High-Precision Lossy Eigenfield Analysis Based on the Finite Element Method

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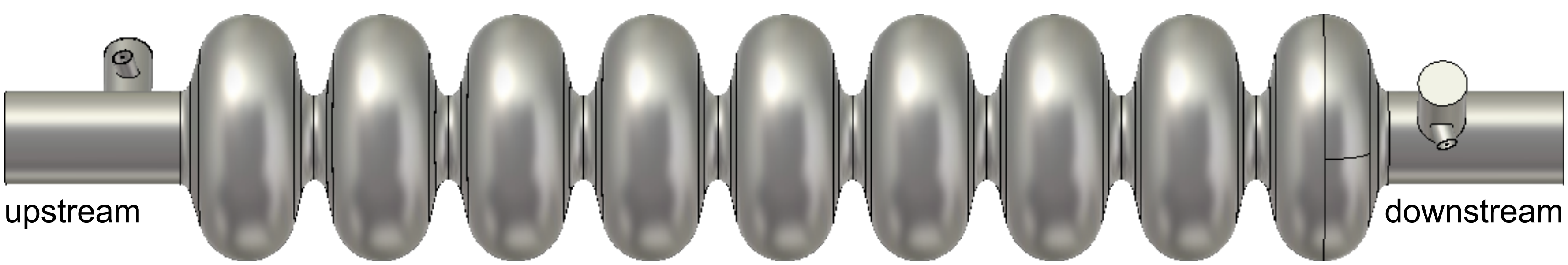
Motivation

- TESLA Type Cavities (1.3 GHz and 3.9 GHz)
  - Photograph 1.3 GHz

http://newsline.linearcollider.org

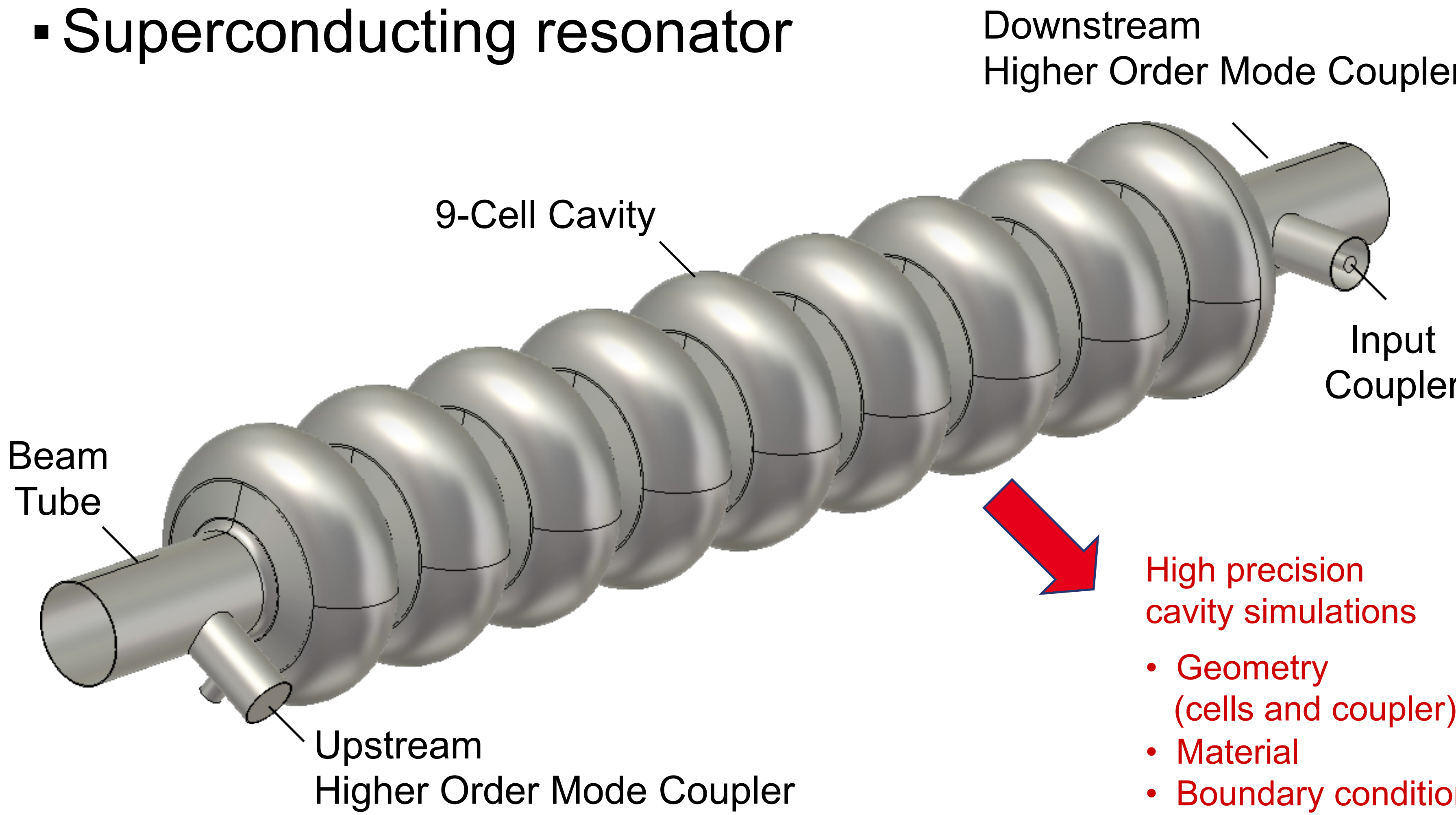
  - Numerical model 1.3 GHz



CST Studio Suite 2018

Motivation

- Superconducting resonator

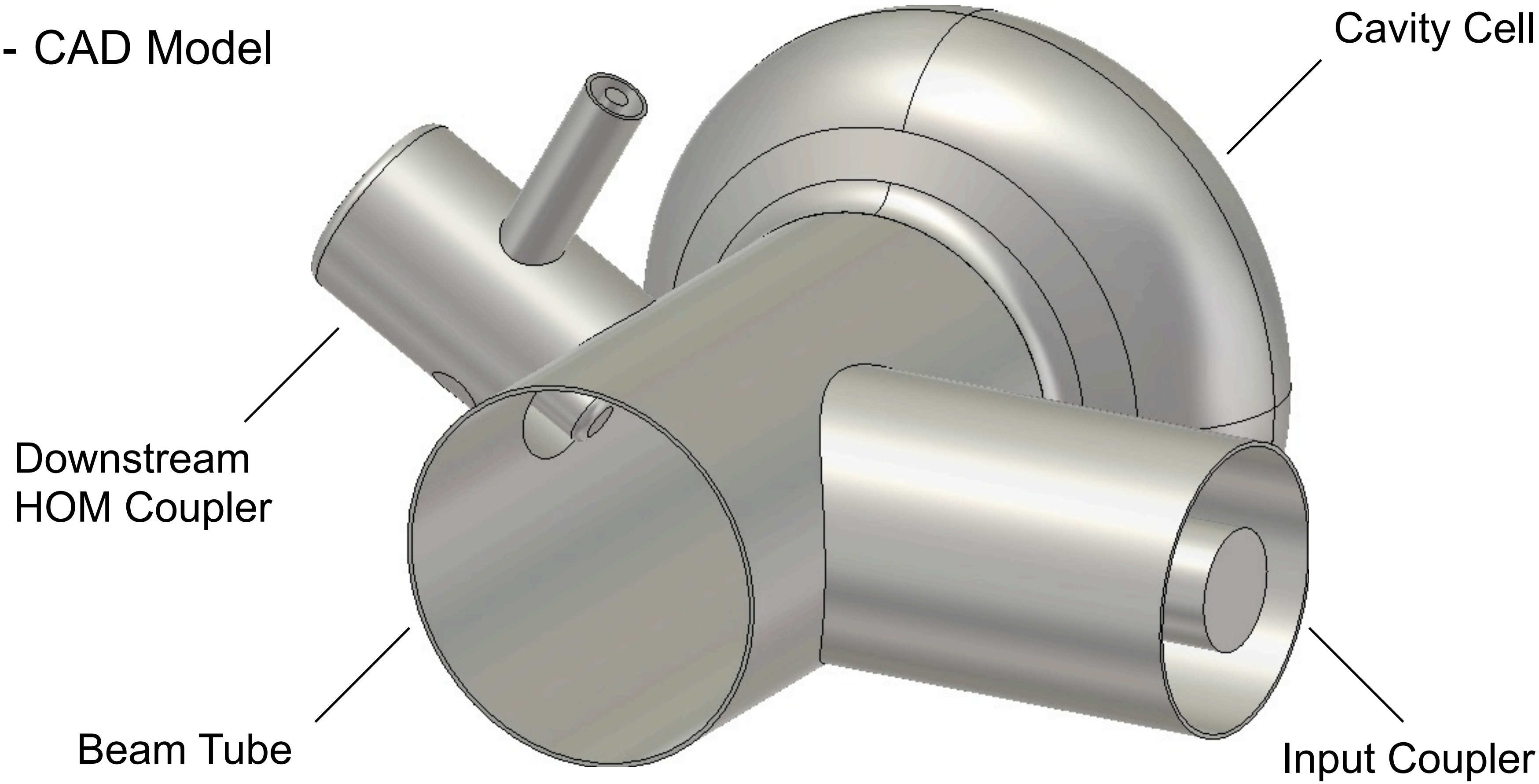


High precision cavity simulations

- Geometry (cells and coupler)
- Material
- Boundary conditions

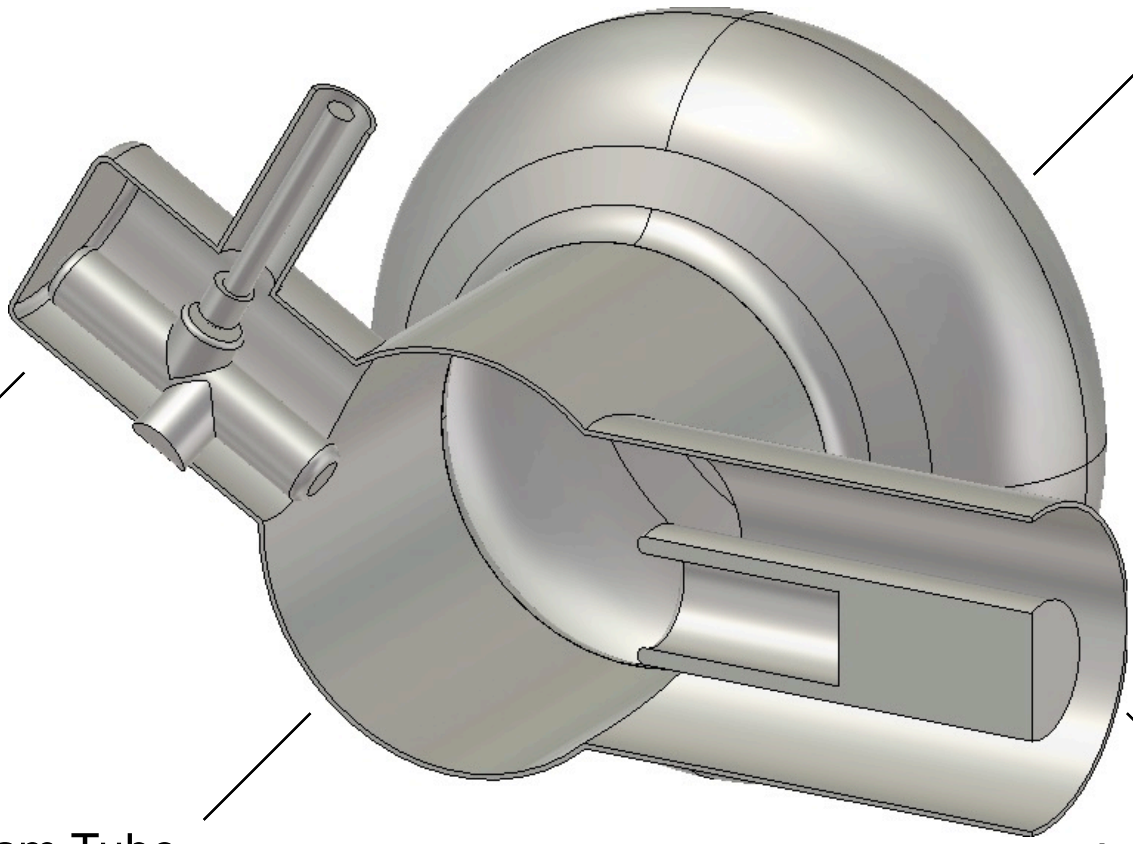
Motivation

- TESLA 3.9 GHz Cavity
  - CAD Model



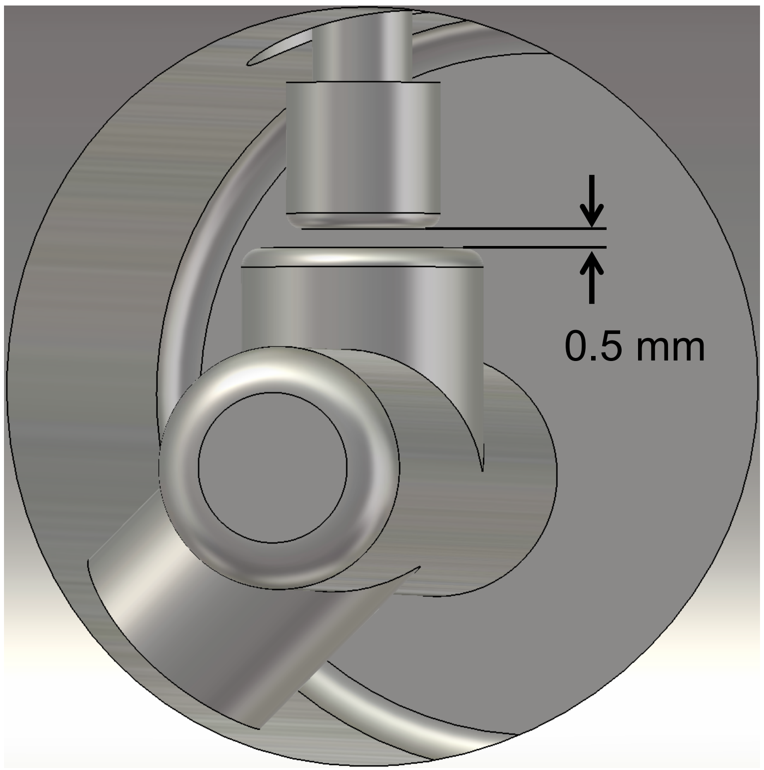
Motivation

- TESLA 3.9 GHz Cavity
  - CAD Model

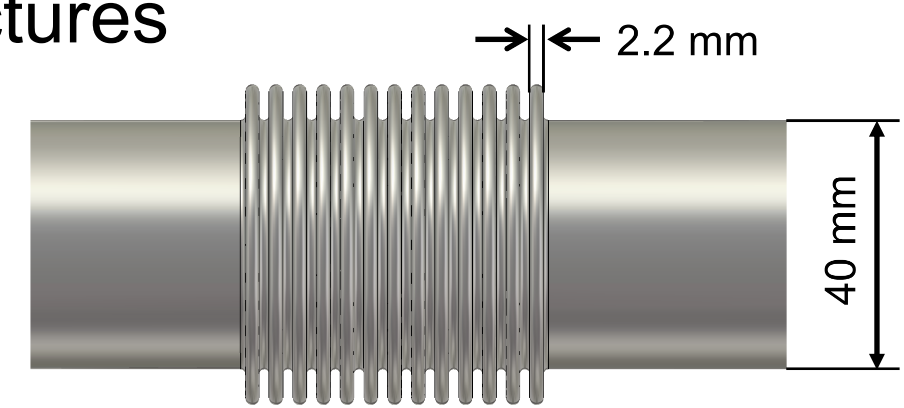


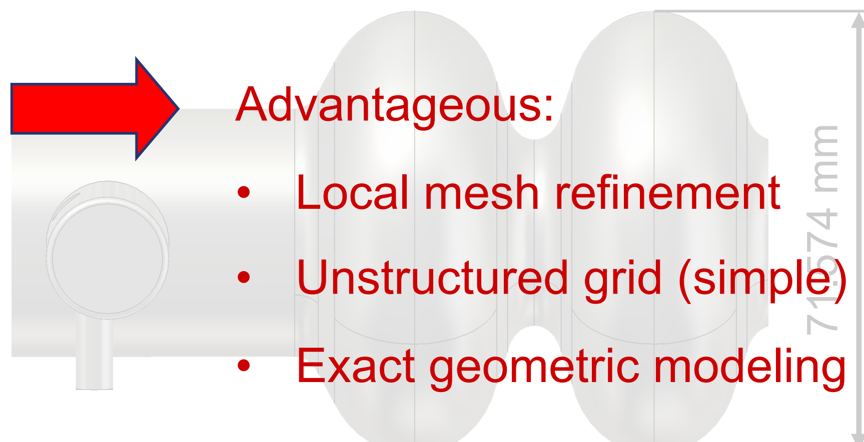
Motivation

- Modeling of detailed structures



TESLA 3.9 GHz structure





Advantageous:

