

Upgrade of MAD-X for HL-LHC project and FCC studies

ICAP'18

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Overview



- MAD-X = Methodical Accelerator Design 10
 - ➔ delivered on Linux, MacOSX and Windows (32 & 64 bits).
 - ➔ standalone all-in-one application (no dependency), open source, CERN copyright.
 - ➔ 3-4 releases / year, built and tested every night (~ 500 test suites).
 - ➔ support, website, e-groups, git repository, issue tracker (~600 tickets).
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- ◎ Single particle beam code.
 - ➔ motion of particles in 5D-6D phase space under external fields.
 - ➔ design, optics, tracking, matching, slicing, orbit steering, orbit correction, orbit measures, fields and alignment errors, aperture offsets, aperture margins, emittance equilibrium, frozen space charge, radiation, survey, plots, etc...

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- ◎ Scripting language.
 - ➔ sequences, commands, variables, macros, tables, files, and more...
 - ➔ special constructs: `if-else`, `while`, `macro`, `line`, `table`.
 - ➔ deferred expressions (`bx := sqrt(betx);`).
 - ➔ global workspace.



Overview (*cont.*)

- MAD-X is an all-in-one application for
 - ➔ machine design (reference @ CERN).
 - ➔ optics and tracking calculation.
 - ➔ validation, tolerances, margins, studies.
 - ➔ optimisations (e.g. from measurements).
 - ➔ legacy physics for large machines (e.g. LHC).
 - ➔ PTC for smaller machines, 6D (also 4D, 5D, 56D), complex topology, high orders.
 - ➔ front-end to Sixtrack (long term tracking, DA studies).
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- ◎ CERN has ~8 millions lines of MAD-X scripts.
 - ➔ +95% are clones or generated (history, variants, layout dump).
- ◎ Used world wide.
 - ➔ community is about 500 users.
 - ➔ used in many projects (often ignored by team).
 - ➔ colliders, boosters, storage rings, linacs, gantries, transfer lines, FFAG, racetracks, ...



Scripting language



- Weak string “parsing”, still good enough for complex macros (=input generator).

