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THEORETICAL AND COMPUTATIONAL MODELING OF A PLASMA WAKEFIELD BBU INSTABILITY*

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Abstract

Plasma wakefield accelerators achieve accelerating gradients on the order of the wave-breaking limit, mc^2k_p/e, so that higher accelerating gradients correspond to shorter plasma wavelengths. Small-scale accelerating structures, such as plasma and dielectric wakefields, are susceptible to the beam break-up instability (BBU), which can be understood from the Panofsky-Wenzel theorem: if the fundamental accelerating mode scales as b^{-1} for a structure radius b, then the dipole mode must scale as b^{-4}, meaning that high accelerating gradients necessarily come with strong dipole wake fields. Because of this relationship, any plasma-accelerator-based future collider will require detailed study of the trade-offs between extracting the maximum energy from the driver and mitigating the beam break-up instability. Recent theoretical work predicts the tradeoff between the witness bunch stability and the amount of energy that can be extracted from the drive bunch, a so-called efficiency-instability relation. We will discuss the beam break-up instability and the efficiency-instability relation and the theoretical assumptions made in reaching this conclusion. We will also present preliminary particle-in-cell simulations of a beam-driven plasma wakefield accelerator used to test the domain of validity for the assumptions made in this model.

INTRODUCTION

A future lepton collider will have to operate at center of mass energies near 10 TeV. Conventional warm copper or superconducting rf structures are limited to around 50 MeV m^{-1} accelerating gradients, meaning that a conventional rf linac would require hundreds of kilometers of accelerating structures. Smaller scale millimeter or THz structures can achieve gradients closer to 300 MeV m^{-1} bring the length of the linac to the scale of ten kilometers. The gradients in both of these structures are primarily limited by breakdown phenomena. Much higher accelerating gradients are possible in accelerating structures that are already broken down, that is to say plasmas. The accelerating gradients available to plasma-based accelerators – beam-driven plasma wakefield accelerators (PWFAs) or laser-driven laser plasma accelerators (LPAs) – have accelerating gradients limited by the wave-breaking limit of the plasma $E_{WB} = mc^2k_p/e \approx 96.\sqrt{n_{pe}[\text{cm}^{-3}]}[\text{V/m}]$ which, for laboratory plasma densities around 1×10^{16} cm^{-3} corresponds to accelerating gradients near 10 GV m^{-1}.

This accelerating gradient comes at the cost of small-scale structures, since decreasing the plasma wavelength $\lambda_p = 2\pi/k_p$ increases the wave-breaking limit. At the same time, this shrinks the size of the accelerating plasma wave, which makes the witness bunch more prone to various transverse instabilities, such as the hosing instability [1], which is similar to the transverse beam break-up (BBU) instability [2] in traditional linear accelerators. In the case of conventional accelerator, BBU is dictated by the transverse size of the beam pipe, while the accelerating gradient depends on the longitudinal length scales of the accelerating cavity, two independent parameters. In plasma accelerators, the "beam pipe" and "accelerating cavity" are both the plasma wave, and the scales are not independent.

Because these scales are related, there exists an *efficiency-instability relationship* [3] predicted to limit how much energy can be extracted from the driver of a plasma accelerator before the BBU instability makes the witness bunch unusable for collider applications. Our ongoing work is to study the domain of validity of this theory, and ways to avoid this limit, to design a high-efficiency plasma accelerator based TeV lepton collider.

BEAM BREAK-UP AND HOSING INSTABILITIES

The dipole beam break-up instability occurs when a beam is off-center from the beam pipe. This excites a transverse dipole field, where betatron oscillations of the head of the bunch can drive the tail of the bunch to larger betatron amplitudes. A description of this instability in conventional linear accelerators is discussed in detail in Chapter 3 of Chao [4], and is illustrated in Fig. 1.

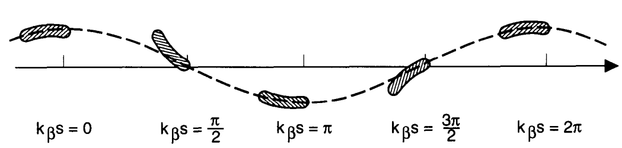

Figure 1: Illustration of the beam break-up instability in Chao [4].