- Local mesh refinement
- Unstructured grid (simple)
- Exact geometric modeling

Motivation

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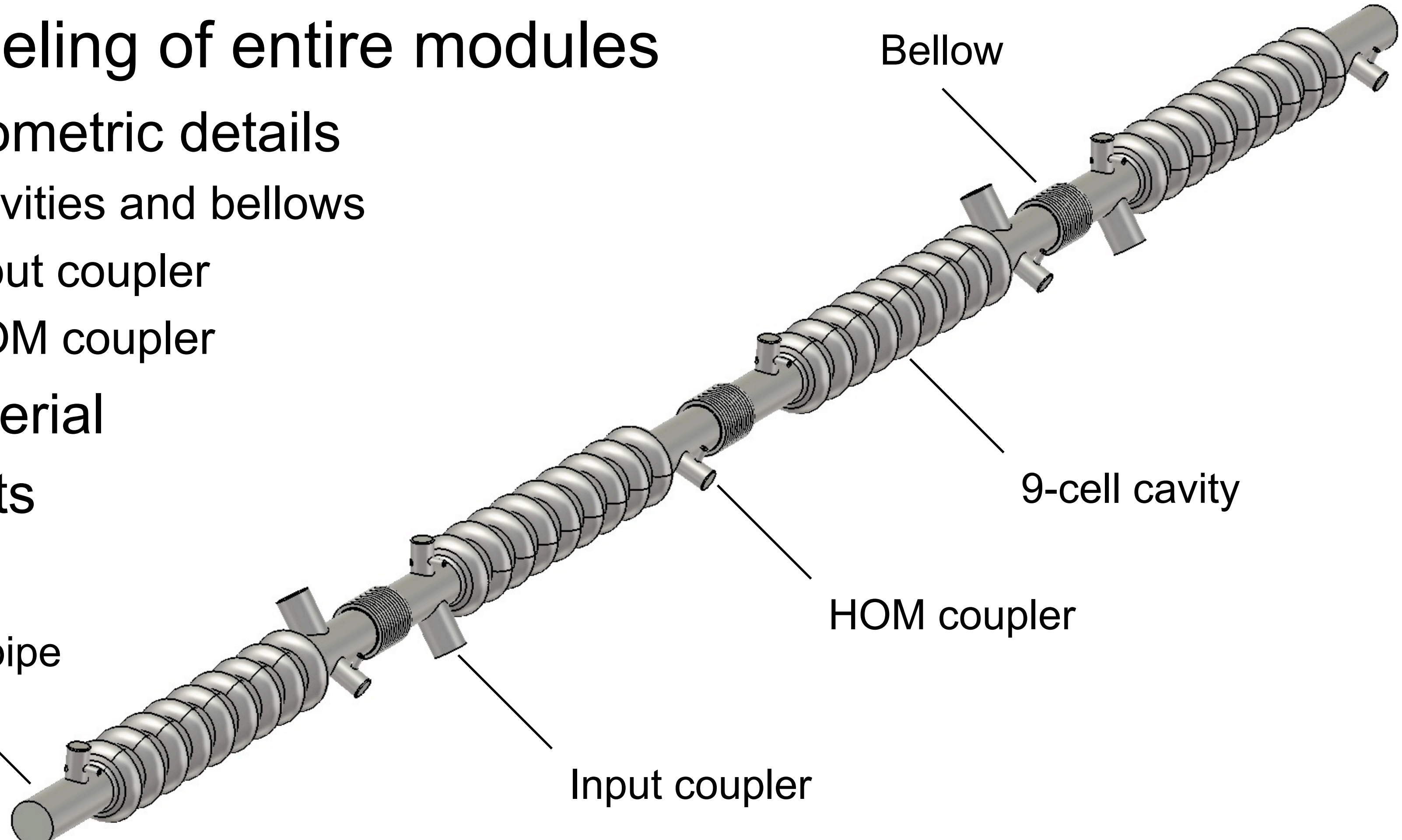
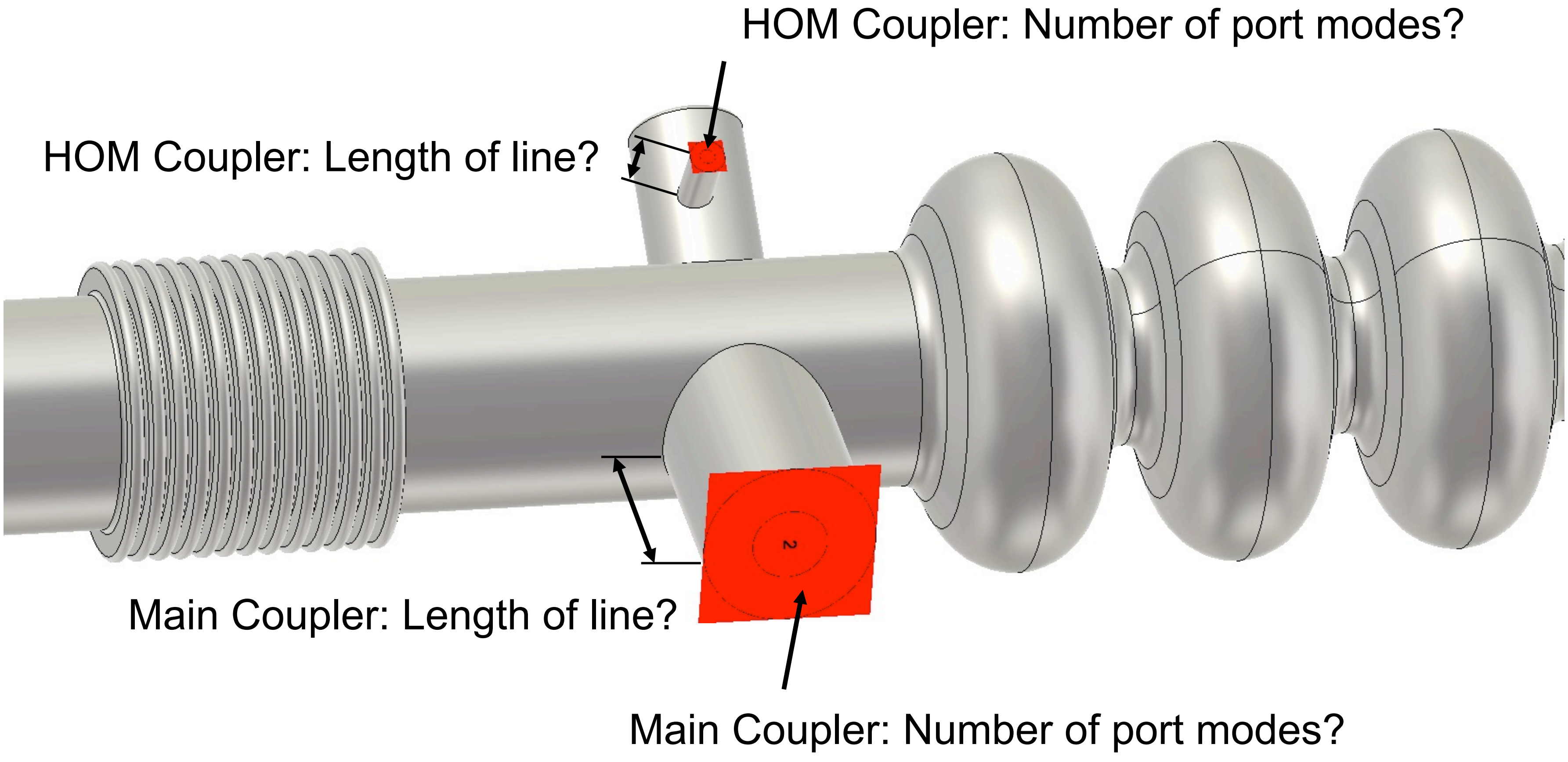
- Modeling of entire modules
  - Geometric details
    - Cavities and bellows
    - Input coupler
    - HOM coupler
  - Material
  - Ports



TESLA 3.9 GHz module

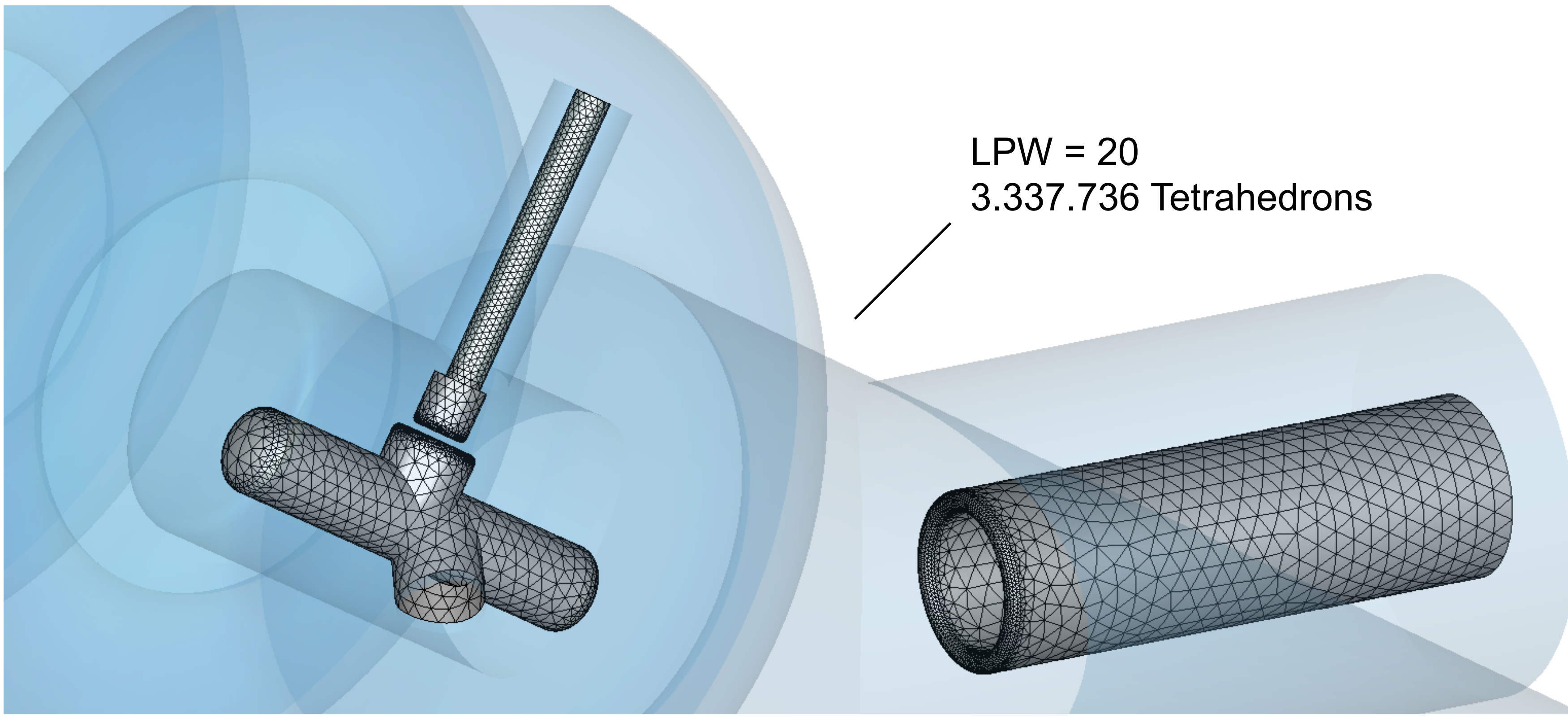
Computational Model

- Boundary conditions



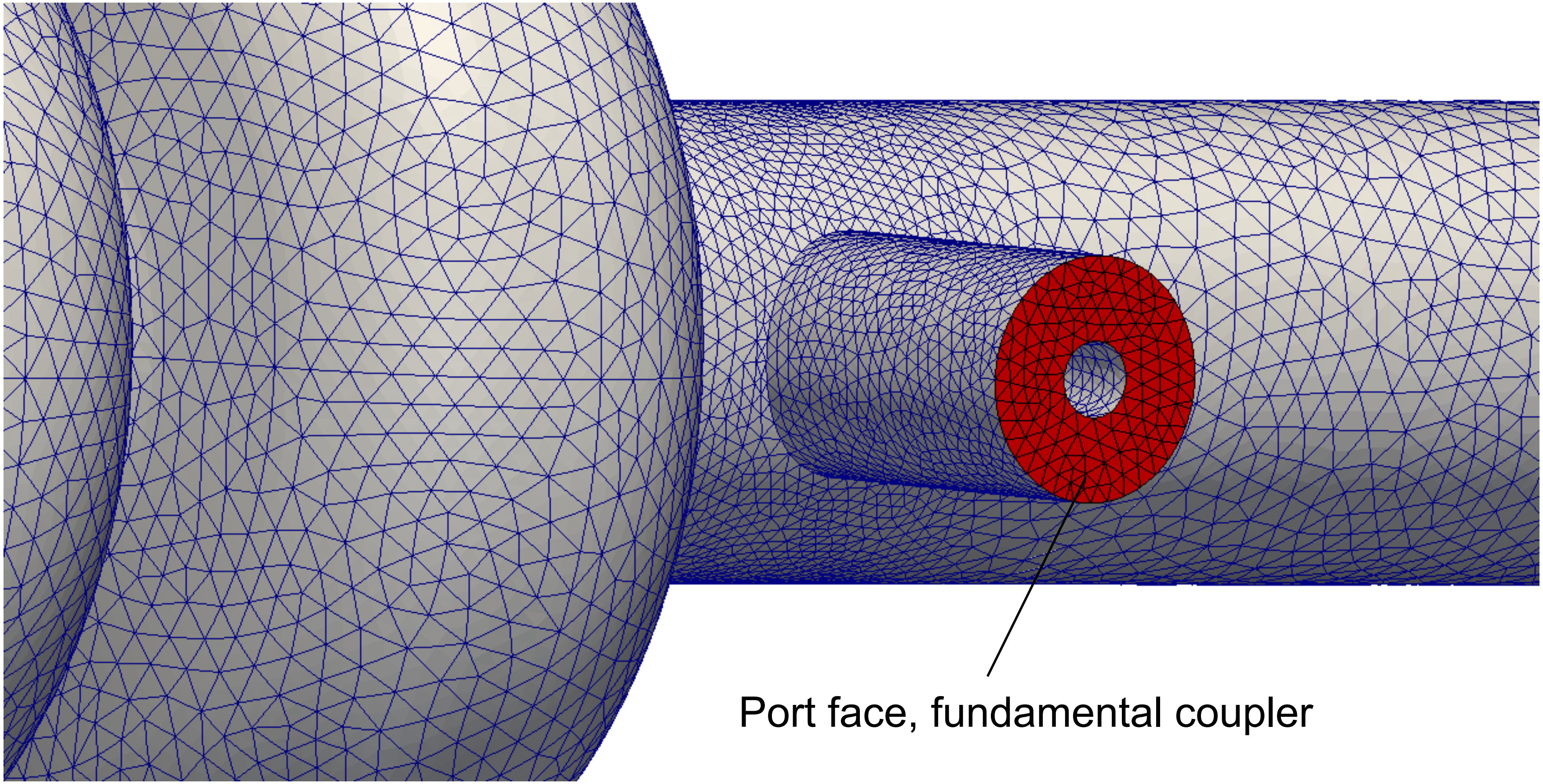
Computational Model

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum with surface mesh on the PEC couplers



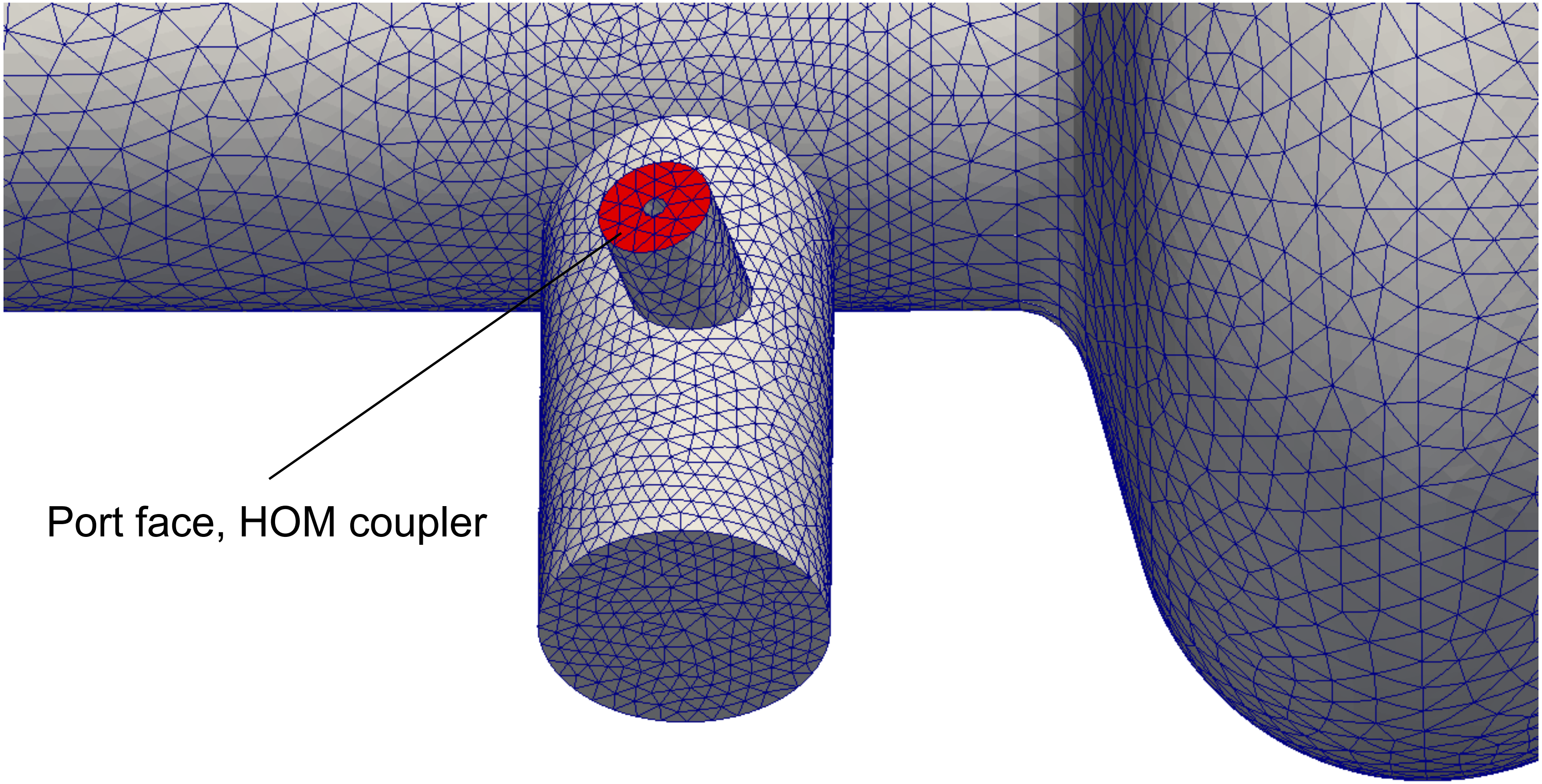
Computational Model

- Port boundary condition



Computational Model

- Port boundary condition



Computational Model

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- Problem formulation
  - Local Ritz approach

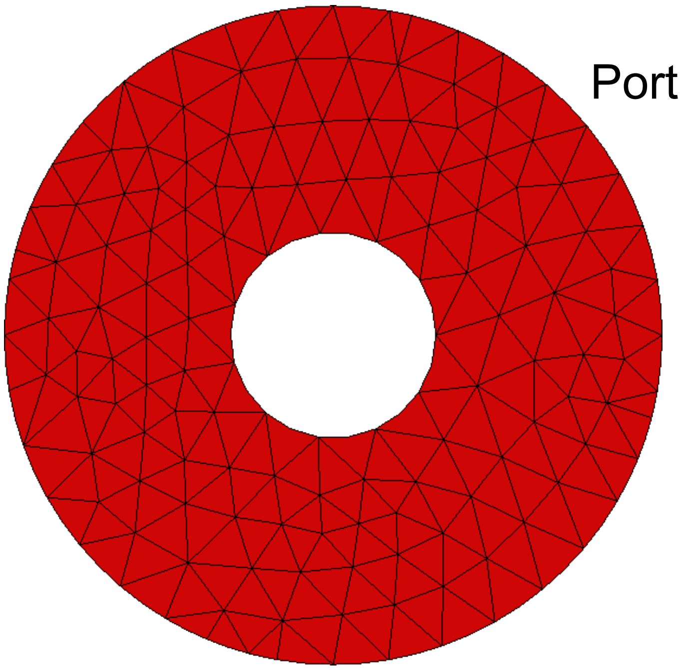
$$\vec{E} = \vec{E}(\vec{r}) = \sum_{i=1}^{n} \alpha_i \, \vec{w}_i(\vec{r})$$