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```
sorttable(tblname, colname, coldir) : macro = { ! optimized shellsort
  sorttable.__g = 1 ; sorttable.count = 0 ;
  sorttable.__n = table(tblname, tablelength)+1 ;
  while (sorttable.__g <= sorttable.__n+1) {
    sorttable.__g = sorttable.__g * 3 + 1 ;
  }
  sorttable.__g = floor(sorttable.__g / 3) ;
  while ( sorttable.__g > 0 ) {
    sorttable.__i = sorttable.__g ;
    while (sorttable.__i < sorttable.__n) {
      setvars table = tblname, row = sorttable.__i ; sorttable.__t = colname*coldir ;
      fill table = tblname, row = sorttable.__n ;
      sorttable.__j = sorttable.__i - sorttable.__g ; sorttable.__f = 1 ;
      while (sorttable.__j > 0 && sorttable.__f > 0) {
        setvars table = tblname, row = sorttable.__j ; sorttable.__v = colname*coldir ;
        if (sorttable.__v < sorttable.__t) {
          sorttable.count = sorttable.count + 1 ;
          fill table = tblname, row = sorttable.__j + sorttable.__g ;
          sorttable.__j = sorttable.__j - sorttable.__g ;
        } else { sorttable.__f = 0 ; }
      }
      sorttable.count = sorttable.count + 1 ;
      setvars table = tblname, row = sorttable.__n ;
      fill table = tblname, row = sorttable.__j + sorttable.__g ;
      sorttable.__i = sorttable.__i + 1 ;
    }
    sorttable.__g = floor(sorttable.__g / 3) ;
  }
  shrink table = tblname ;
}
```



Functions (op), commands, factories (elem)



- ☞ Functions (and unary/binary operators) return a number
- ☞ unary operator: $(-x)$
- ☞ binary operators: $x+y$, $x-y$, $x*y$, x/y , x^y (power)
- ☞ relational operators: $x==y$, $x<=y$, $x>=y$, $x<y$, $x>y$, $x<>y$
- ☞ logical operators: $lexpr \ \&\& \ lexpr$ (and), $lexpr \ || \ lexpr$ (or)
- ☞ **abs**(x), **sqrt**(x), **exp**(x), **log**(x), **log10**(x)
- ☞ **sin**(x), **cos**(x), **tan**(x), **asin**(x), **acos**(x), **atan**(x),
- ☞ **sinh**(x), **cosh**(x), **tanh**(x)
- ☞ **erf**(x), **erfc**(x) (error functions)
- ☞ **round**(x), **floor**(x), **ceil**(x), **frac**(x) (integral and fractional parts)
- ☞ **ranf**(), **gauss**(), **tgauss**(x) (random number generators)

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➡ relational ➡ **exit**, **quit**, **stop** stop execution

➡ logical op ➡ **call** runs (load) scripts, **return** returns from current script

➡ **abs**(x), **sq** ➡ **exec** expands macros in current script (defined with **macro**)

➡ **sin**(x), **co** ➡ **system** runs shell commands (platform & shell specific)

➡ **sinh**(x), **c** ➡ **value**, **show**, **help** query global environment

➡ **erf**(x), **er** ➡ **option**, **title**, **set** setup global environment

➡ **round**(x), ➡ **beam** setups physics environment

➡ **ranf**(), **ga** ➡ **use**, **select** setup specific environment (commands, modules)

➡ **assign** sets output file for echoing

➡ **print**, **printf** print raw text and formatted text

➡ **copyfile**, **renamefile**, **removefile** portable files manipulation

➡ **plot**, **setplot**, **resplot** plotting facilities