In the beam break-up instability, the head of the bunch begins with some transverse offset. This excites a dipole wake field, which oscillates at the betatron wavelength. The oscillating dipole wake acts as a resonant driving term for the

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tail of the bunch. A two-macroparticle model that includes acceleration has the set of ODEs for a head and tail particle:

$$\frac{d}{ds}\left[\gamma(s)\frac{dy_1}{ds}\right] + k_\beta^2\gamma(s)y_1 = 0, \tag{1a}$$

$$\frac{d}{ds}\left[\gamma(s)\frac{dy_2}{ds}\right] + k_\beta^2\gamma(s)y_2 = -\frac{Nr_eW}{2\gamma(s)L}y_1, \tag{1b}$$

where here W is the dipole wake function, which scales with the structure radius as b^{-4}. The acceleration mitigates the instability to some extent, but asymptotically for slow acceleration the growth factor

$$\Upsilon = -\frac{Nr_eW_1(z)L_0}{4k_\beta\gamma_iL}\ln\frac{\gamma_f}{\gamma_i} \tag{2}$$

leads to unacceptably large growth in the tail emittance for a collider without mitigating the instability. The hosing instability in plasma accelerators described by Whittum *et al.* [1] has a similar structure to the beam break-up instability for conventional linear accelerators.

One established approach to damping this instability in conventional linacs is to use BNS damping [5]. In BNS damping, a correlated energy spread across the bunch, combined with the chromaticity of the lattice, tunes the betatron frequency of the tail away from the resonance. The instability is completely stabilized if

$$\frac{\Delta k_\beta}{k_\beta} = \frac{\Upsilon}{k_\beta L}. \tag{3}$$

Driving this frequency spread with linear chromaticity, $\Delta k_\beta = \xi\Delta p/p$, for a plasma accelerator would require correlated energy spread on the order of 10%, far too large for a final focus in a linear collider.

Another possibility for mitigating the instability is to use the octupolar focusing that comes from ion motion in the plasma channel [6]. The space charge fields of the drive and witness bunch can cause the background ions to move, which can in some cases lead to "ion collapse" and the complete spoiling of the witness bunch emittance [7]. However, if this is carefully balanced so that the ion motion is only slightly perturbed, the perturbed ion density can introduce sufficient nonlinear focusing to introduce a tune shift with amplitude that detunes the resonance without the need for a large correlated energy spread. The nonlinear transverse focusing will increase the emittance in the tail of the witness bunch, but the emittance growth could be acceptable for collider applications if the nonlinear ion focusing is sufficient to prevent the instability.

EFFICIENCY-INSTABILITY RELATIONSHIP

Without these mitigation schemes, the efficiency of a single stage of a plasma accelerator is related to the same parameters of the nonlinear wake as the instability growth rate. Under a handful of assumptions, this leads to an efficiency-instability relationship [3].

We can define the efficiency of an accelerator stage as the ratio of the beam power transferred to the witness bunch to the power in the drive bunch, $\eta_t = P_t/P_b$. The strength of the instability can be described as the ratio of the transverse focusing force to the strength of the forcing term, $\eta_p = -F_t/F_r$. Because the focusing and the acceleration are both directly related to the plasma wake parameters, the efficiency and instability are related approximately as

$$\eta_t \approx \frac{\eta_p}{4(1-\eta_p)}, \tag{4}$$

so, from this relationship, achieving high efficiency makes the witness bunch particularly susceptible to the beam break-up instability. If this conclusion holds, and the instability cannot be mitigated, it would put a hard limit on the efficiency available for a plasma accelerator for collider applications.

There are a handful of assumptions that go into this relationship:

1. The sheath of the plasma wake is a good conductor;
2. The transverse witness bunch wake in the variable-radius plasma bubble will satisfy a particular relationship, described in [3]; and
3. The plasma wake is not strongly deformed by the witness bunch.