$\vec{w}$ vectorial function

α_i scalar coefficient

i global index

n number of DOFs



Mixed 2-D vector and scalar basis

$$\vec{\omega}_i = \begin{cases} \vec{\omega}_i^{\mathrm{2D}} & \text{tangential} \\ \vec{n}\,\varphi_i & \text{normal} \end{cases}$$

Computational Model

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$$\text{curl } 1/\mu_{\text{r}} \, \text{curl}\vec{E} = \left(\frac{\omega}{c_0}\right)^2 \varepsilon_{\text{r}} \, \vec{E} \,\Big|_{\vec{r}\in\Omega}$$

$$\text{div}(\varepsilon\vec{E})\Big|_{\vec{r}\in\Omega} = 0 \qquad \text{+ boundary conditions}$$

continuous eigenvalue problem, loss-free

Galerkin

$$A_{ij} = \iint_A 1/\mu_{\text{r}} \, \text{curl}\vec{w}_i \cdot \text{curl}\vec{w}_j \, \text{d}\Omega$$

$$B_{ij} = \iint_A \varepsilon_{\text{r}} \, \vec{w}_i \cdot \vec{w}_j \, \text{d}\Omega$$

$$\vec{w}_i(x,y,z) = \vec{w}_i(x,y) \cdot e^{-ik_z z}$$

$$A\vec{\alpha} = \left(\frac{\omega}{c_0}\right)^2 B\vec{\alpha}$$

discrete eigenvalue problem

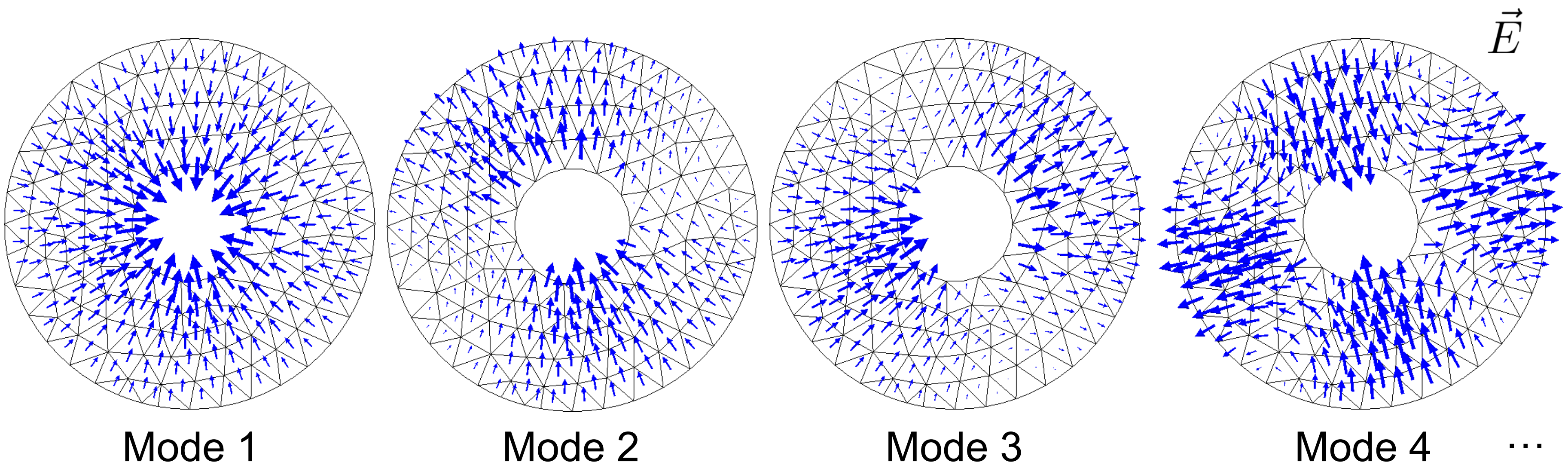
Computational Model

- Problem formulation
  - Determine propagation constant for a fixed frequency

$$\begin{pmatrix} A_{11} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{y}_t \\ \vec{y}_z \end{pmatrix} = -k_z^2 \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} \vec{y}_t \\ \vec{y}_z \end{pmatrix}$$

algebraic eigenvalue problem

→ eigenvector and eigenvalue



Computational Model

TECHNISCHE UNIVERSITÄT DARMSTADT

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continuous eigenvalue problem

Galerkin

$$A_{ij} = \iiint_\Omega 1/\mu_\mathrm{r}\,\mathrm{curl}\vec{w}_i \cdot \mathrm{curl}\vec{w}_j\ \mathrm{d}\Omega$$

$$B_{ij} = \iiint_\Omega \varepsilon_\mathrm{r}\,\vec{w}_i \cdot \vec{w}_j\ \mathrm{d}\Omega$$

$$C_{ij} = \iiint_\Omega Z_0\,\sigma\,\vec{w}_i \cdot \vec{w}_j\ \mathrm{d}\Omega$$

$$A\vec{\alpha} + j\,\frac{\omega}{c_0}C\vec{\alpha} + (j\,\frac{\omega}{c_0})^2 B\vec{\alpha} = 0$$

discrete eigenvalue problem

Computational Model

- Jacobi-Davidson method
  - Important properties
    - Direct solution difficult because of dense matrix in correction equation.
    - Iterative solution not immediately applicable because vectors $\Delta\vec{x}$ with $\Delta\vec{x} \in R\{(V_B)_\perp\}$ are not mapped back onto $R\{(V_B)_\perp\}$ again.
  - Preconditioning
    - The JD - preconditioner

$$
\begin{aligned}
PC &= \{I - M^{-1}V_B[(M^{-1}V_B)^T V_B]^{-1} V_B^T\} M^{-1} \\
&= M^{-1} - M^{-1}V_B[(M^{-1}V_B)^T V_B]^{-1} V_B^T M^{-1}
\end{aligned}
$$

retains the property $\Delta\vec{x} \in R\{(V_B)_\perp\}$ for any preconditioner M^{-1}.

⇒ Simplest case: $M^{-1} = I \quad \hookrightarrow \quad PC = I - VV_B^T = P$

Numerical Examples

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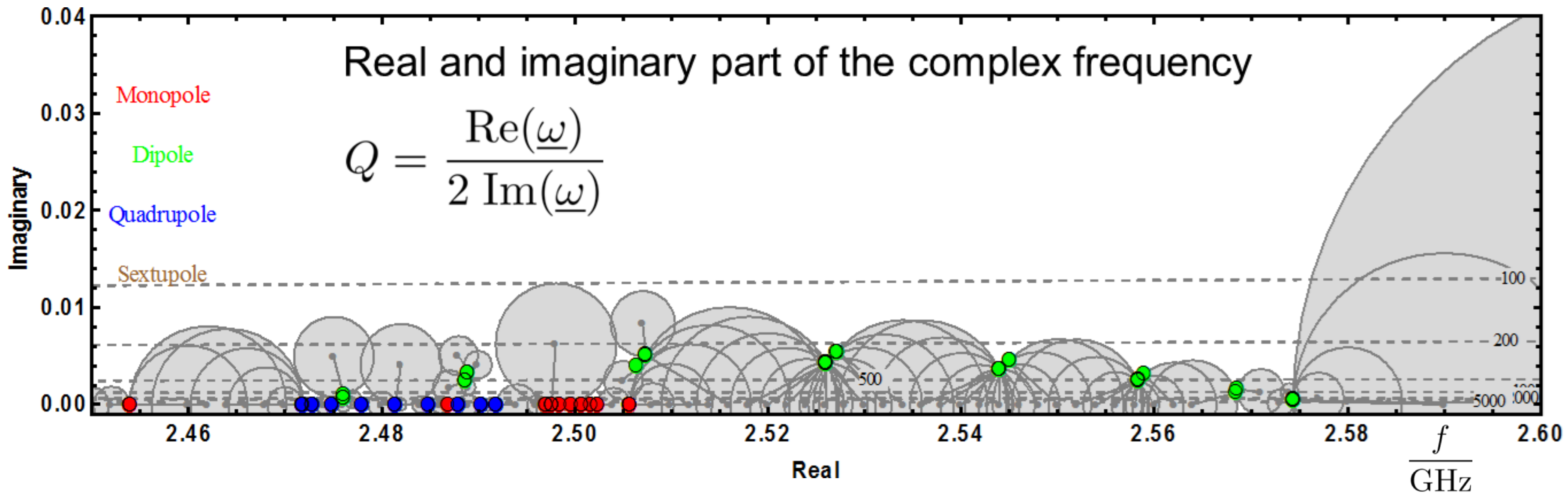
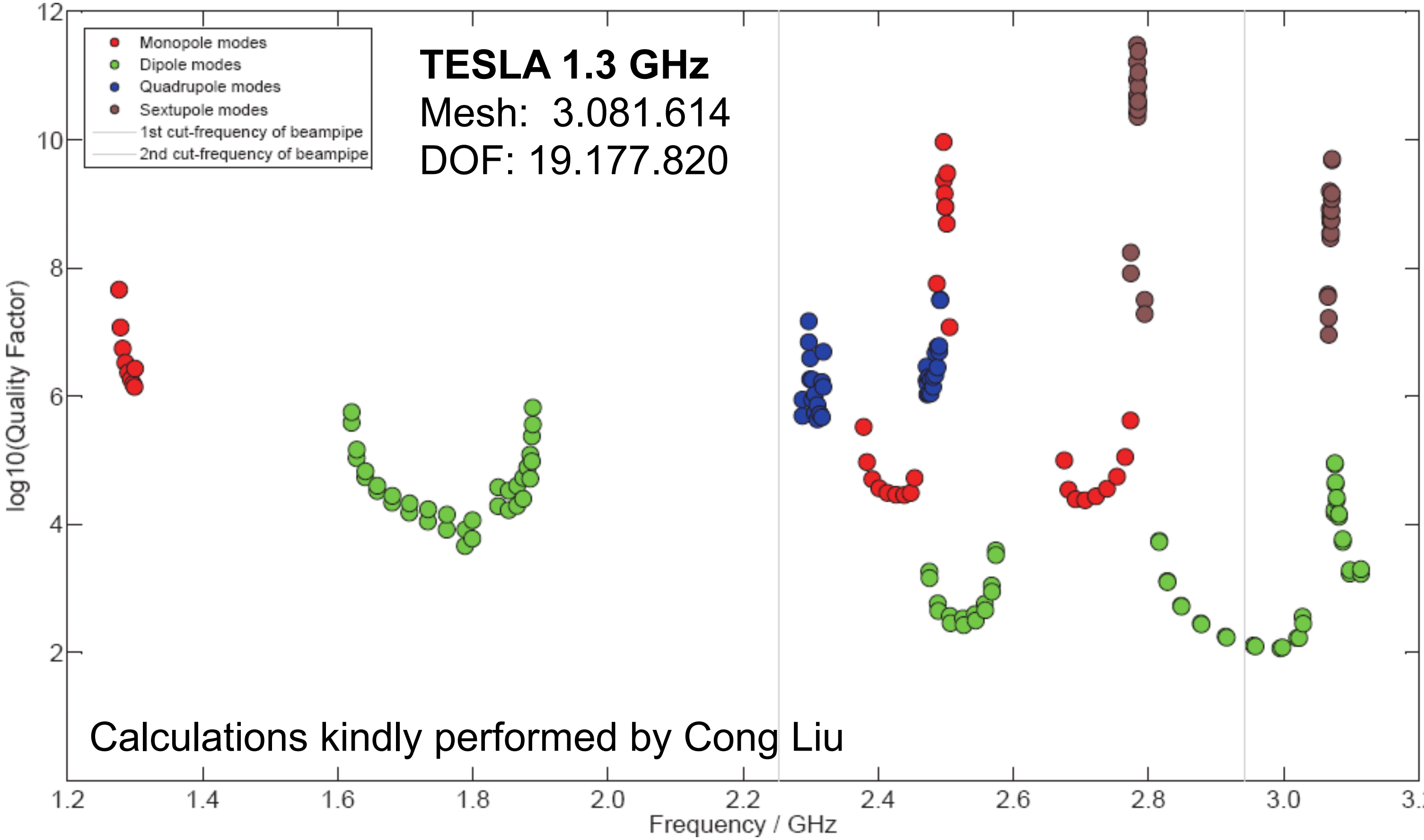
- Controlling the Jacobi-Davidson eigenvalue solver
  - Evaluation in the complex frequency plane
  - Select best suited eigenvalues in circular region around user-specified complex target



Numerical Examples

- Quality factor versus frequency



Numerical Examples

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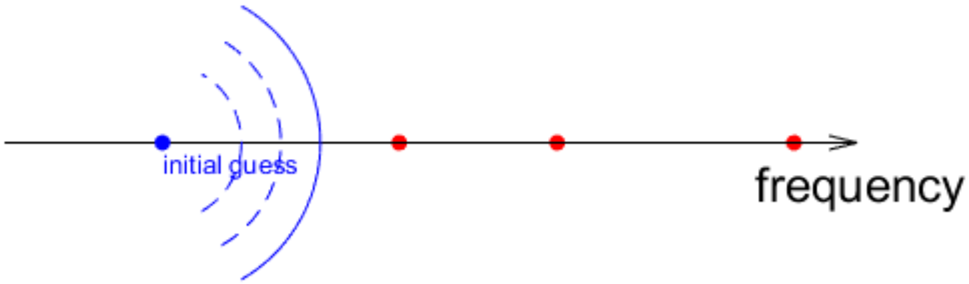
- Lossless accelerator cavity
  - Eigenvalues on the real axis



- choose an initial guess
- expand the search space …
- until an approximate solution is found
- the solution becomes the new initial guess
- continue expanding the search space …

Numerical Examples

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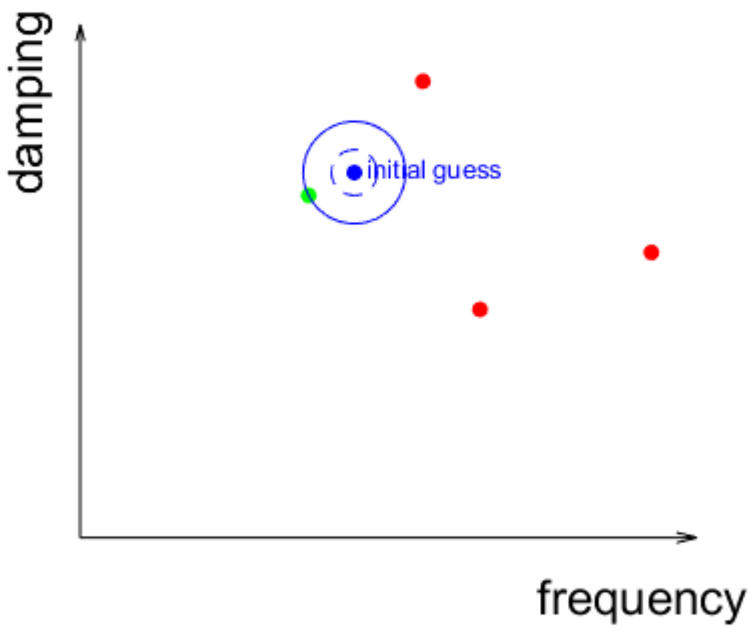
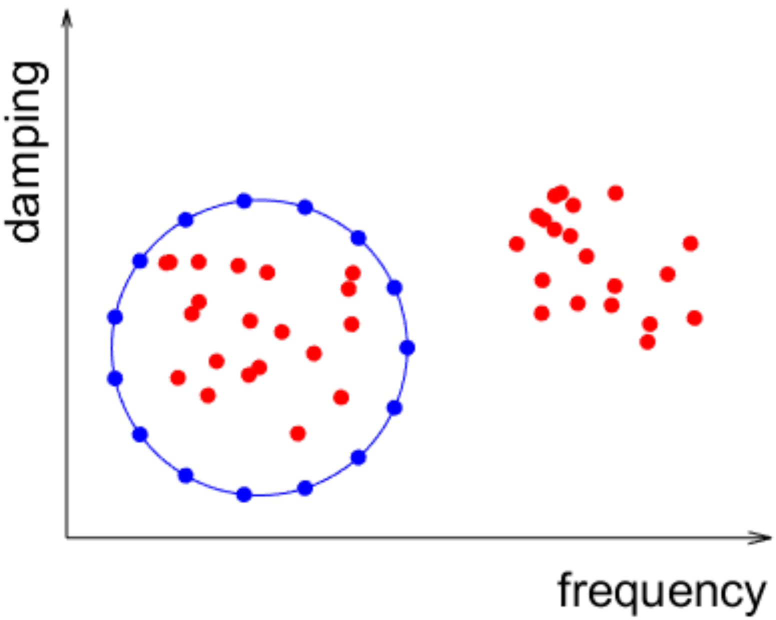
- Lossy accelerator cavity
  - Eigenvalues in the complex plane



- choose an initial guess
- expand the search space …
- until an approximate solution is found
- choose another initial guess
- continue expanding the search space …
- find another approximate solution
- if we choose an unsuitable initial guess
- the algorithm will converge to …
- an already determined eigenvalue!!!

Numerical Examples

- Accurate computation of eigenpairs inside a region enclosed by a non-self-intersecting curve



- choose a region to look for eigenvalues
- the region can be of any shape, e.g rectangle …
- circle/ellipse
- most computation is spent to solve linear equation systems at different interpolation points which can be parallelized.