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☞ **erf**(x), **er** ☞ **option**, ☞ **marker**: marker

☞ **round**(x), ☞ **beam se** ☞ **drift** : drift

☞ **ranf**(), **ga** ☞ **use**, **sel** ☞ **sbend**, **rbend**, **quadrupole**, **sextupole**, **octupole**, **multipole**, **solenoid**, **dipedge**, **rfmultipole**: magnets

☞ **assign s** ☞ **rfcavity**, **twcavity**, **crabcavity**, **rfmultipole**: cavities

☞ **print**, **p** ☞ **kicker**, **hkicker**, **vkicker**, **tkicker**: correctors

☞ **copyfile** ☞ **ecollimator**, **rcollimator**: collimators*

☞ **plot**, **se** ☞ **monitor**, **hmonitor**, **vmonitor**, **blmonitor**: monitors

☞ **instrument**, **placeholder**: placeholders

☞ **srotation**, **yrotation**: rotations

☞ **elseparator**, **beambeam**, **matrix**: others

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angle ($k0l$), $k1$, [$k2$]

sbend with $e1=e2=\theta/2$

$k1$

$k2$

$knl[]$, $ksl[]$, $lrad$

orbit correction

X,R,T



Survey, Track, Twiss, Match



- SURVEY: 3D geometrical tracking (global frame, design orbit)

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 - ➔ thin lens tracking (D-K-D), 6D closed orbit.
 - ➔ can track thick elements: ***dipole, quadrupole and solenoid.***
 - ➔ can track thousand particles (no hard limits, ***OMP parallelization***).
 - ➔ synchrotron radiation (***distribution and quantum***).
 - ➔ can (re-)evaluates time dependent expressions every turn ($\text{freq} = F_0/N$).

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- TWISS: 5D optical tracking (almost symplectic) up to 2nd order (i.e. X,R,T)
 - ➔ thick lens tracking, optical functions (Courant-Snyder), 4D-5D closed orbit, beta blocks.
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 - ➔ synchrotron radiation (**distribution**). NEW
 - MATCH: global non-linear optimiser
 - ➔ handle multiple sequences and beta blocks.
 - ➔ local/global constraints, ranges, equality or inequality.
 - ➔ direct (run TWISS) or indirect (run macro, PTC_TWISS).
 - ➔ can match at any positions inside elements. NEW

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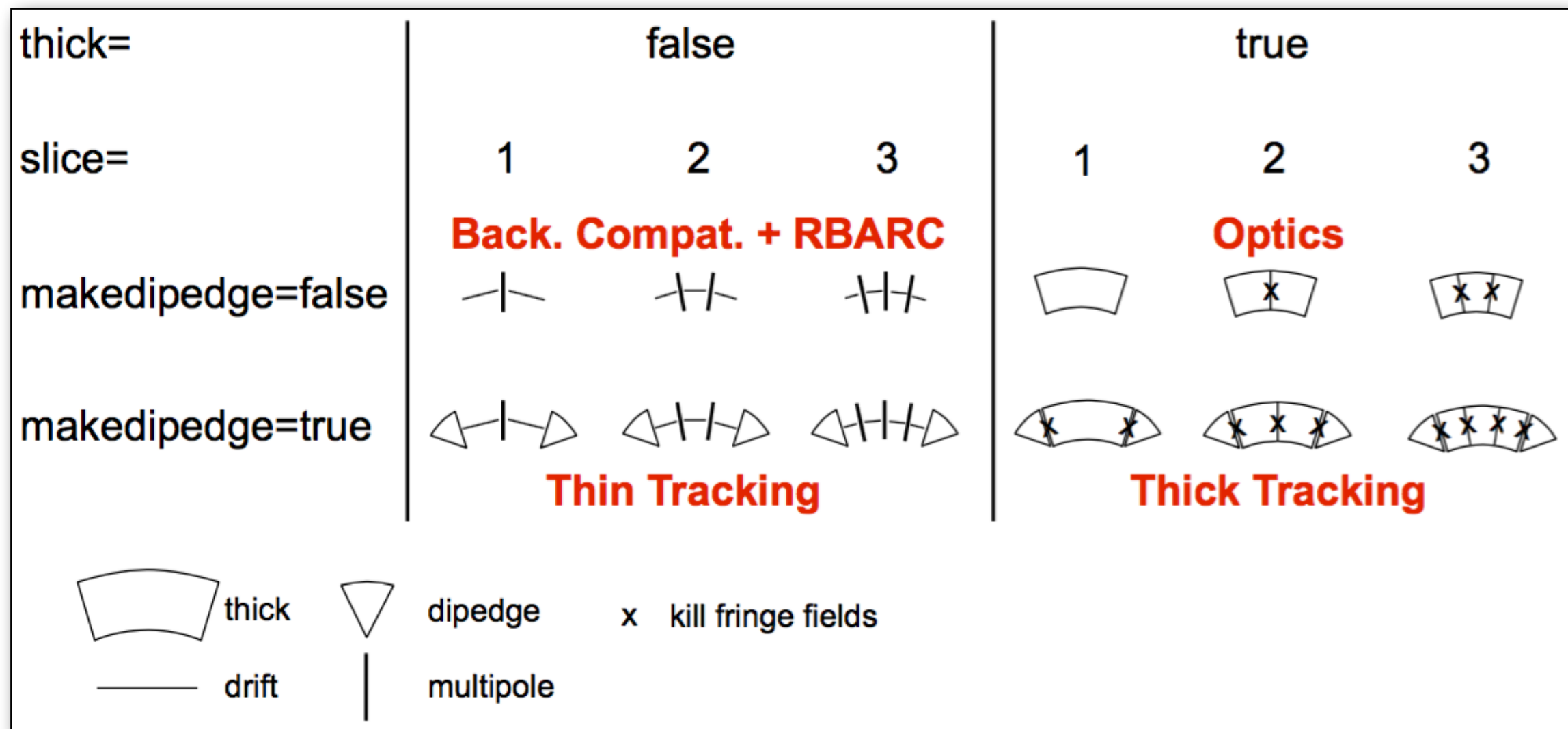
```
match use_macro, ... ;
... vary statements ...
macro1: macro = { ... madx statements ... }
constraint expr= "lhs1 < | = | > rhs1" ;
constraint expr= "lhs2 < | = | > rhs2" ;
... constraint statements ...
macro2: macro = { ... madx statements ... }
... constraint statements ...
macro3: macro = { ... madx statements ... }
... constraint statements ...
... methods statements ...
endmatch ;
```



Makethin improvement

- Convert thick sequence into thin sequence (in place).
 - ➔ new module (2015-) in C++ by H. Burkhardt.
 - ➔ generalise TEAPOT symplectic integration scheme for $n > 4$ slices to minimise beta beating using better interpolation (i.e. than SIMPLE) of thick quadrupole.
 - ➔ can keep dipole, quadrupole and solenoid thick.

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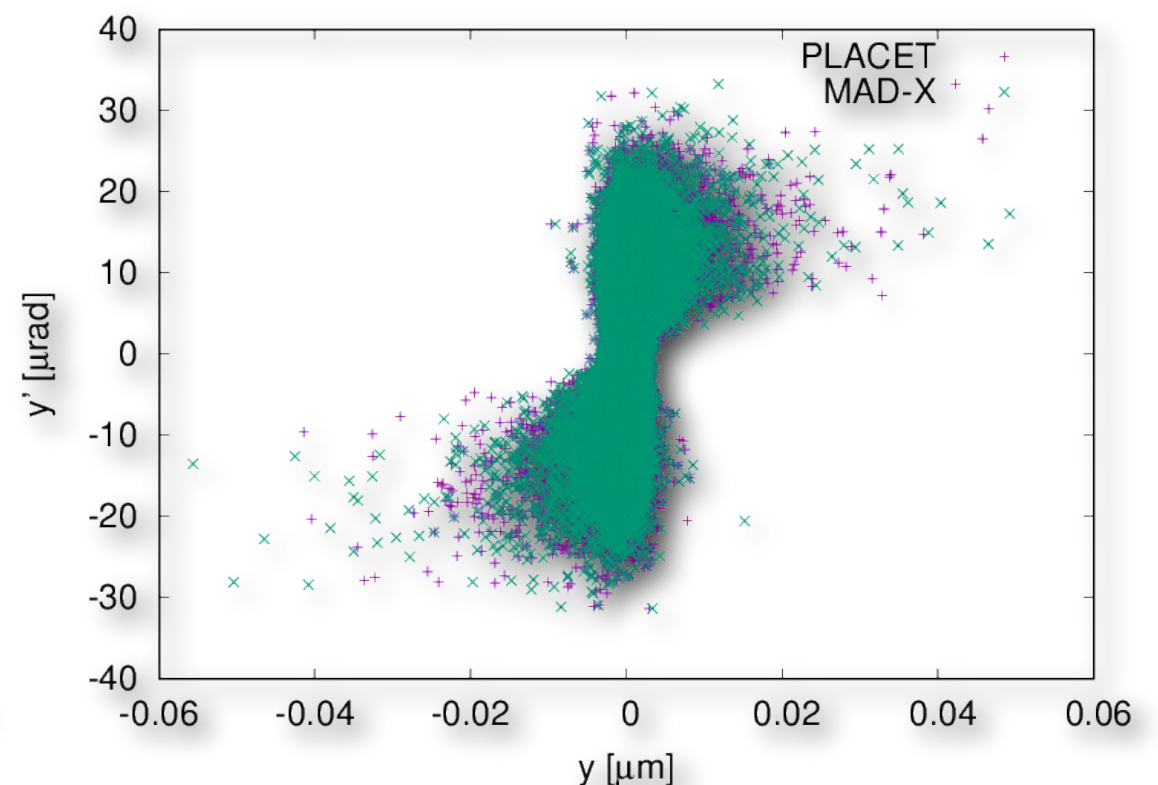
