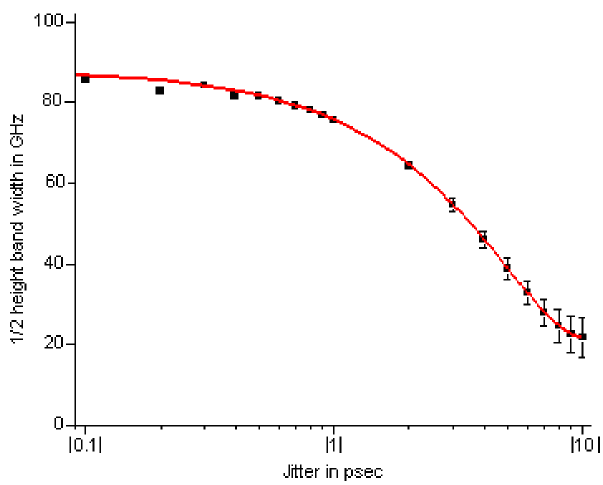

Figure 7: The spectrum width at half-height of the envelope of the main maximum

Thepresented results allow applying well-developed technique of spectral and correlation analysis [1-3] to detecttimingjitter of quasi-periodic bounded sequence of electron bunches.

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