Computational Model

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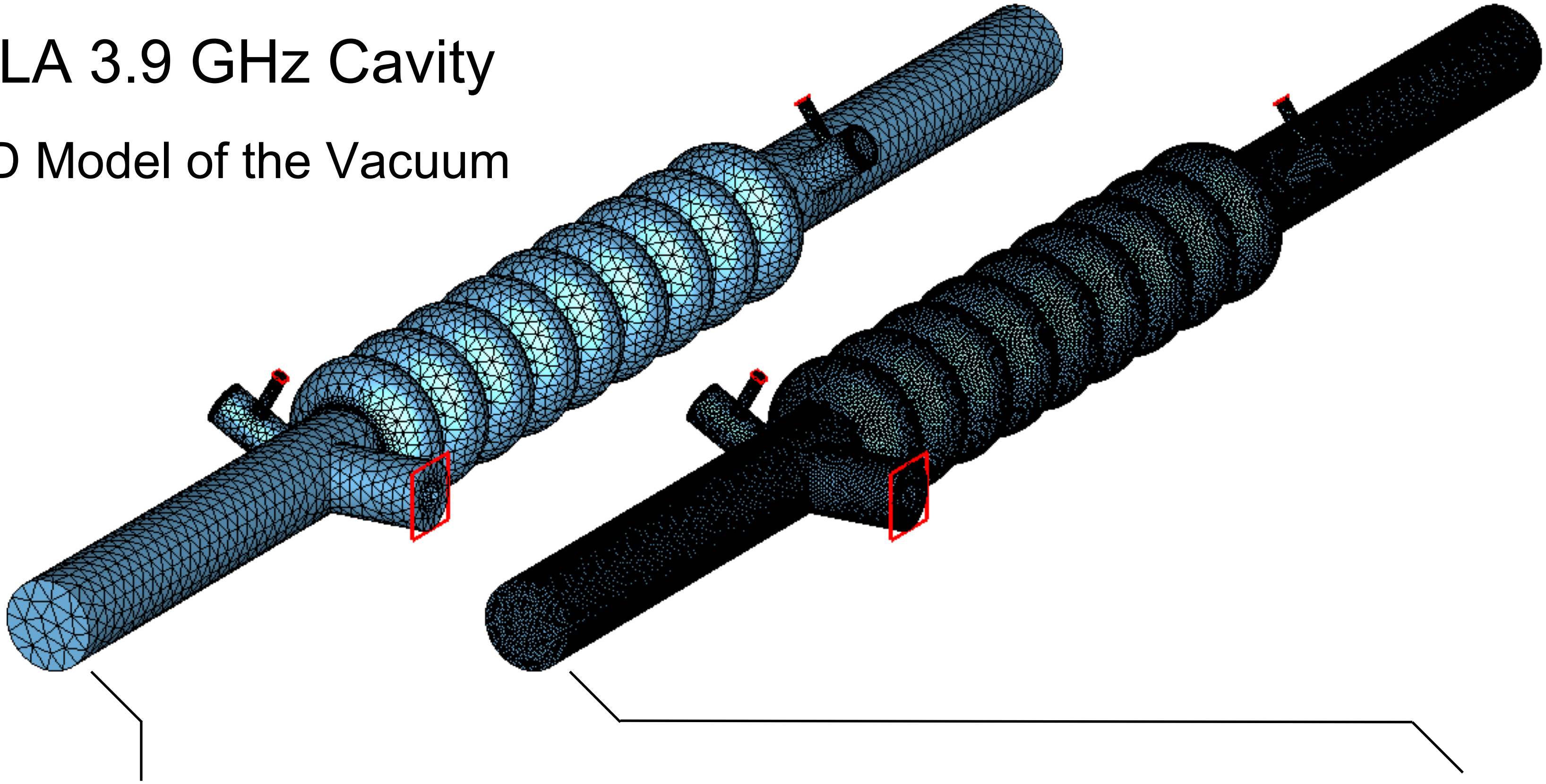
- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum



LPW (9 GHz)	4	6	8	10	12	14	16	18	20
Tetrahedrons	136.443	187.435	304.833	480.376	767.271	1.177.883	1.704.528	2.432.978	3.337.736
Complex DOF	761.820	1.079.488	1.802.314	2.885.154	4.668.072	7.227.096	10.509.404	15.064.232	20.721.334

Simulation Results

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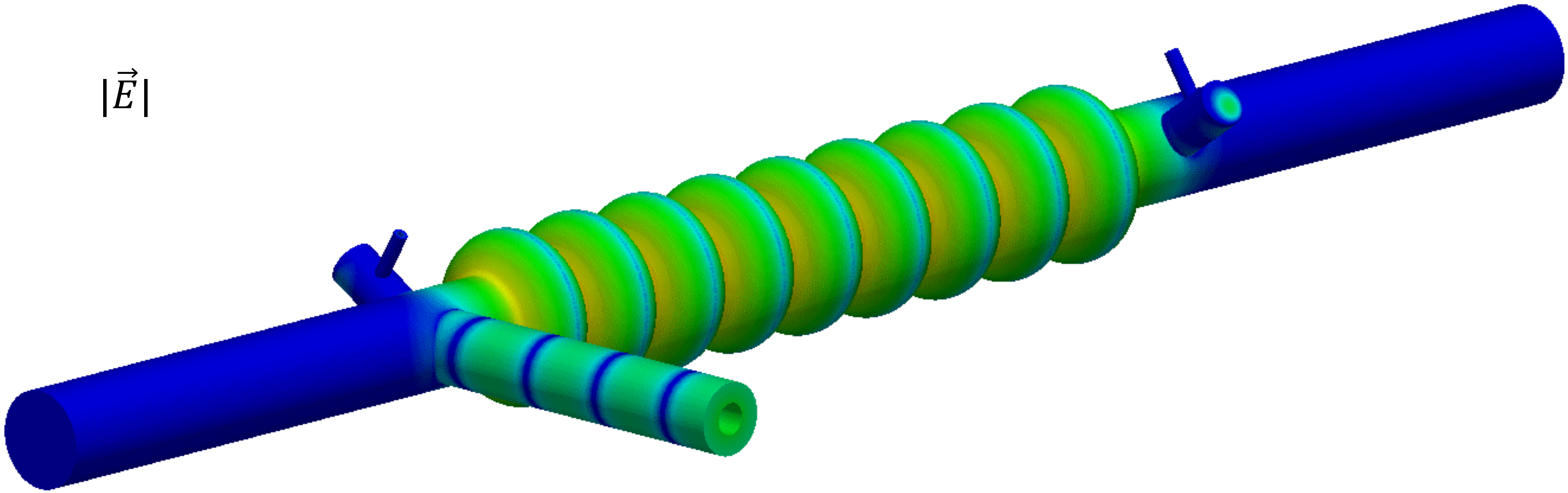
- TESLA 3.9 GHz Cavity
  - Fundamental mode

    Absolute value of the electric field strength

$|\vec{E}|$

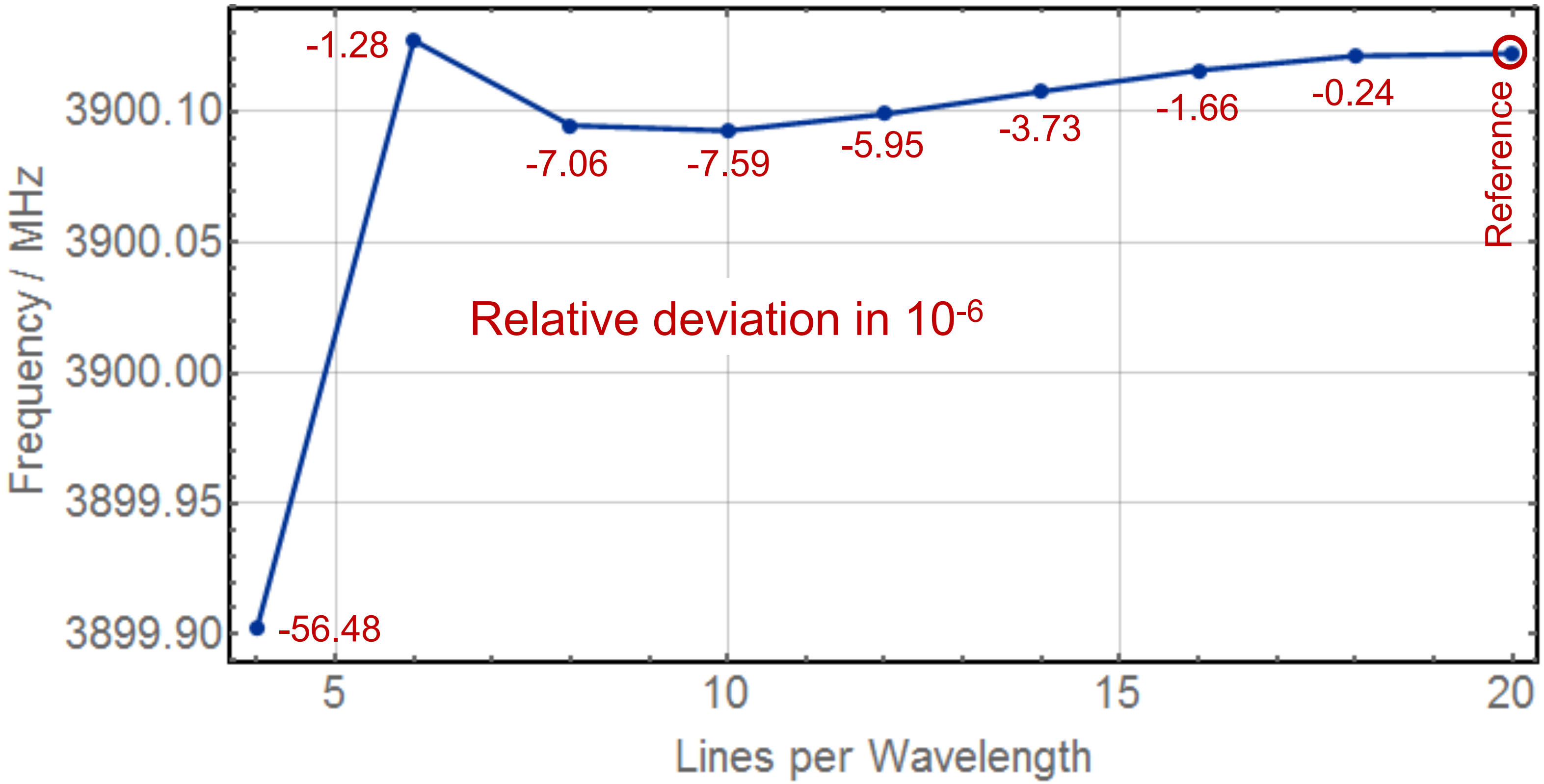


LPW = 20
3.337.736 Tetrahedrons

Logarithmic scale from 1e4 to 1e7 V/m

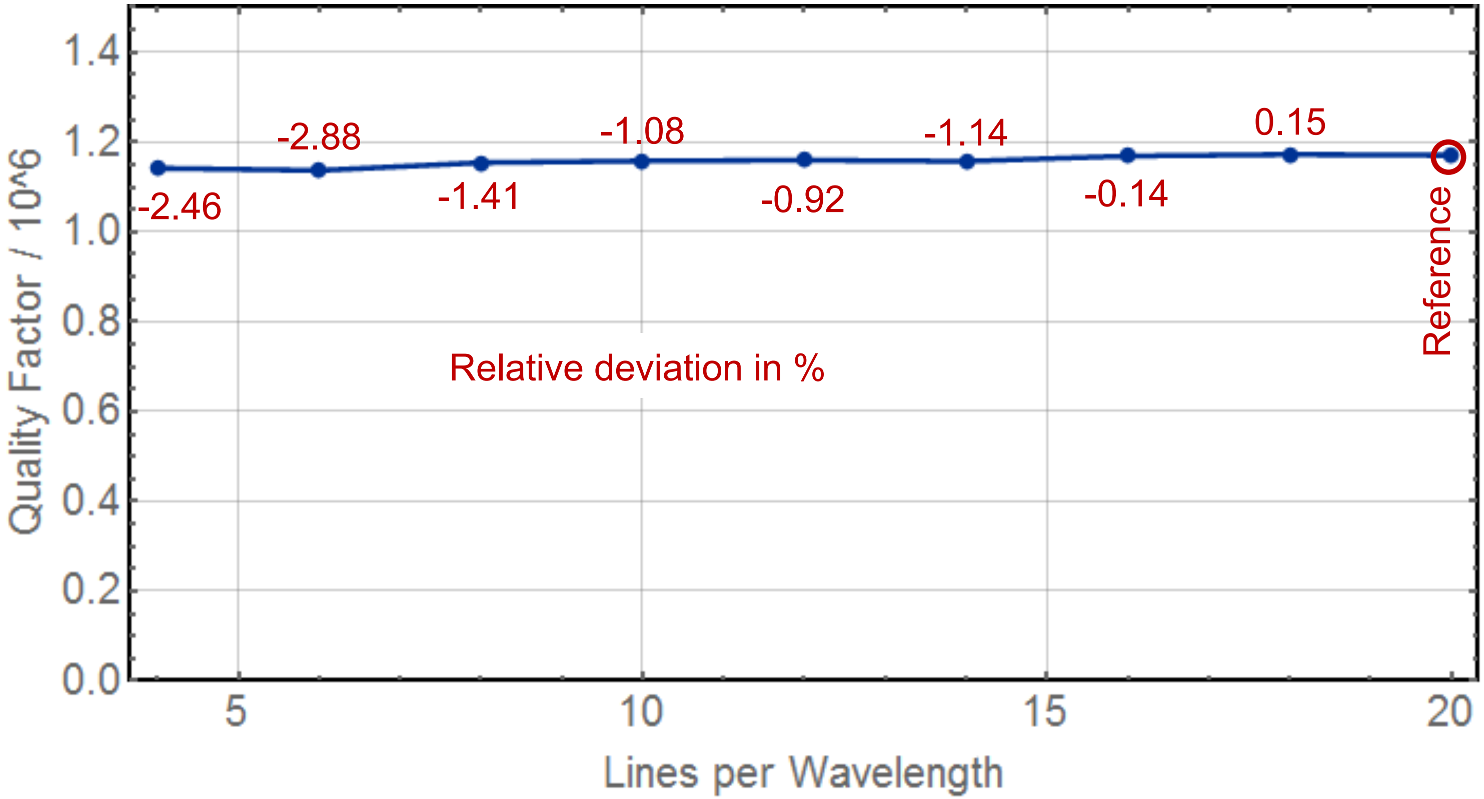
Simulation Results

- Convergence study for global quantities
  - Resonance frequency



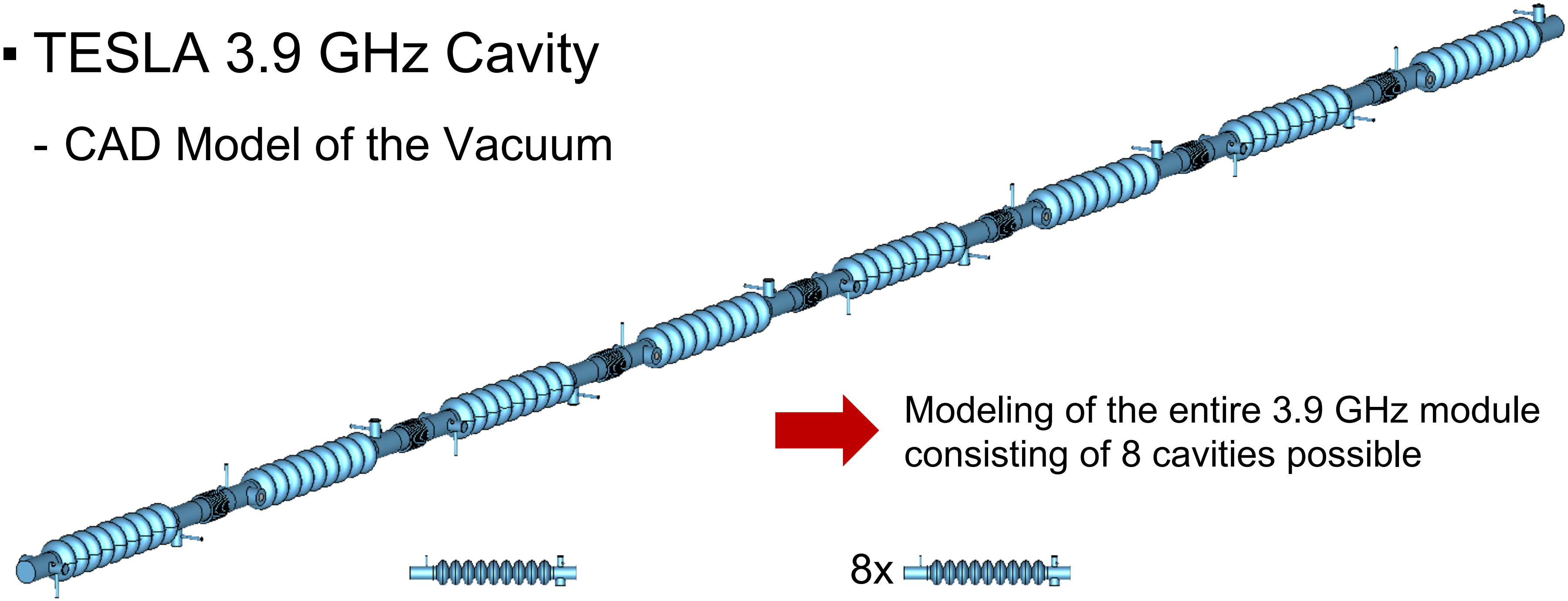
Simulation Results

- Convergence study for global quantities
  - Quality factor



Simulation Results

- TESLA 3.9 GHz Cavity
  - CAD Model of the Vacuum



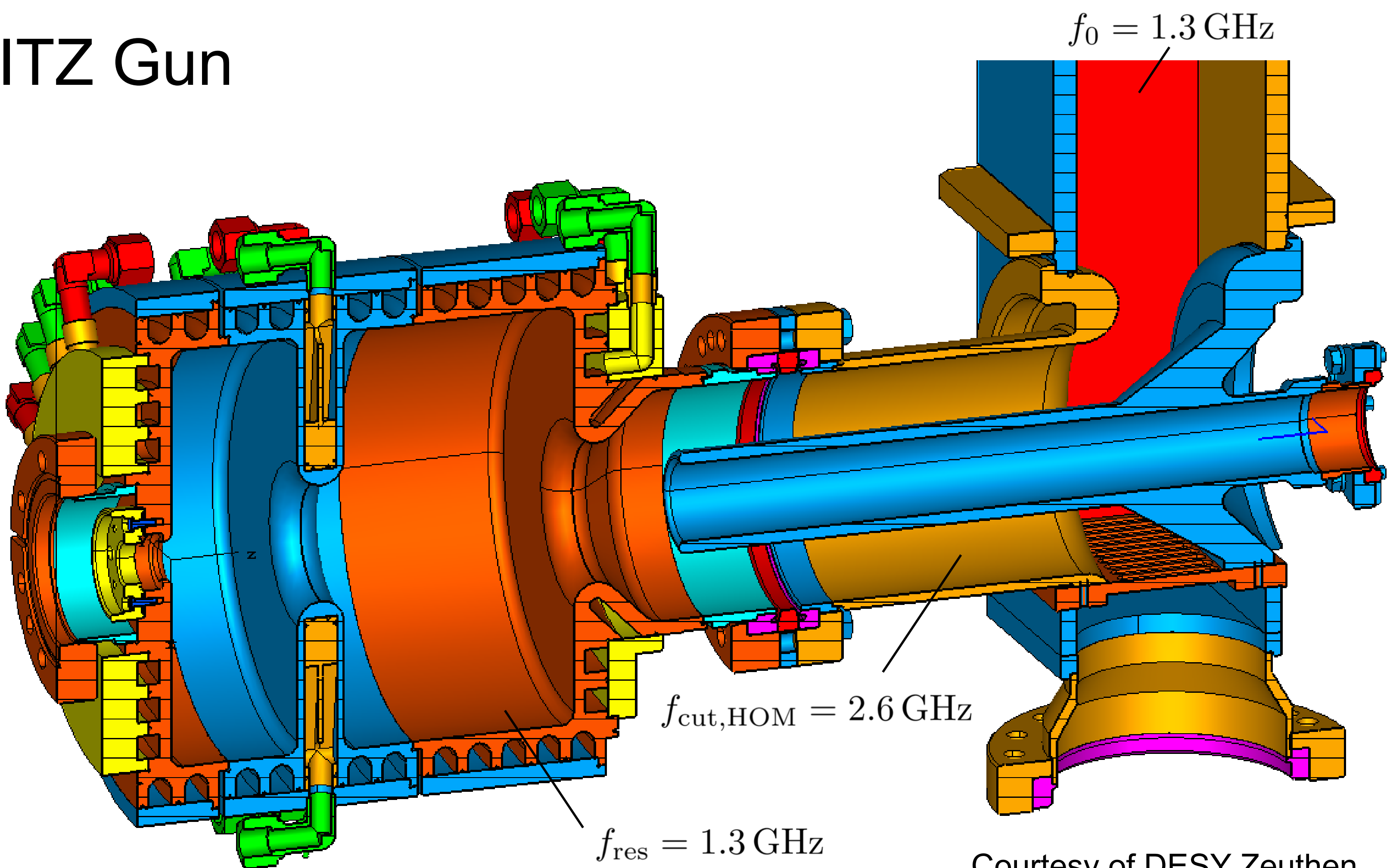
Modeling of the entire 3.9 GHz module consisting of 8 cavities possible