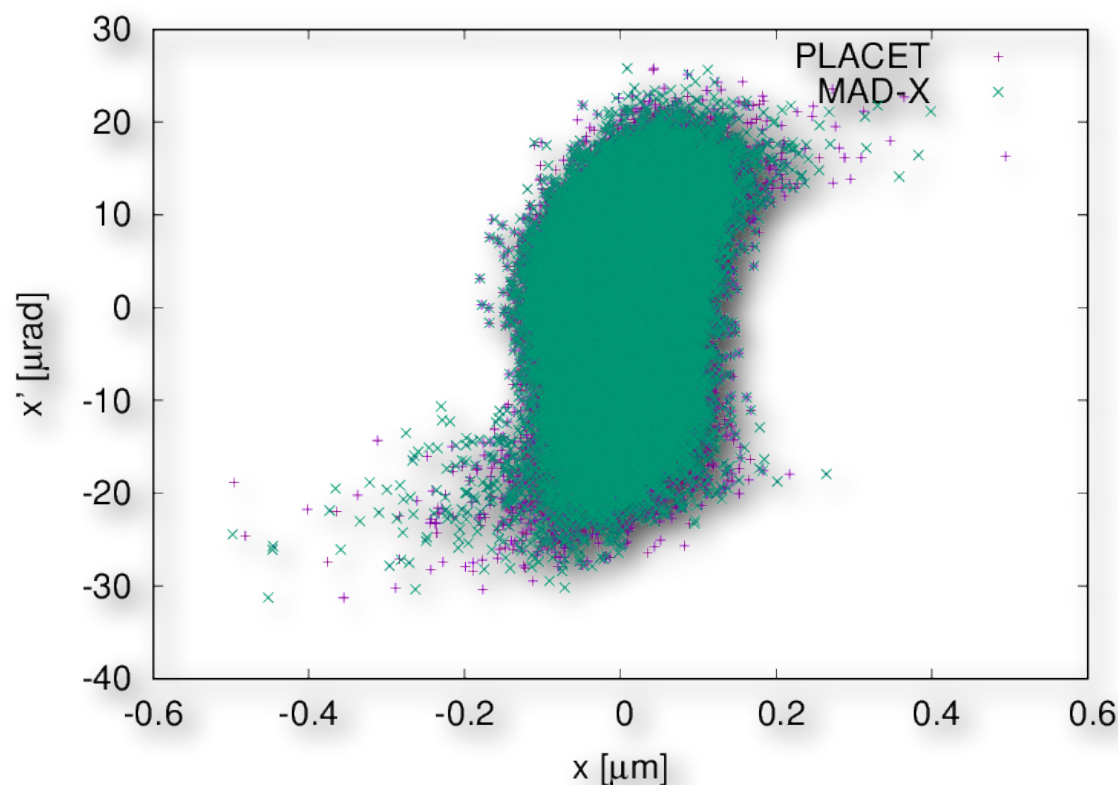


Synchrotron radiation effects

- Fixed and improved in Twiss, Track and Emit modules.
 - ➔ originally implemented in MAD8 for high-energy e^+e^- collider (LEP) at CERN.

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- ◎ 4 levels of activation
 - ➔ *no radiation*, corresponding to the usual Hamiltonian dynamics.
 - ➔ *deterministic radiation*, all particles radiate as a single particle on the closed-orbit.
 - ➔ *deterministic radiation*, with full dependence on canonical coordinates to generate natural radiation damping (Track and Emit only).
 Preferred method for dynamic aperture calculations in high-energy lepton rings.
 - ➔ *Individual quantum excitation* with stochastic photon emissions, provides particle distributions and equilibrium emittances (Track only).

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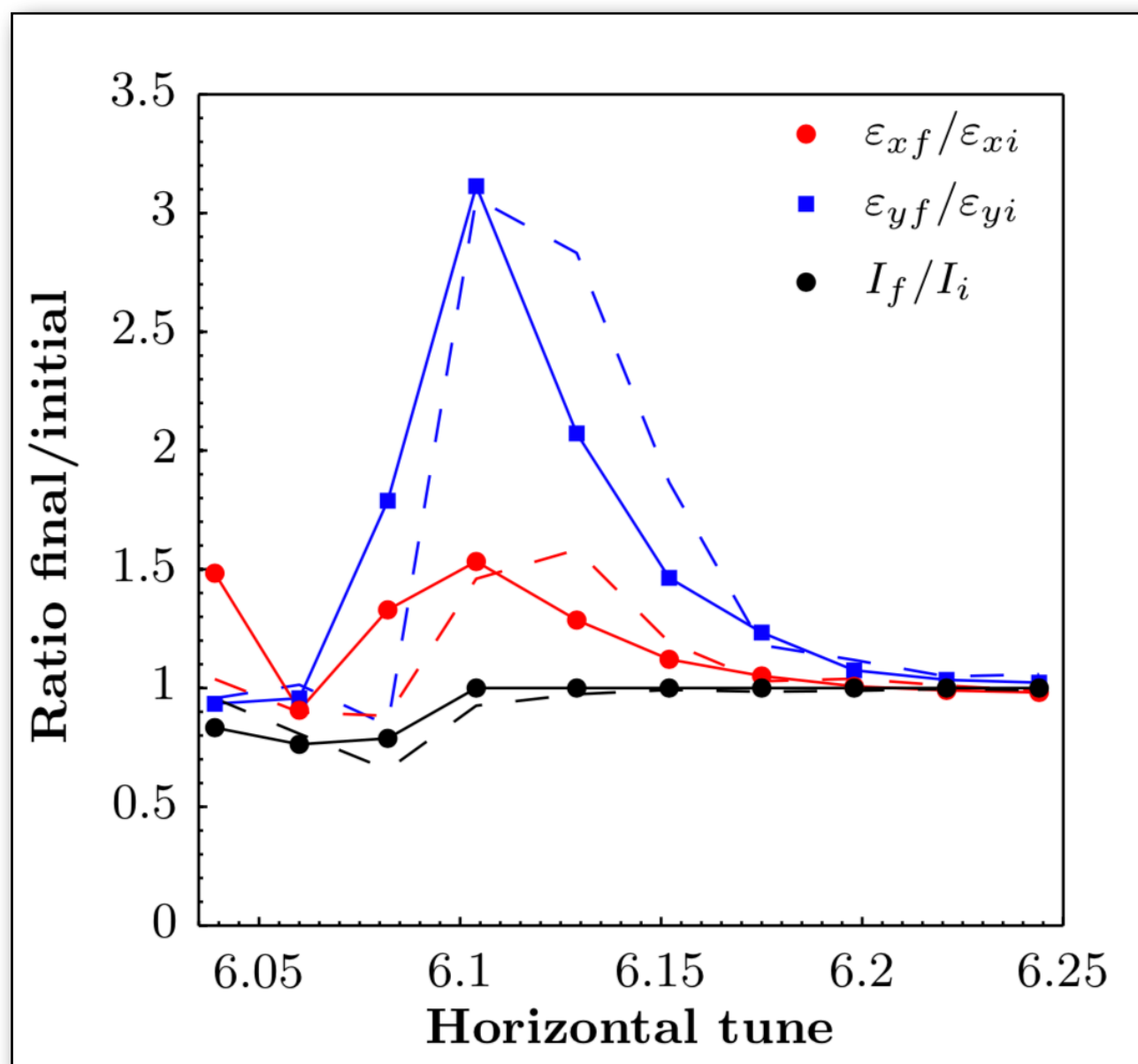




Frozen space charge

- ◎ The code has undergone a complete refactoring
 - ➔ new separate Fortran module.
 - ➔ simplify support and extension without changing the model.
 - ➔ could become a new separate MAD-X command in the future.

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Emittance growth and beam intensity as computed with MAD-X (adaptive mode). Experimental data are shown with dashed lines.

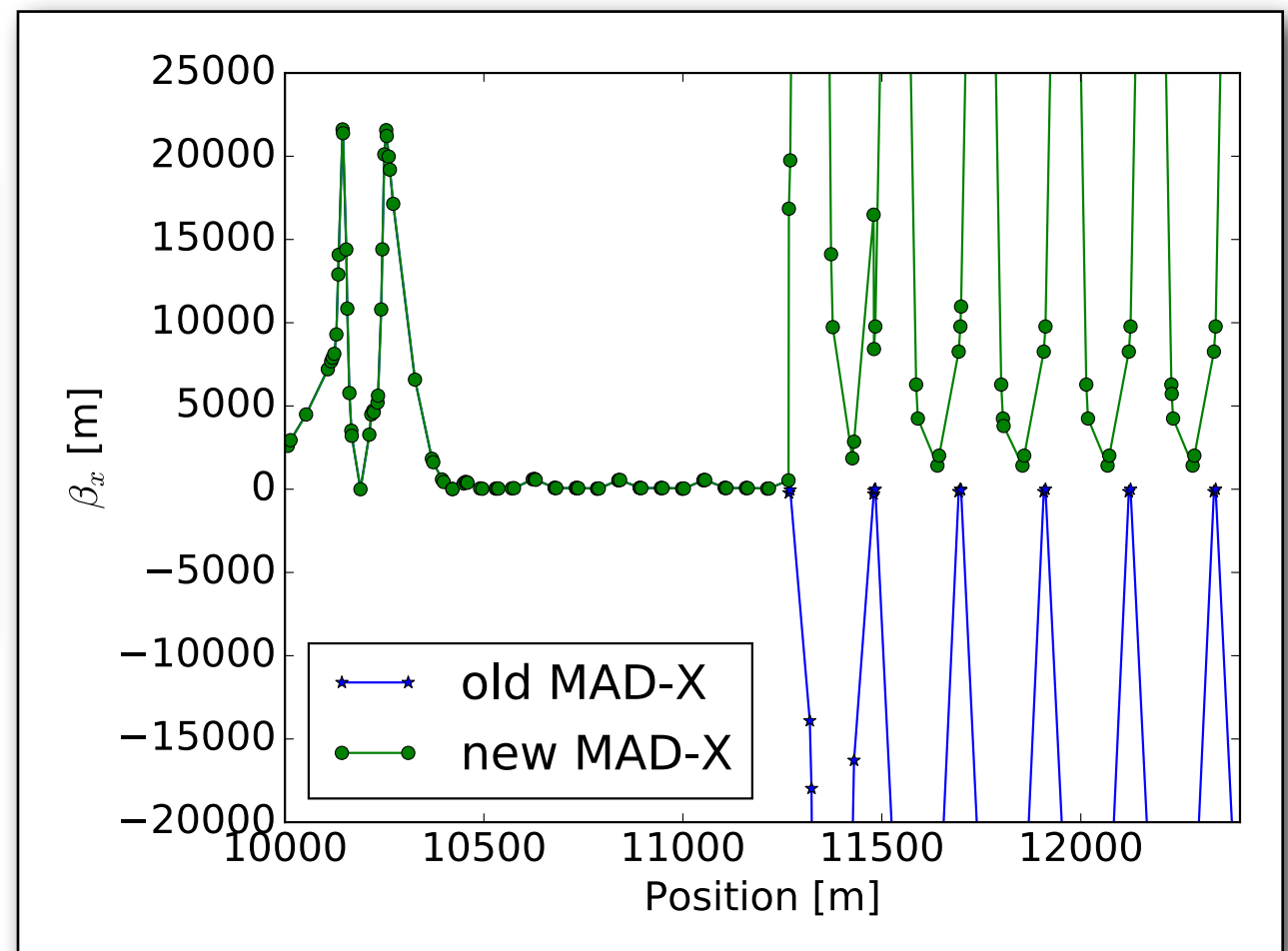


Linear coupling review

- ⦿ Review triggered by negative beta functions in the presence of strong coupling
 - ➔ occurred for a single seed leading to strong misalignment in HL-LHC studies.
 - ➔ tried two alternate implementations without success (Sagan & Rubin, Talman).
 - ➔ added the flip mode to the implementation (fixed the problem).
 - ➔ added a couple missing checks to validate assumptions.
 - ➔ review the theory and the implementation from scratch (lost knowledge).
 - ➔ review fixed typos in the manual and restored the understanding of the method.

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Negative (blue) beta functions in HL-LHC studies, right after the skew quadrupole starting from IP (left). Coupling is so strong that optics is not stable. Fixed in recent released (green).





Linear coupling review (initialisation)

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$$\vec{X}(s_2) = M \vec{X}(s_1),$$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = R_M \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix} R_M^{-1} = R_M M_{\perp} R_M^{-1}.$$

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$M =$

$$\text{eig}(M) = \text{eig}(M^{-1}) = \{(\lambda_i, \lambda_i^{-1}), i = 1..2\},$$

$$\text{eig}(M + \bar{M}) = \{\Lambda_i = \lambda_i + \lambda_i^{-1}, i = 1..2\},$$

$$\det(M + \bar{M} - \Lambda I) = (\text{tr } A - \Lambda)(\text{tr } D - \Lambda) - |C + \bar{B}| = 0,$$

$$\Lambda_{A,D} = \frac{1}{2}(\text{tr } A + \text{tr } D) \pm \frac{1}{2} \text{sign}(\text{tr } A - \text{tr } D) \sqrt{\Delta}.$$

Eigenmodes

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\det