It is important to determine when these assumptions are correct, to determine what range of parameters the efficiency-instability relationship applies to, how the wake field picture is modified when one or more of these assumptions is not valid, and if we can develop a modified picture for transverse beam break-up instabilities when the assumptions are valid.

Because the blowout regime in plasma accelerators is so complicated, it is unlikely we will be able to probe these limits using an analytical or semi-analytical theory. We will therefore have to resort to computing wake functions from self-consistent electromagnetic particle-in-cell simulations.

COMPUTING WAKE FUNCTIONS IN PLASMA ACCELERATORS

It is important to note that much of the preceding analysis—the beam break-up/hosing instability, the need for BNS damping, and the efficiency-instability relationship these imply—rely on the assumption that the wake fields generated by the witness bunch are well-described by a linear response function. However, the domain of validity for this approximation in a plasma accelerator in the blowout regime has not been established. To determine whether the linear response is valid, we are using FBPIC to run particle-in-cell simulations of first the hollow channel.

FBPIC [8] uses the quasi-cylindrical spectral Lifschitz algorithm for electromagnetic computations. The algorithm decomposes the fields into $e^{im\theta}$ azimuthal modes, which makes it ideal for isolating individual wake field components,

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since for example simulating only the $m = 0$ and $m = 1$ modes will preclude quadrupolar and higher-order wake functions. Because FBPIC uses a Fourier-Bessel PSATD algorithm, it is less susceptible to various numerical instabilities, making it easier to interpret the data without having to filter high-frequency numerical Čerenkov and other artifacts.

The theoretical treatment of the beam break-up instability and the conclusions that are applied to plasma accelerators rely on the existence of a wake function, that is that the transverse and longitudinal forces can be described as a linear response to the witness bunch current:

$$\mathbf{F} \propto \int d\zeta' \, \mathbf{W}(\zeta - \zeta') I(\zeta'). \tag{5}$$

We will call this relationship, as applied to plasma accelerators, the "wake ansatz" or, in the frequency domain, the "impedance ansatz". This is a linear response approximation; in a conventional accelerator with a stainless steel or other conducting beam pipe, it is the assumption that the material obeys Ohm's law.

To determine if and when the "impedance ansatz" is valid, we are running self-consistent particle-in-cell simulations of plasma-based accelerator structures with the FBPIC code. From these simulations, we will attempt to extract wake functions for a given configuration of drive bunch charge, and vary the parameters of the drive bunch to see if those wake functions are predictive for a range of parameters and not just the specific parameters of one scenario.

Our benchmark problem is the hollow plasma channel, which has analytic solutions for its wake functions (see Schroeder et al. [9]). This provides a benchmark for short lengths behind the drive bunch, after which surface plasma waves at the hollow channel edge distort the wake fields [10]. However, in the limit that the surface plasma waves are linear, a wake function should still exist.

To test the existence of a meaningful wake function, we will compute a wake function from the response to one drive bunch in an FBPIC simulation, and compare the fields predicted by that wake function for a different set of parameters for the drive bunch. To compute the wake function, we follow a four-step process: (1) compute **F** along the axis from the simulations; (2) take the Fourier transform of **F** and then (3) divide out the Fourier transform of the beam current to give the impedance; (4) take the inverse Fourier transform of the impedance to compute the wake function. In this way, we can determine the domain of validity for the wake function model. We illustrate this process with some preliminary results for the hollow channel below.

For simplicity in demonstrating the concept, we are focusing our efforts on the accelerating fields. Once we have benchmarked this approach, we will then turn our attention to studying the dipole wake fields, which are of critical interest for the beam break-up instability.