8x

LPW (9 GHz)	4	6	8	10	12	14	16	18	20
Tetrahedrons	136.443	187.435	304.833	480.376	767.271	1.177.883	1.704.528	2.432.978	3.337.736
Complex DOF	761.820	1.079.488	1.802.314	2.885.154	4.668.072	7.227.096	10.509.404	15.064.232	20.721.334

Motivation

- PITZ Gun



Courtesy of DESY Zeuthen

Computational Model

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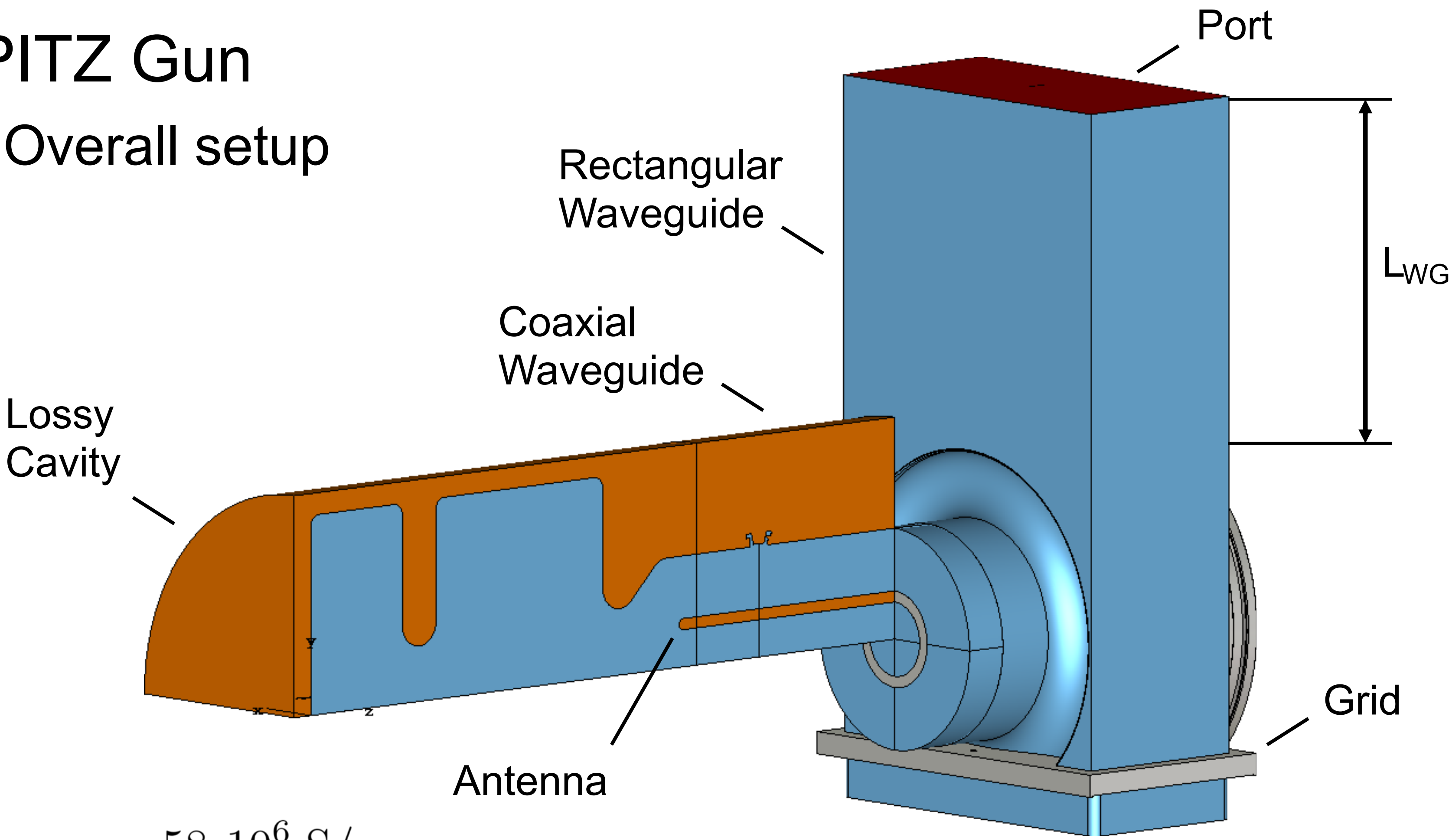
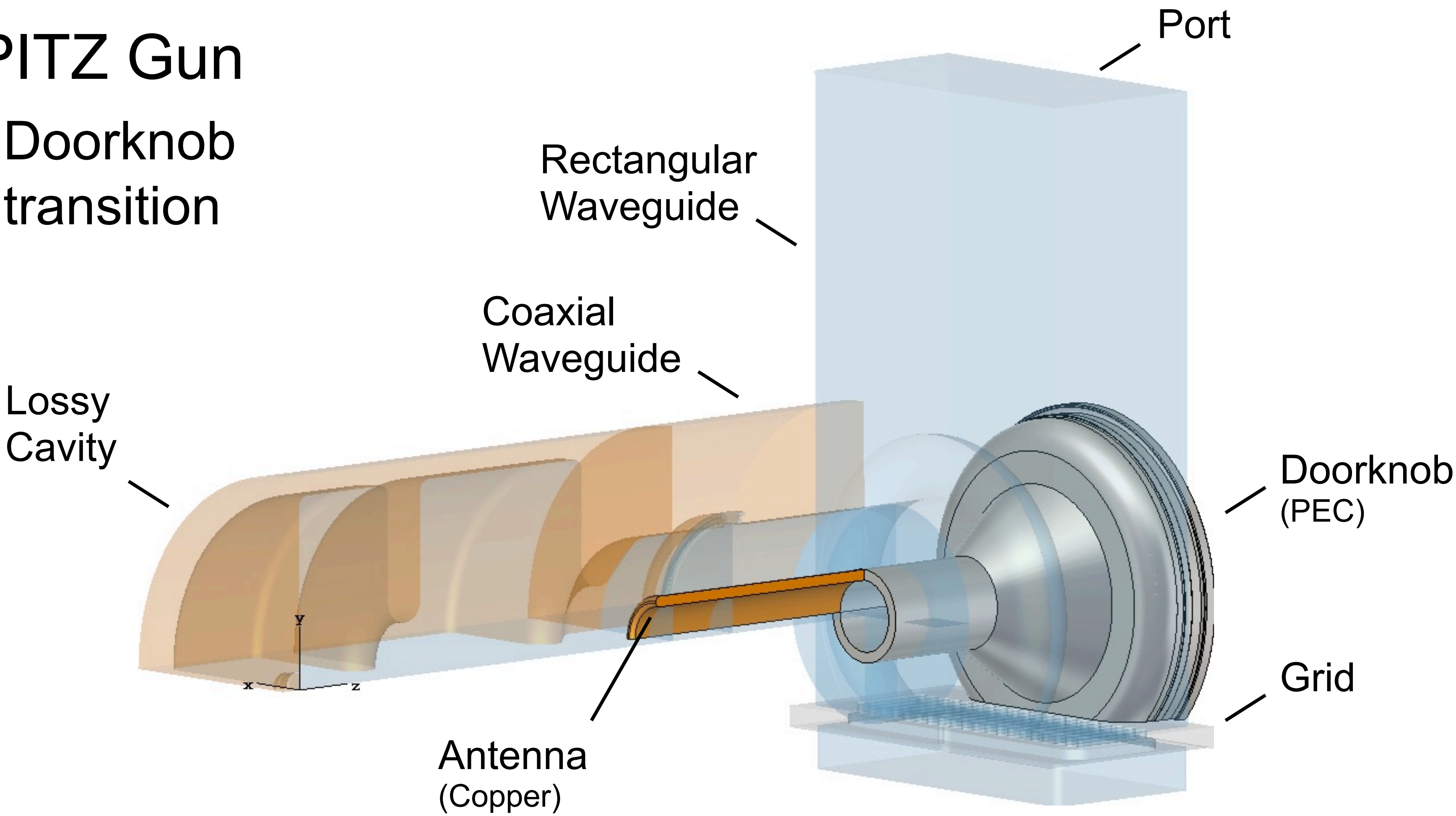
- PITZ Gun
  - Overall setup



$$\sigma_{\mathrm{Copper}} = 58 \cdot 10^6 \ \mathrm{S/m}$$

Computational Model

- PITZ Gun
  - Doorknob transition



Computational Model

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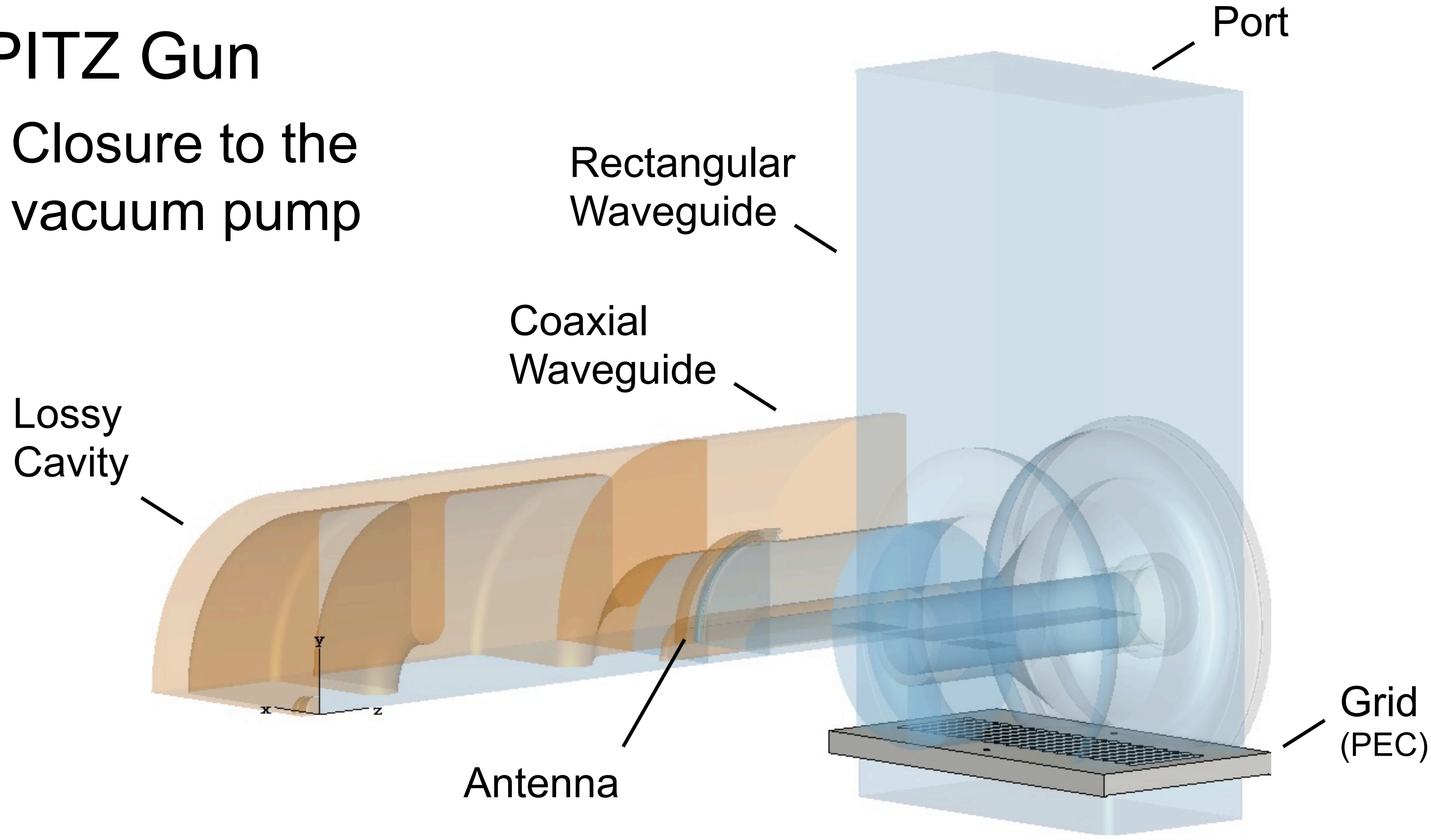
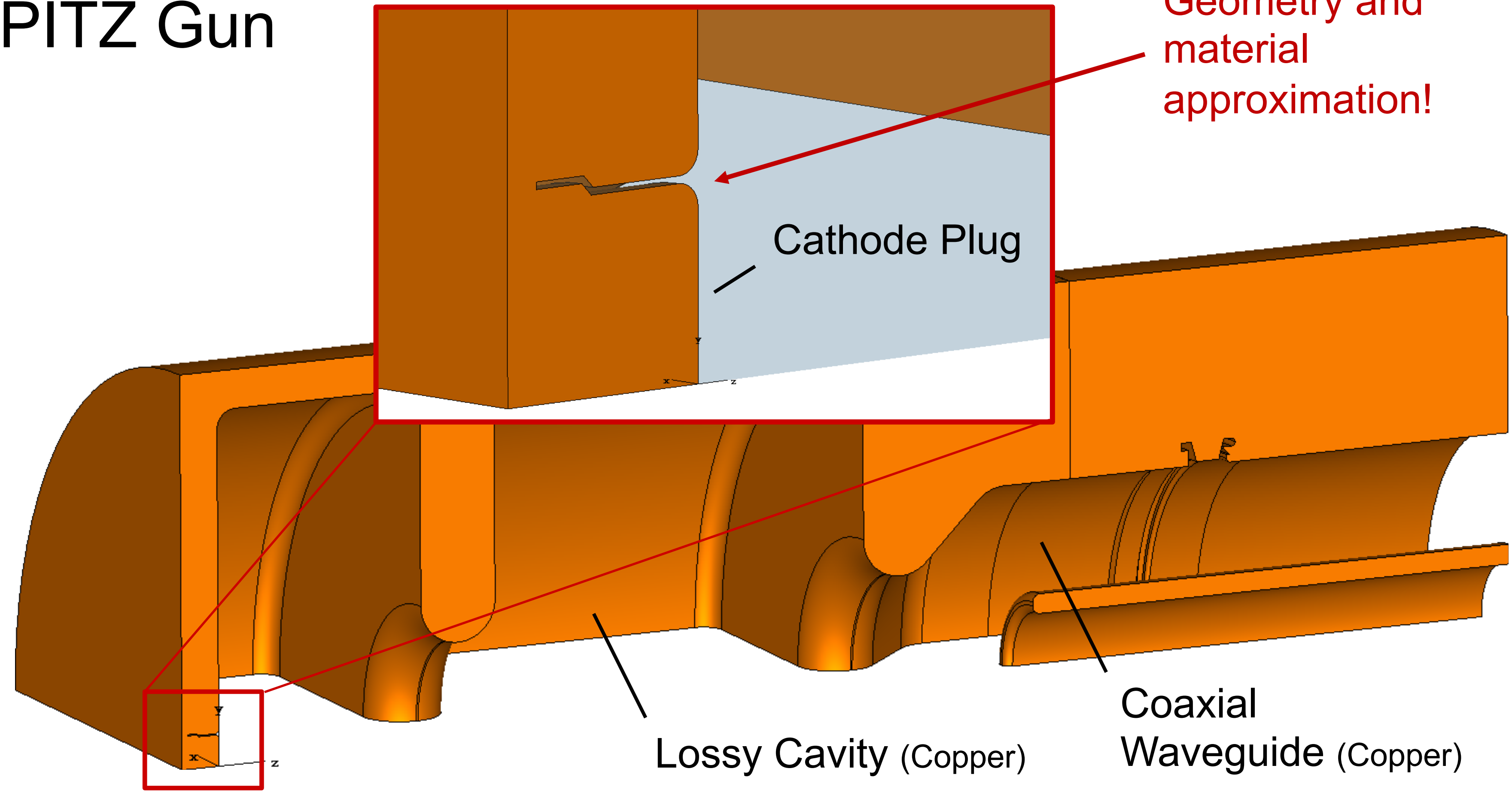
- PITZ Gun
  - Closure to the vacuum pump