$$\begin{pmatrix} (\text{tr } A - \Lambda_A)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_A)I \end{pmatrix} \begin{pmatrix} X \\ R_A X \end{pmatrix} = 0,$$

$$\begin{pmatrix} (\text{tr } A - \Lambda_D)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_D)I \end{pmatrix} \begin{pmatrix} R_D Y \\ Y \end{pmatrix} = 0,$$

$$R = - \left(\frac{1}{2}(\text{tr } A - \text{tr } D) + \frac{1}{2} \text{sign}(\text{tr } A - \text{tr } D) \sqrt{\Delta} \right)^{-1} (C + \bar{B}),$$

$$\text{with } R_A = -\bar{R}_D = R.$$

Eigenmodes

Eigenvectors

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<i>Eigenmodes</i>	$\det \begin{pmatrix} (\text{tr } A - \Lambda_A)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_A)I \end{pmatrix} \begin{pmatrix} X \\ R_A X \end{pmatrix} = 0,$ $\begin{pmatrix} (\text{tr } A - \Lambda_D)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_D)I \end{pmatrix} \begin{pmatrix} R_D Y \\ Y \end{pmatrix} = 0,$
<i>Eigenvectors</i>	$R = -$ $M_{\perp} = R_M^{-1} M R_M = g^2 \bar{R}_M M R_M$ $= g^2 \begin{pmatrix} I & -\bar{R} \\ R & I \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I & \bar{R} \\ -R & I \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix},$
<i>Normal form</i>	$E = g^2 (A - \bar{R}C - (BR - \bar{R}DR)) = A - \bar{R}C = A - BR,$ $F = g^2 (D + RB + (C\bar{R} + RA\bar{R})) = D + RB = D + C\bar{R}.$

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Eigenmodes

$$\vec{X}(s_2) = M \vec{X}(s_1),$$

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Eigenvectors

$$R = - \begin{matrix} M_{\perp} = R_M^{-1} M R_M = g^2 \bar{R}_M M R_M \\ = g^2 \begin{pmatrix} I & -\bar{R} \\ R & I \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I & \bar{R} \\ -R & I \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix}, \end{matrix}$$

Normal form

$$\begin{matrix} E = \\ F = \end{matrix} E = \begin{pmatrix} E_{1,1} & E_{1,2} \\ E_{2,1} & E_{2,2} \end{pmatrix} = \begin{pmatrix} \cos \mu_A + \alpha_A \sin \mu_A & \beta_A \sin \mu_A \\ -\gamma_A \sin \mu_A & \cos \mu_A - \alpha_A \sin \mu_A \end{pmatrix},$$

Optical parameters

$$\cos \mu_A = \frac{1}{2} \text{tr } E, \quad \sin \mu_A = \text{sign}(E_{1,2}) \sqrt{-E_{1,2} E_{2,1} - \left(\frac{E_{1,1} - E_{2,2}}{2} \right)^2}$$

$$\beta_A = \frac{E_{1,2}}{\sin \mu_A}, \quad \gamma_A = -\frac{E_{2,1}}{\sin \mu_A}, \quad \alpha_A = \frac{E_{1,1} - E_{2,2}}{2 \sin \mu_A},$$



Linear coupling review (propagation)

- Restore the derivation of the equations, provide intermediate validity checks and more stable formula (see the links in refs for details).

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$$\begin{aligned}
 M_2 &= M_{12} M_1 M_{12}^{-1} = M_{12} (R_{M_1} M_{1\perp} R_{M_1}^{-1}) M_{12}^{-1}, \\
 M_{2\perp} &= R_{M_2}^{-1} M_2 R_{M_2} = (R_{M_2}^{-1} M_{12} R_{M_1}) M_{1\perp} (R_{M_1}^{-1} M_{12}^{-1} R_{M_2}) \\
 &= W_{12} M_{1\perp} W_{12}^{-1}.
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$$R_{M_2} W_{12} = M_{12} R_{M_1},$$

Normal form