First, we compute the electric field for an axisymmetric drive bunch using FBPIC. In Fig. 2 we can see the wake amplitude collapsing due to the surface plasma waves in the hollow channel edge. If this is the result of a purely linear

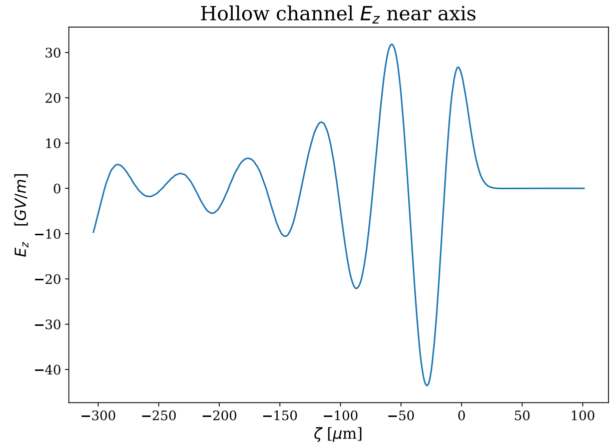

Figure 2: The trailing electric field in the plasma hollow channel, measured from a reference location.

response, we should be able to capture that effect using an impedance model.

To deconvolve the bunch current from the wake function, we take a Fourier transform of the fields, Fig. 3, and then

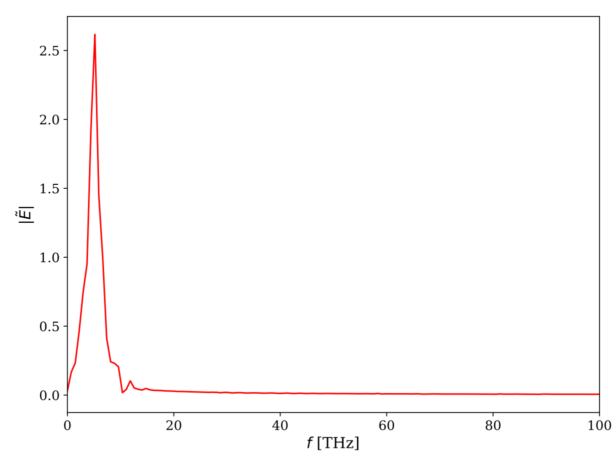

Figure 3: The Fourier transform of the electric field generated by the drive bunch.

divide by the Fourier transform of the bunch current density to obtain the impedance. The numerically computed impedance has several clear spikes with some line width, and some residual lower-amplitude features that are likely the result of noise in the particle-in-cell simulation.

It remains to determine the best way to extract the wake function from this data. The computed impedance in Fig. 4 has a considerable amount of noise that must be filtered, as well as a handful of smaller peaks that may or may not be physical. The inverse Fourier transform of this will provide the wake function as a Fourier series.

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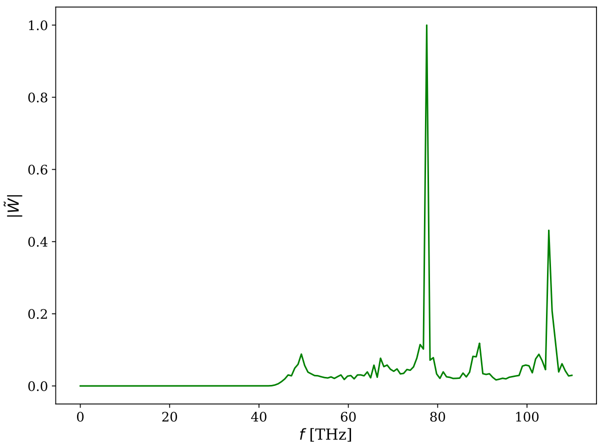

Figure 4: The numerically computed impedance.

FUTURE WORK

We are currently developing and benchmarking an approach to extracting wake functions from particle-in-cell simulations of plasma accelerating structures. In the future, we will apply these techniques to a blowout plasma wakefield accelerator with a loaded wake, such as depicted in Fig. 5, and determine where this approach is valid for describing the difference between a loaded and unloaded wake, such as in Fig. 6.