Computational Model

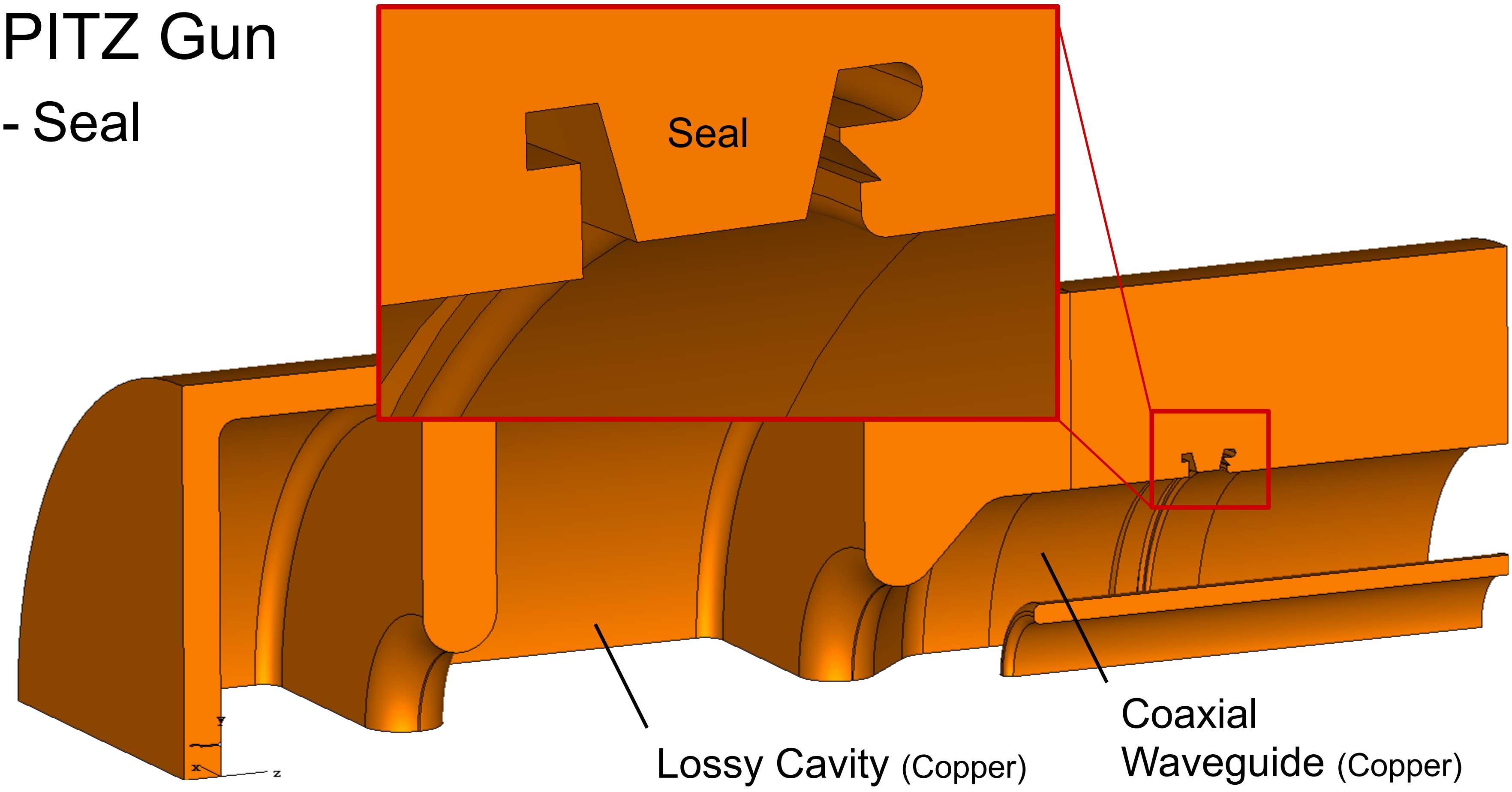
- PITZ Gun



M. Otevrel, „Report on Gun Conditioning Activities at PITZ in 2013“

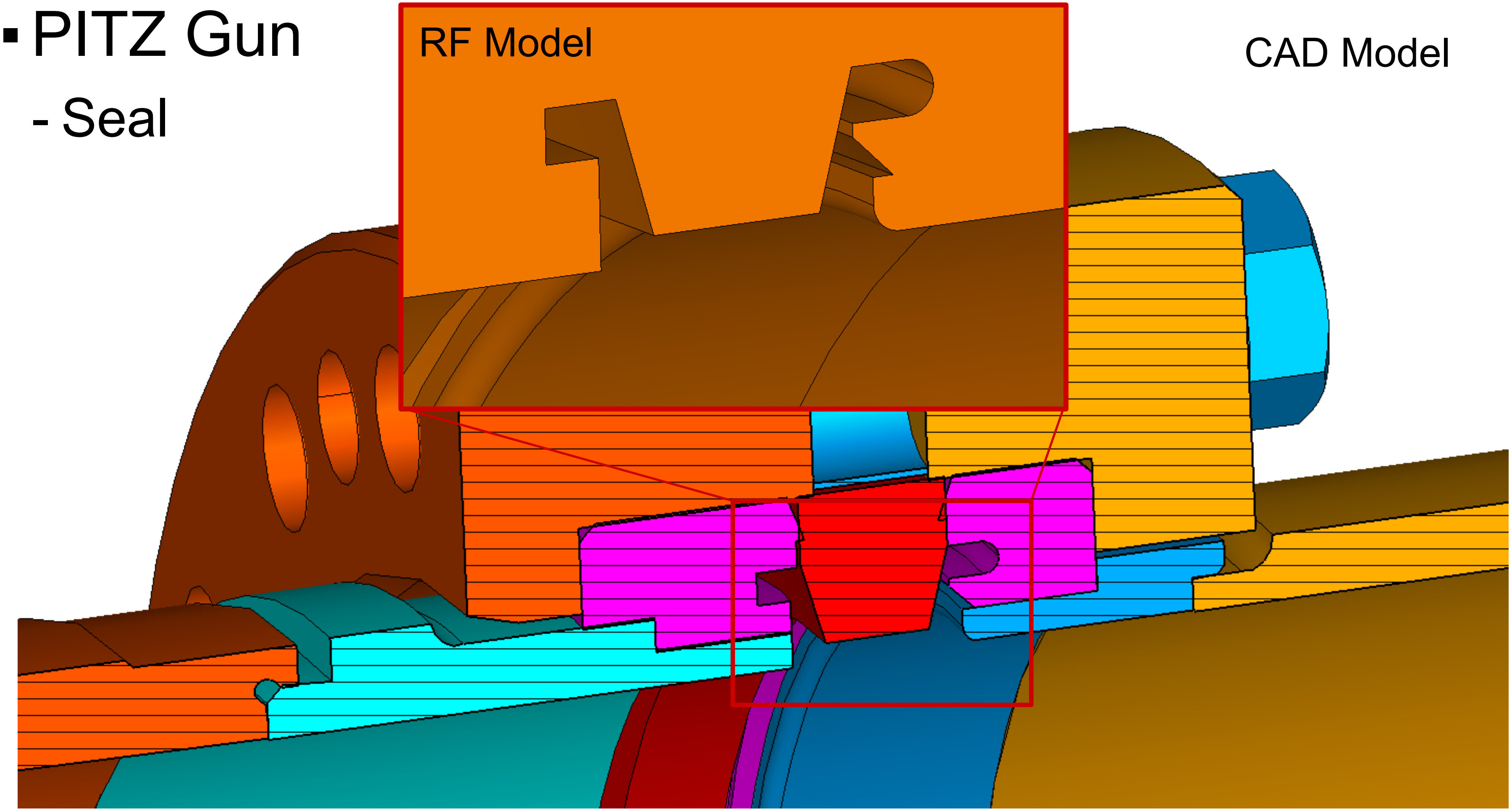
Computational Model

- PITZ Gun
  - Seal



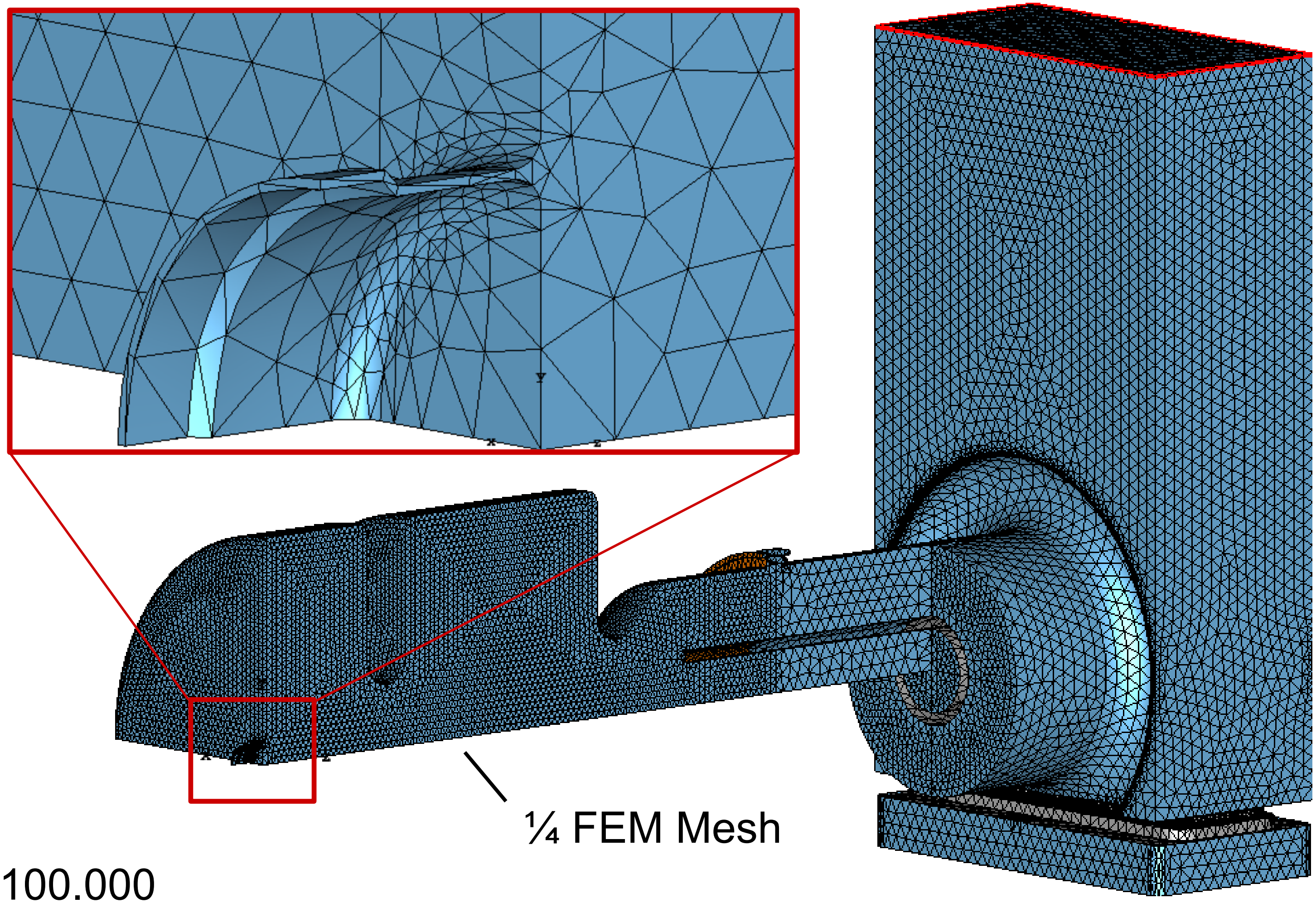
Computational Model

- PITZ Gun
  - Seal



Computational Model

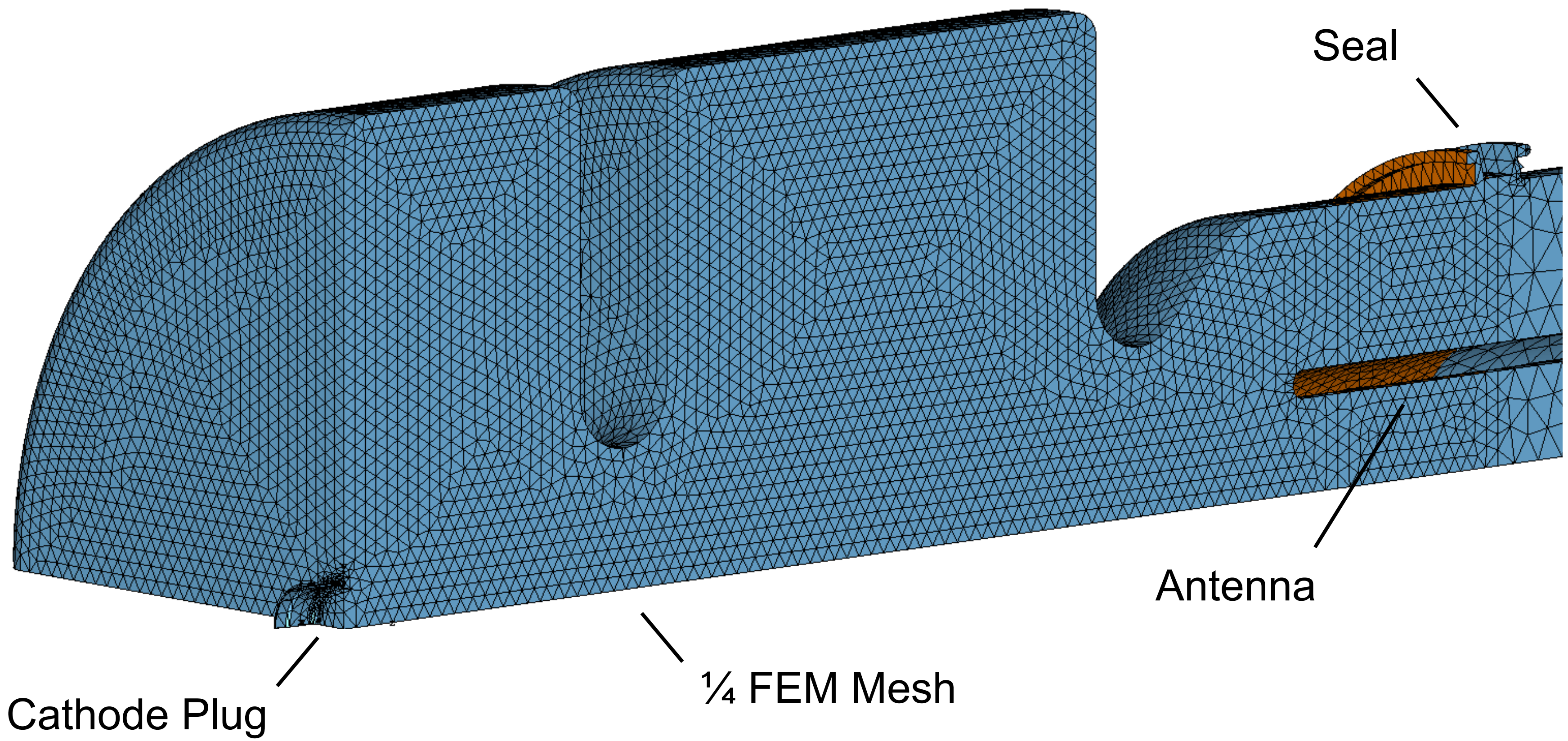
- Gun



$N_{tetra} \approx 2.100.000$

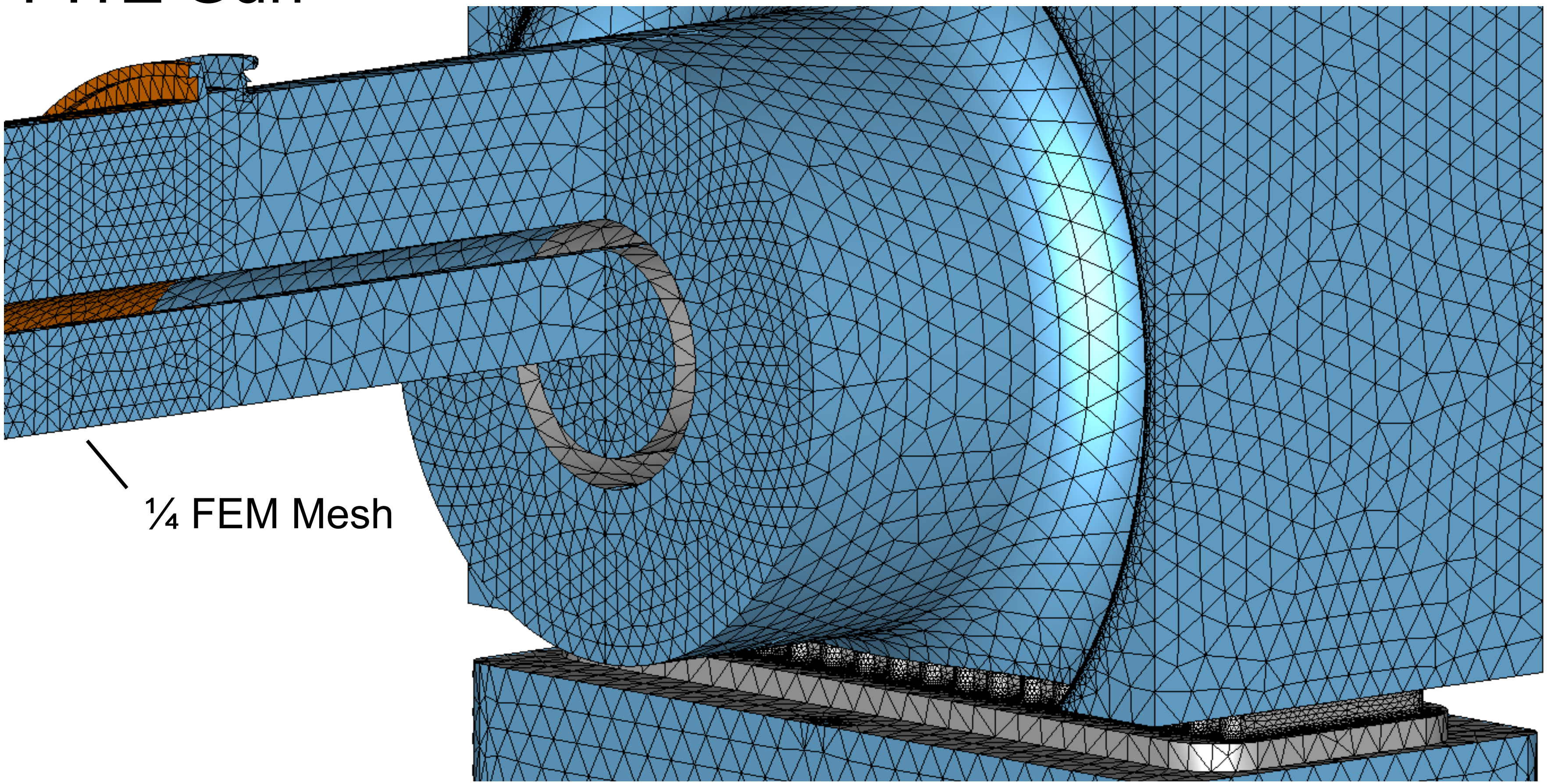
Computational Model

- PITZ Gun



Computational Model

- PITZ Gun

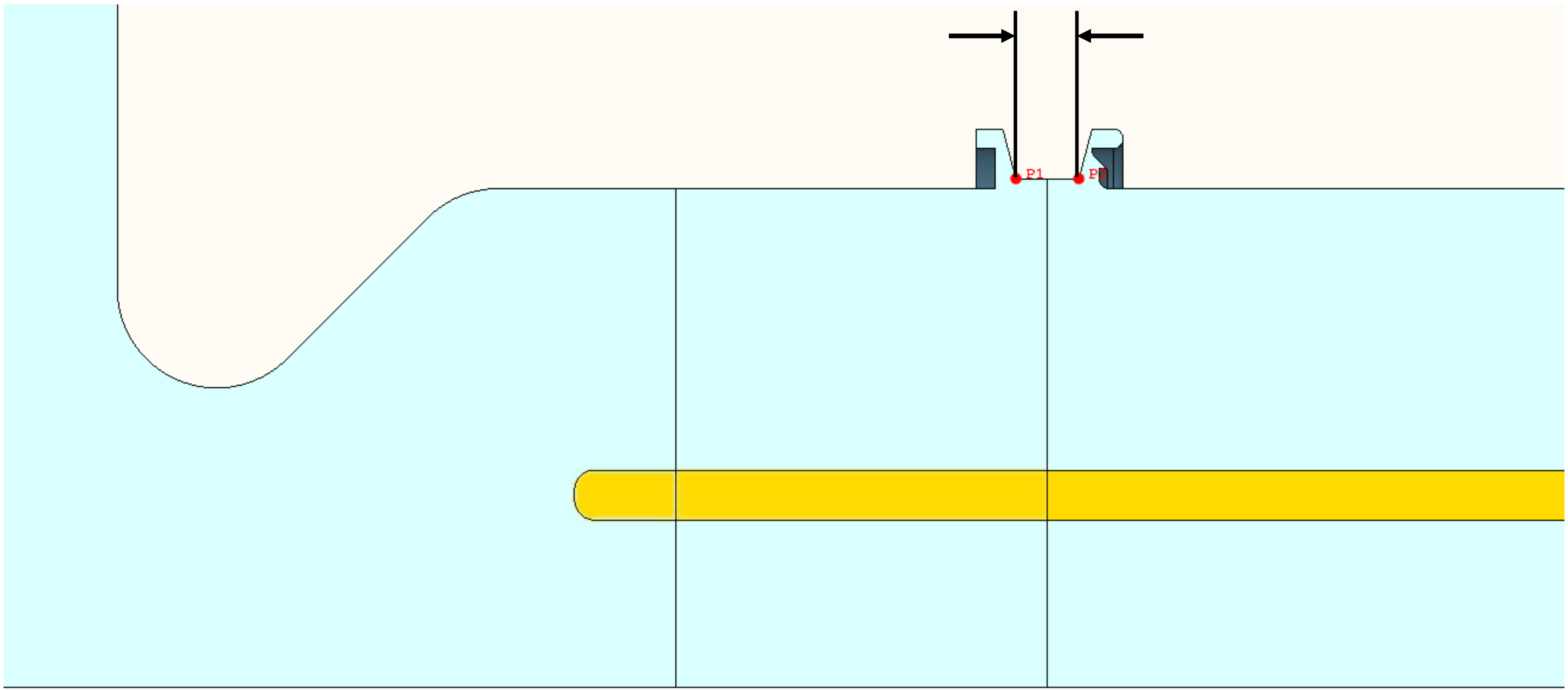


Simulation Results

- Antenna Tuning

Parameter: Seal Thickness

$$6.32\,\mathrm{mm} + L_{\mathrm{coax}}$$



Simulation Results