$$g_2 \begin{pmatrix} I & \bar{R}_2 \\ -R_2 & I \end{pmatrix} \begin{pmatrix} E_{12} & 0 \\ 0 & F_{12} \end{pmatrix} = g_1 \begin{pmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{pmatrix} \begin{pmatrix} I & \bar{R}_1 \\ -R_1 & I \end{pmatrix},$$

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$$g_2 \begin{array}{l|l} E_{12} = g_{12}(A_{12} - B_{12}R_1) & E_{12} = g_{12}(B_{12} + A_{12}\bar{R}_1) \\ F_{12} = g_{12}(D_{12} + C_{12}\bar{R}_1) & F_{12} = g_{12}(C_{12} - D_{12}R_1) \\ R_2 = -g_{12}(C_{12} - D_{12}R_1)E_{12}^{-1} & R_2 = -g_{12}(D_{12} + C_{12}\bar{R}_1)E_{12}^{-1} \end{array}$$

Flip mode

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--	--

Flip mode

$R_2 \begin{cases} E_2 = E_{12}E_1\bar{E}_{12}/ E_{12} \\ F_2 = F_{12}F_1\bar{F}_{12}/ F_{12} \\ R_2 = -(C_{12} - D_{12}R_1)\frac{\bar{E}_{12}}{ E_{12} } \end{cases}$	$\begin{cases} E_2 = E_{12}F_1\bar{E}_{12}/ E_{12} \\ F_2 = F_{12}E_1\bar{F}_{12}/ F_{12} \\ R_2 = -(D_{12} + C_{12}\bar{R}_1)\frac{\bar{E}_{12}}{ E_{12} } \end{cases}$
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Solution

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---	---

Flip mode

$R_2 \begin{array}{l} E_2 = E_{12} E_1 \bar{E}_{12} / E_{12} \\ F_2 \end{array}$	$E_2 = E_{12} F_1 \bar{E}_{12} / E_{12} $
--	--

Solution

$$T_2 = W_{12} T_1 W_{12}^{-1}, \quad T^{(E,F)} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix},$$

$$\alpha_2 = -((E_{2,1}\beta_1 - E_{2,2}\alpha_1)(E_{1,1}\beta_1 - E_{1,2}\alpha_1) + E_{1,2}E_{2,2})/(|E_{12}|\beta_1),$$

$$\beta_2 = ((E_{1,1}\beta_1 - E_{1,2}\alpha_1)^2 + E_{1,2}^2)/(|E_{12}|\beta_1),$$

$$\mu_2 = \mu_1 + \tan^{-1}(E_{1,2}, (E_{1,1}\beta_1 - E_{1,2}\alpha_1)),$$

Optical parameters

$$\gamma_2 = (1 + \alpha_2^2)/\beta_2.$$



Element specific extensions

- ◎ Patches added to SURVEY, TWISS and TRACK
 - ➔ s-rotation, x-rotation, y-rotation and translation elements.
 - ➔ used to change the reference frame.
 - ➔ used to misalign magnets in HL-LHC studies.
 - ➔ used to keep “flat” non-flat beam lines (gantry).

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x-rotation tracking map

$$x^f = x^i + \frac{yp_x^i \tan \theta}{p_z^i - p_y^i \tan \theta}$$

$$p_x^f = p_x^i$$

$$y^f = \frac{y^i}{\cos \theta - p_y^i \tan \theta / p_z^i}$$

$$p_y^f = p_y^i \cos \theta + p_z^i \sin \theta$$

$$t^f = t_i - \frac{y^i (1/\beta_0 + p_t) \tan \theta}{p_z^i - p_y^i \tan \theta}$$

$$p_z = \sqrt{1 + 2p_t/\beta_0 + p_t^2 - p_x^2 - p_y^2}$$

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- ⊙ SELECT updated and used by TWISS and MATCH
 - ➔ extended with INTERPOLATE to specify points of interpolation (output) within elements.
 - ➔ matching constraints can refer to arbitrary positions.
 - ➔ still under testing (will be in end-of-the-year release)

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```
SELECT, FLAG=INTERPOLATE, RANGE=mq1, AT={0.5, 1};
MATCH, SEQUENCE=seq;
VARY, NAME=k1; # vary strength of quadrupole mq1
CONSTRAINT, RANGE=mq1, IINDEX=0, BETX=5;
LMDIF; # match betx at centre of mq1 varying k1
ENDMATCH;
```