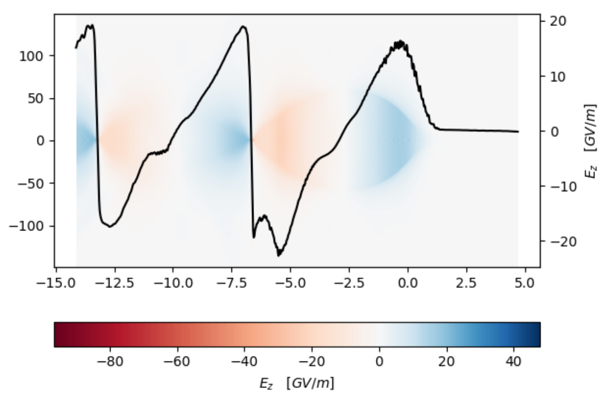

Figure 5: Blowout PWFA with a loaded wake.

Once we have computed the longitudinal and transverse dipole wake functions for a blowout PWFA with a short, low-charge witness bunch, we will be able to compare predictions for the fields as a function of witness bunch charge to determine when the impedance ansatz breaks down. This will provide critical insights into the domain of validity of the efficiency-instability relationship, the exact nature of the beam break-up instability in loaded blowout plasma wakefield accelerators, and the details required for a mitigation technique for the instability.

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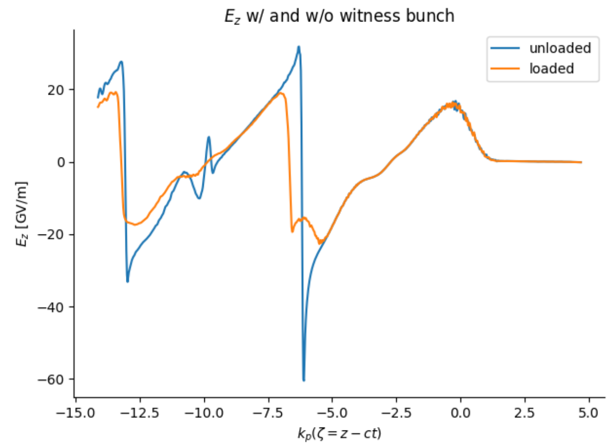

Figure 6: A comparison of a loaded and unloaded blowout PWFA wake.

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REFERENCES

[1] D. H. Whittum, W. M. Sharp, S. S. Yu, M. Lampe, and G. Joyce, “Electron-hose instability in the ion focused regime”, *Phys. Rev. Lett.* **67**, 991 (1991).

[2] R. B. Neal and W. K. H. Panofsky, *Science* **152**, 1353 (1966); W. K. H. Panofsky and M. Bander, *Rev. Sci. Instr.* **39**, 206 (1968).

[3] V. Lebedev, A. Burov, and S. Nagaitsev, "Efficiency versus instability in plasma accelerators", *Phys. Rev. Acc. Beams* **20**, 121301 (2017).

[4] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators*, J. Wiley & Sons, New York (1993).

[5] V. Balakin, A. Novokhatsky, and V. Smirnov, *Proc. 12th Int’l. Conf. High Energy Accel.* Fermilab (1983).

[6] A. Burov, S. Nagaitsev, and V. Lebedev, “Beam Breakup Mitigation by Ion Mobility in Plasma Acceleration”, arXiv:1808.03860 [physics.acc-ph].

[7] J. B. Rosenzweig, A. M. Cook, A. Scott, M. C. Thompson, and R. B. Yoder, “Effects of Ion Motion in Intense Beam-Driven Plasma Wakefield Accelerators”, *Phys. Rev. Lett.* **95**, 195002 (2005).

[8] R. Lehe, M. Kirchen, I. A. Andriyash, B. B. Godfrey, and J.-L. Vay, “A spectral, quasi-cylindrical and dispersion-free Particle-In-Cell algorithm”, *Comp. Phys. Comm.* **203** pp. 66-82 (2015).

[9] C. B. Schroeder, D. H. Wittum, and J. S. Wurtele, “Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator”, *Phys. Rev. Lett.* **82**, 1177 (1999).

[10] C. A. Lindstrøm *et al.*, “Measurement of Transverse Wakefields Induced by a Misaligned Positron Bunch in a Hollow Channel Plasma Accelerator”, *Phys. Rev. Lett.* **120**, 124802 (2018).