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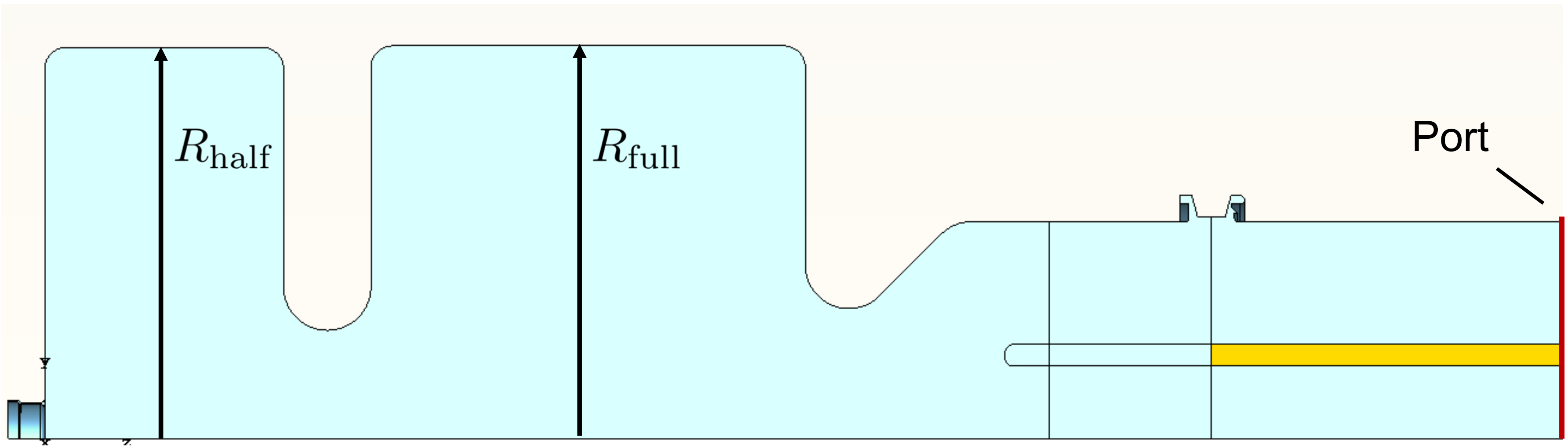
- Cavity Tuning
  - Rotationally symmetric model

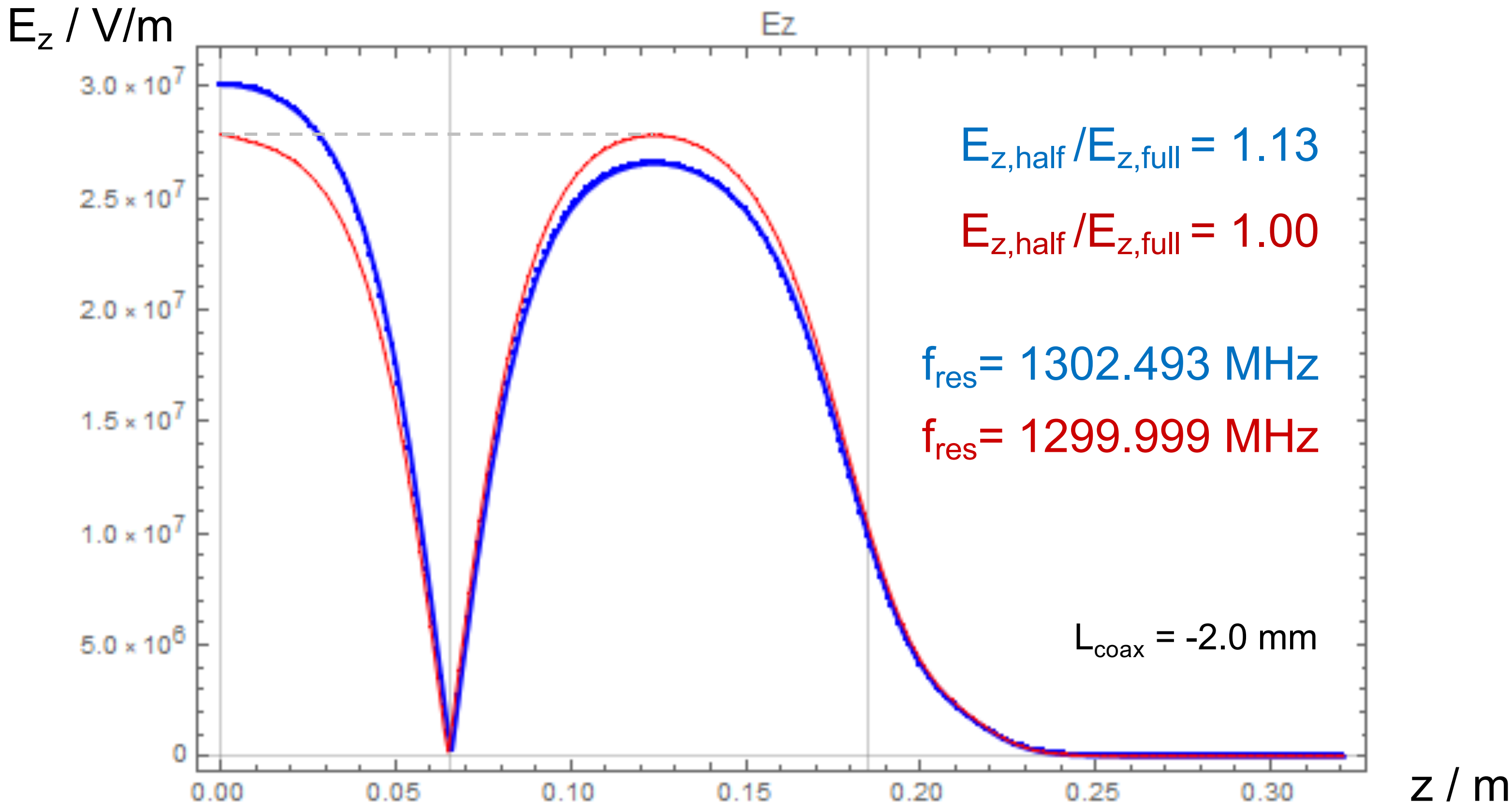


Simulation Results

- Cavity Tuning

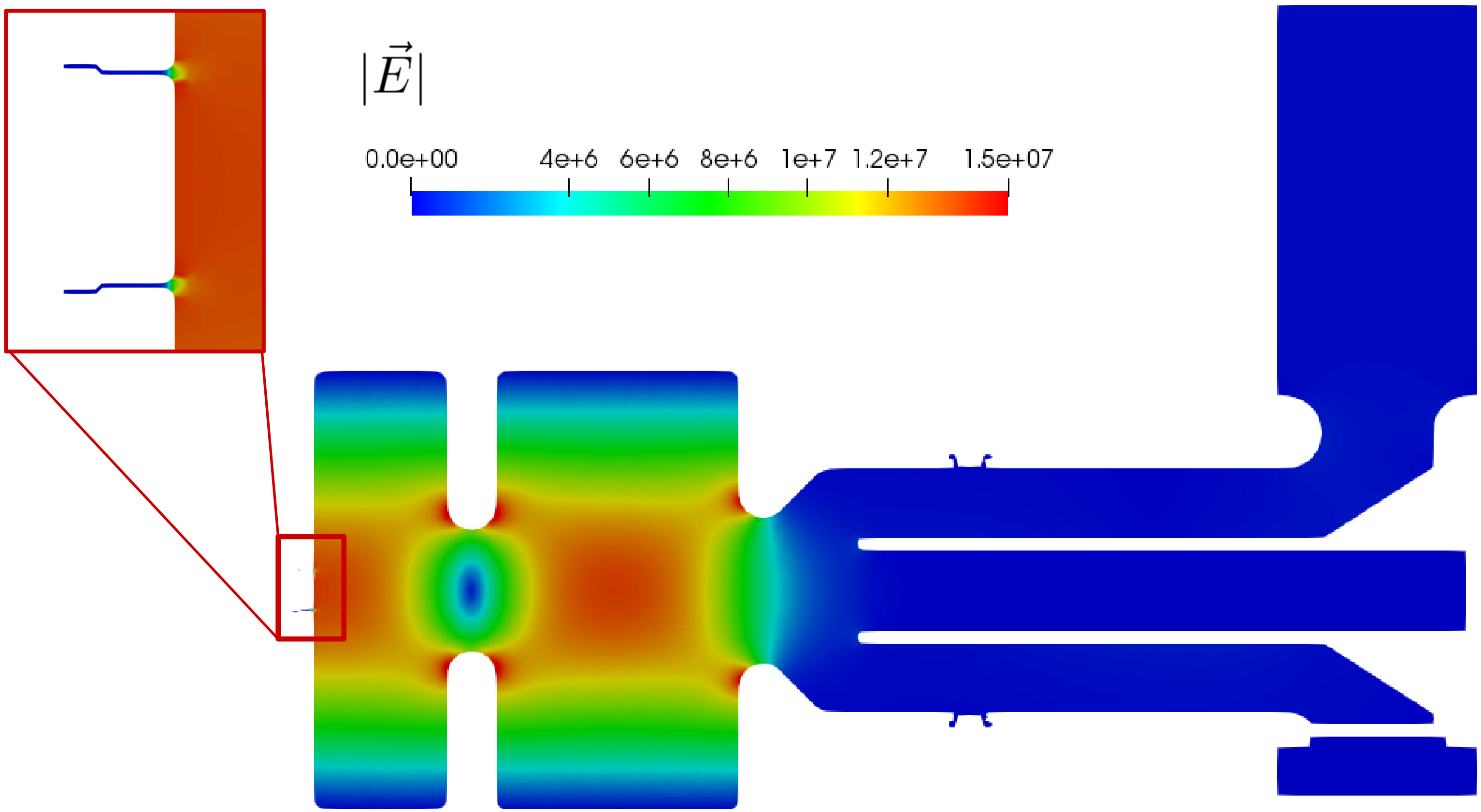
R_{half} = (89.95+0.234-0.048+0.012) mm = 90.148 mm

R_{full} = (90.32+0.172-0.021+0.002) mm = 90.473 mm



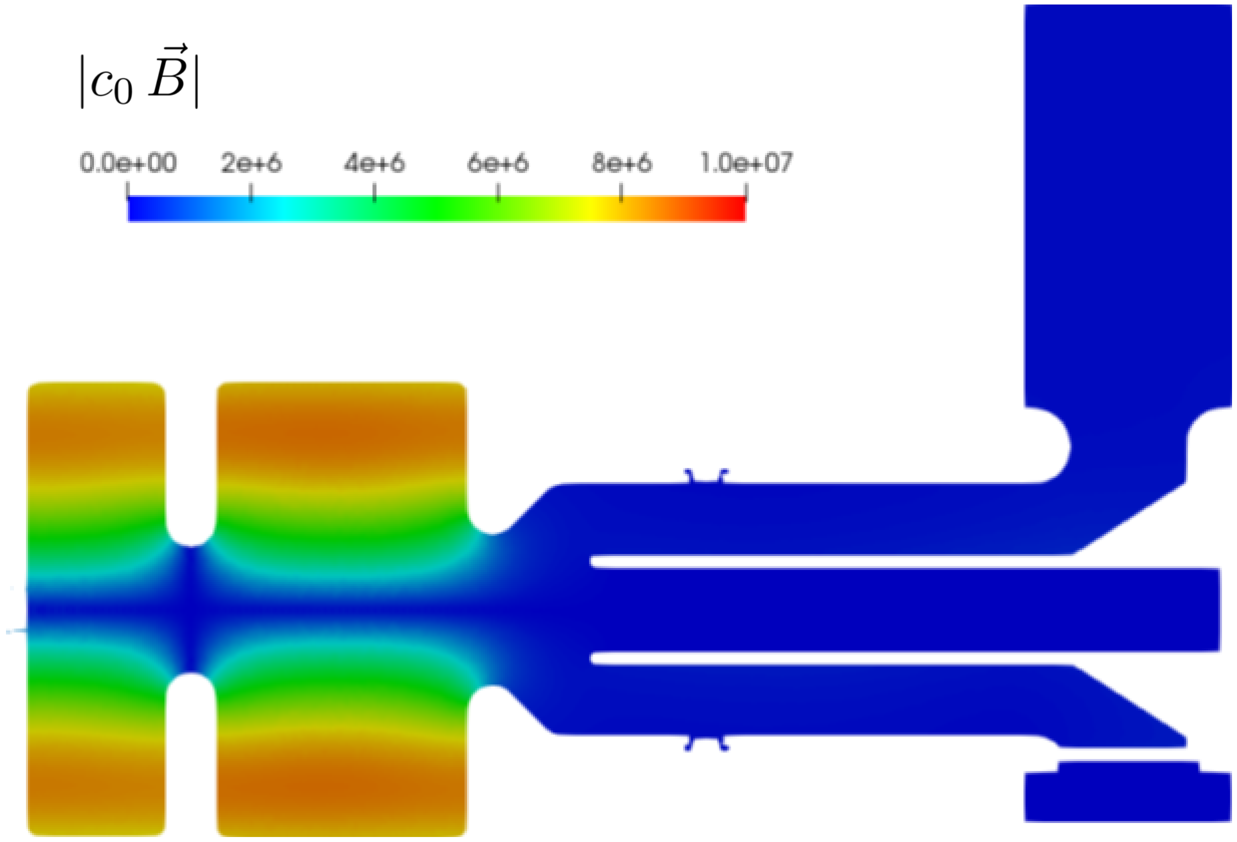
Simulation Results

- Electric Field Strength $|\vec{E}| = \sqrt{\vec{E} \cdot \vec{E}^*}$



Simulation Results

- Magnetic Flux Density $|\vec{B}| = \sqrt{\vec{B} \cdot \vec{B}^*}$

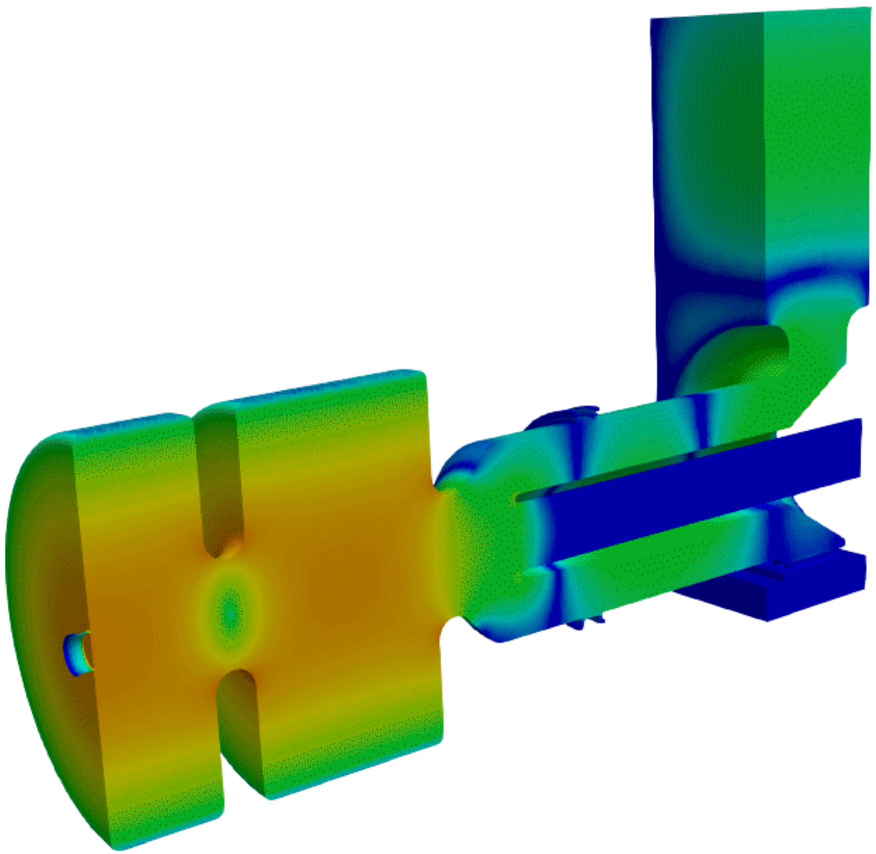


Simulation Results

TECHNISCHE UNIVERSITÄT DARMSTADT

- Electric Field Strength $\vec{E}(t) = \mathrm{Re}(\vec{E} \cdot e^{i\omega t})$