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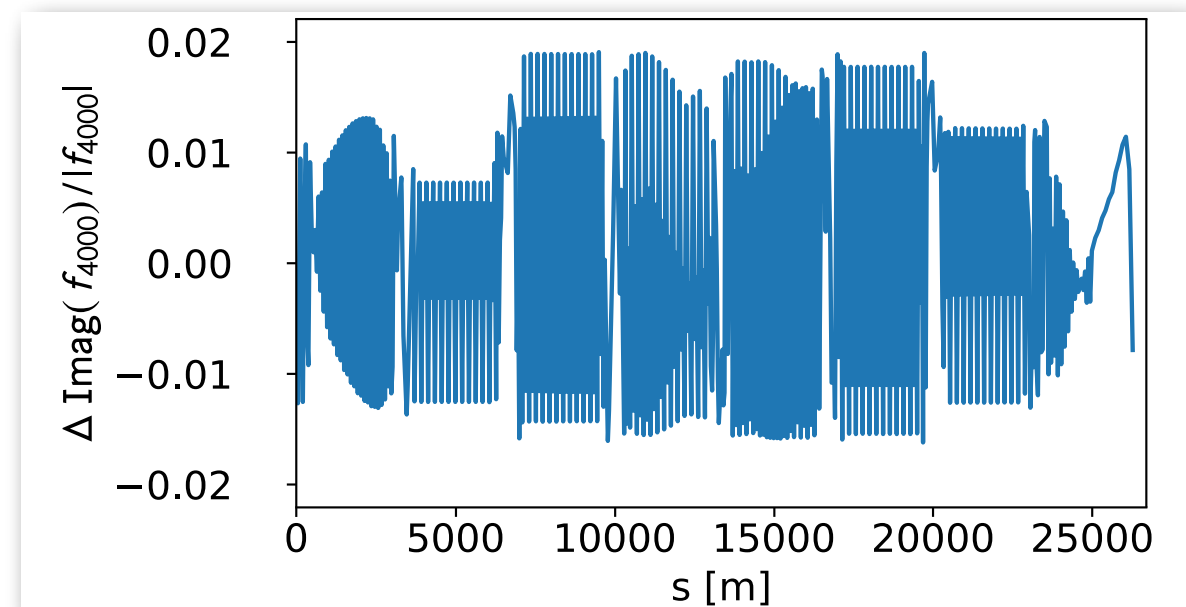
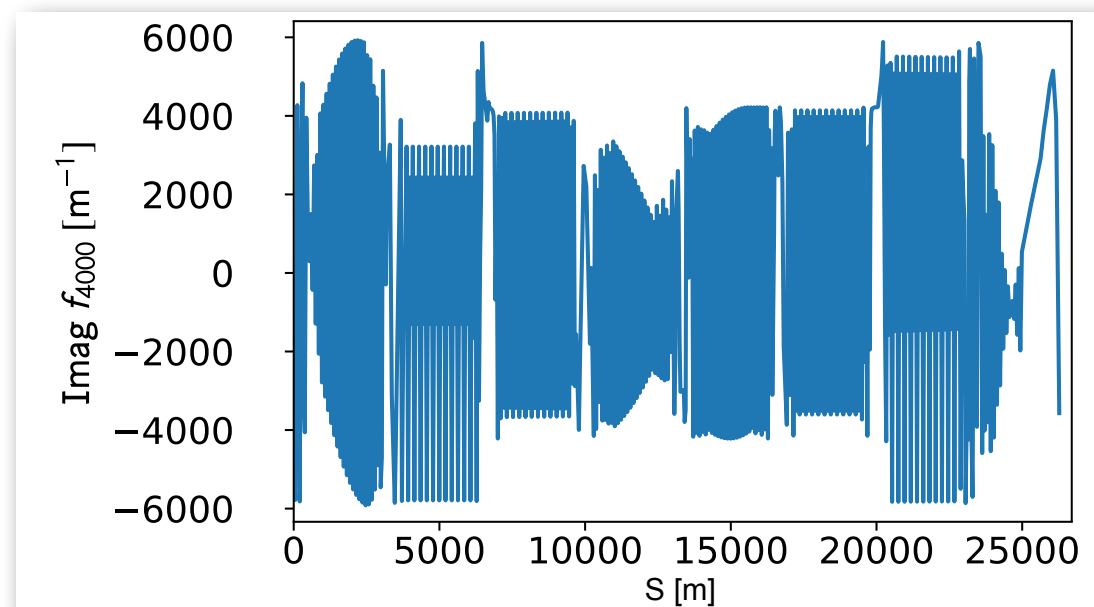
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MAD-X PTC extensions

- Interface between MAD-X and PTC/FPP embedded library is constantly improved.
 - ➔ follow-up of new releases of PTC/FPP from E. Forest for update in MAD-X.
 - ➔ synchrotron radiation effects connected to MAD-X, PTC_TWISS outputs damping times and equilibrium emittances now.
 - ➔ optimise the sector-bend maps with the exact Hamiltonian by automatic detection of maximum multipole order required, speed-up PTC_TWISS on LHC by factor x3.
 - ➔ added RECLOSS to record lost particles in table by PTC_TRACK.
 - ➔ added 6D closed orbit search with TOTALPATH, and correctly calculates the dependence of the beam momentum on RF frequency.
 - ➔ added NORMAL and TRACKRDTS options to PTC_TWISS to output the three tunes, dispersions, eigenvectors, RDTs (generating functions), Hamiltonian, and one-turn map to the new tables NONLIN and TWISSRDT and hence become available for MATCHing.

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Many thanks to

MAD-X team: A. Latina, T. Persson P. Skowronski, *I. Shreyber*, *G. Roy*.

MAD-X contributors: H. Burkhardt, R. De Maria, F. Schmidt, T. Gläße,
and many others...

PTC/FPP author: E. Forest.

CERN BE/ABP: M. Giovannozzi, G. Arduini, P. Collier.



MAD season 4 (advertisement)



And the story continue,

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MAD8, MAD9, MAD-X, ... MAD-NG

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MAD *Next Generation* is coming soon!



Extra slides



Polymorphic Tracking Code (in MAD-X)

- Developed by E. Forest (KEK) since the 90's (+2 books)
 - ➔ continuously updated in MAD-X since 2015 (following Etienne's releases), new connection from P. Skowronsky.
 - ➔ advanced Fortran 90 Library for beam dynamics, require to develop a Fortran program for your studies.

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- Principle: track high order maps using TPSA from Berz

TPSA = Truncated Power Series Algebra.

 - ➔ provide true 3D geometry for lattice design (i.e. patches).
 - ➔ provide true 6D physics for beam dynamics (small machines),
`icase = 4, 5, 56, 6` (resp. 4D, 5D dp, 6D wo-cav, 6D w-cav),
`model = 1, 2, 3` (resp. D-K-D, M-K-M, D-M-K-M),
`method = 2, 4, 6` (integrator order, Forest & Ruth, Yoshida scheme).
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- Integrated into MAD-X by F. Schmidt, E. Forest et al. in 2002
 Directly available from MAD-X scripting language.
 Weak connection with only a subset of PTC...