$\mathrm{Log}(|\vec{E}(t)|)$



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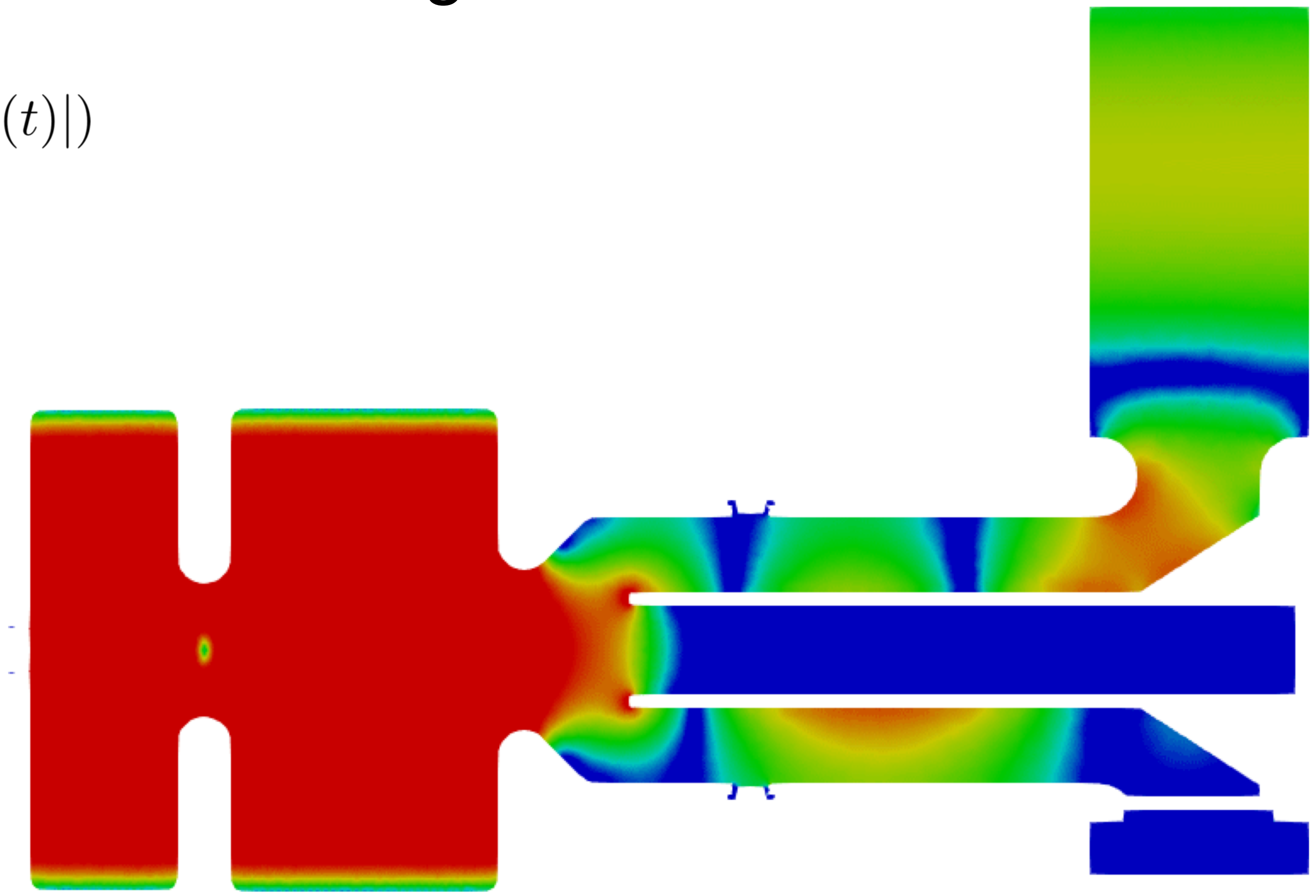

Simulation Results

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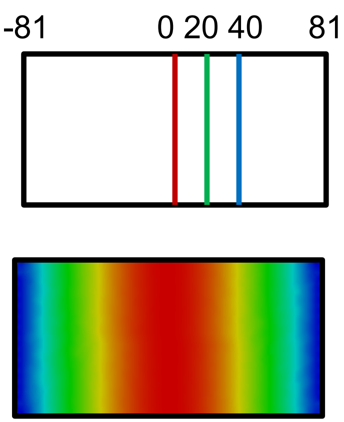
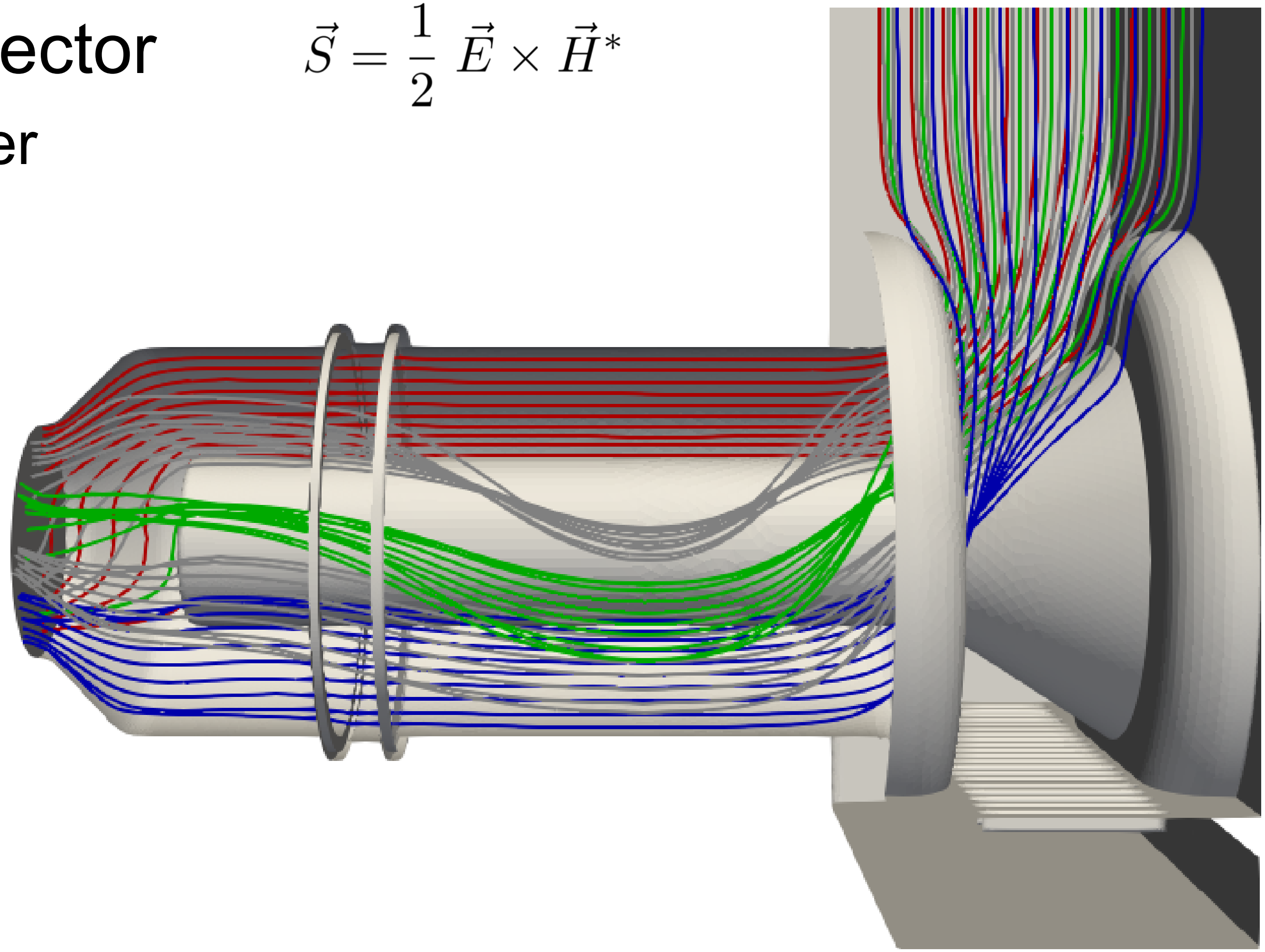
- Poynting Vector
  - Active power

$$\vec{S} = \frac{1}{2}\,\vec{E} \times \vec{H}^*$$

$\mathrm{Re}(\vec{S}\,)$





Simulation Results

TECHNISCHE UNIVERSITÄT DARMSTADT

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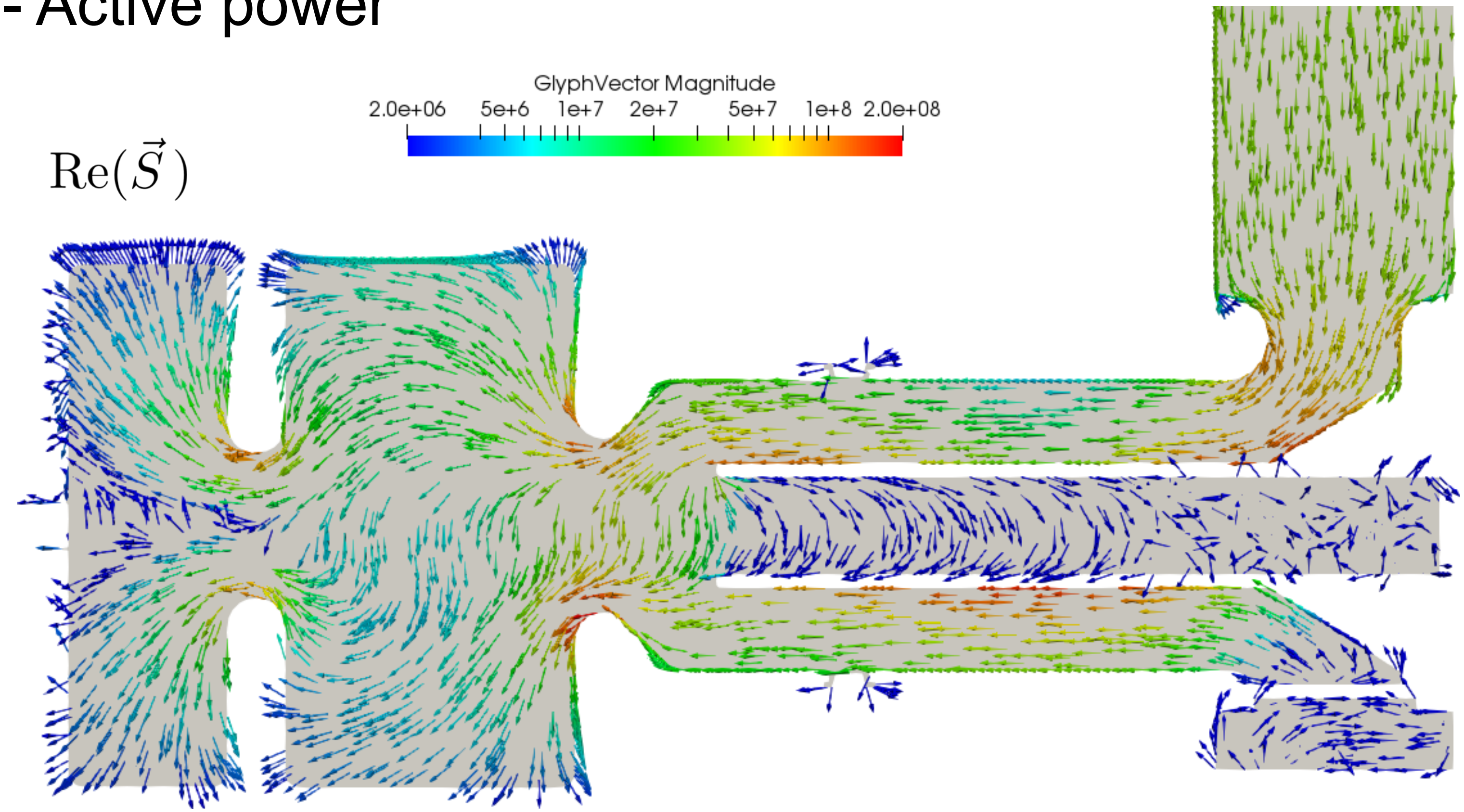
  - Active power

$\mathrm{Re}(\vec{S}\,)$

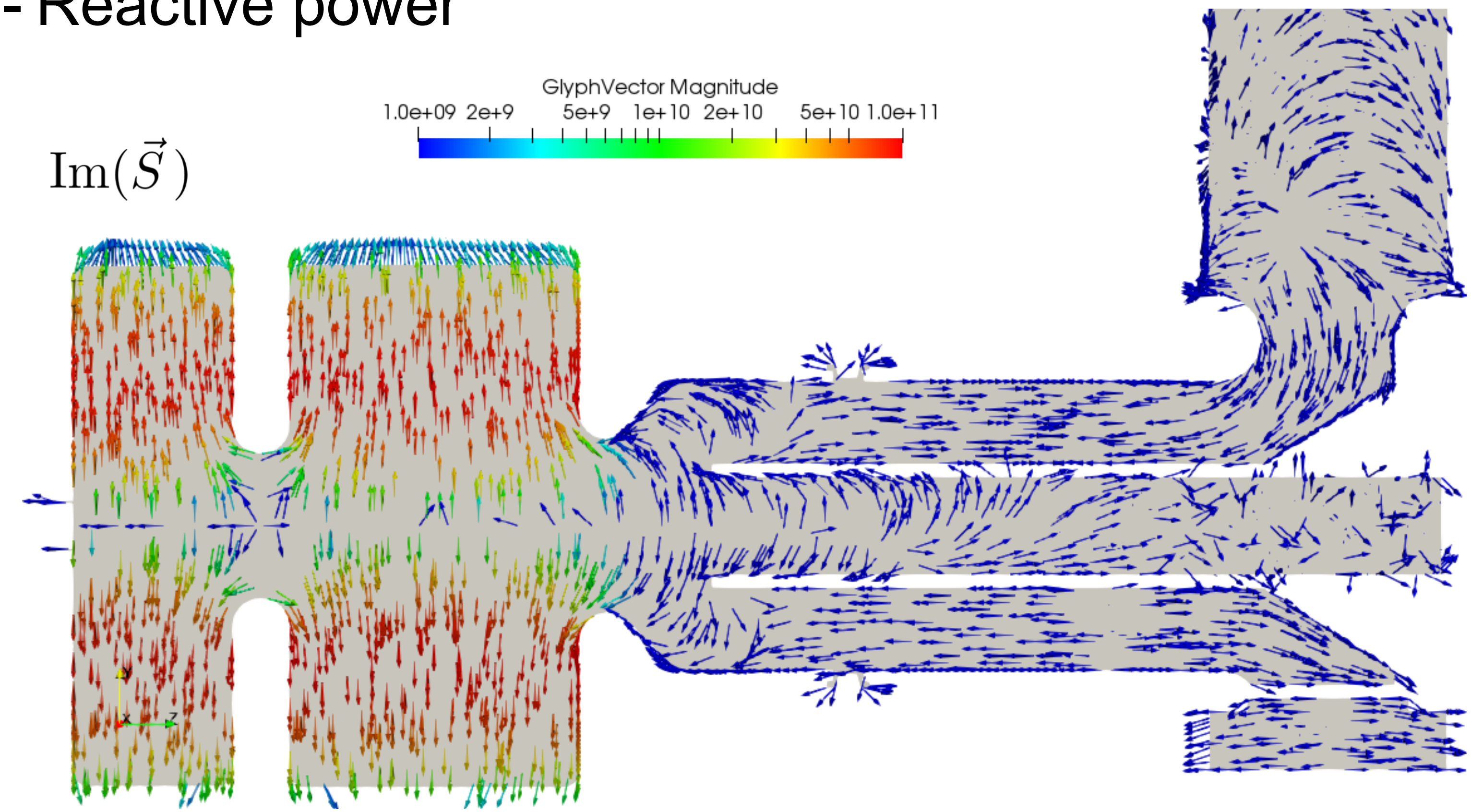


Simulation Results

TECHNISCHE UNIVERSITÄT DARMSTADT

- Poynting Vector $\vec{S} = \frac{1}{2}\,\vec{E} \times \vec{H}^*$
  - Reactive power

$\mathrm{Im}(\vec{S}\,)$



Outline

- Summary
  - High-Precision Lossy Eigenfield Analysis based on the FEM available
  - Lossy mechanism may include volume losses and surface losses due to materials as well as surface losses due to port boundary conditions
  - Any number of ports and any number of modes per port are possible
  - Two types of eigensolver applicable (JDM and CIM)

- Outlook
  - Merge JDM and CIM into a single code to efficiently take advantage of both methods