STATISTICAL ANALYSIS OF THE EIGENMODE SPECTRUM IN THE SRF CAVITIES WITH MECHANICAL IMPERFECTIONS*

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Abstract

The superconducting radio frequency (SRF) technology is progressing rapidly over the last decades toward high accelerating gradients and low surface resistance making feasible the particle accelerators operation with high beam currents and long duty factors. However, the coherent RF losses due to high order modes (HOMs) excitation becomes a limiting factor for these regimes. In spite of the operating mode, which is tuned separately, the parameters of HOMs vary from one cavity to another due to finite mechanical tolerances during cavities fabrication. It is vital to know in advance the spread of HOM parameters in order to predict unexpected cryogenic losses, overheating of beam line components and to keep stable beam dynamics. In this paper we present the method of generating the unique cavity geometry with imperfections while preserving operating mode frequency and field flatness. Based on the eigenmode spectrum calculation of a series of randomly generated cavities, we can accumulate the data for the evaluation of the HOM statistics. Finally, we describe the procedure for the estimation of the probability of the resonant HOM losses in the SRF resonators. The study of these effects leads to specifications of SRF cavity and cryomodule and can significantly impact the efficiency and reliability of the machine operation.

INTRODUCTION

Over recent decades the progress in SRF technology has made it feasible for a number of applications of the particle accelerators to operate in the continuous wave (CW) regime with a high beam current. There is an active demand on such machines based on multiple projects in the industry, high energy physics and material science, such as developing subcritical fission reactors based on an accelerator driven system (ADS), next generations of neutrino facilities and neutron spallation sources (PIP-II, ESS), radioactive ion beam facilities (RIBs) and free electron lasers (FELs) [1-6]. Variety of experimental programs often require a complex beam pattern and an ultra-short bunch length. Figure 1 shows typical examples of the dense beam frequencies spectrum in the PIP-II proton linac and the broadband power spectrum of wake fields generated by a series of 25 μm rms bunches in the LCLS-II cryomodule. Evidently a combination of a large average beam current, a high bunch repetition rate and a broadband generated wake fields might result in significant cavity rf losses. The most danger comes out of the trapped HOMs in a case of

their coherent excitation by the beam. The later causes excessive cryogenic loads, overheating of beam line components and beam emittance dilution.
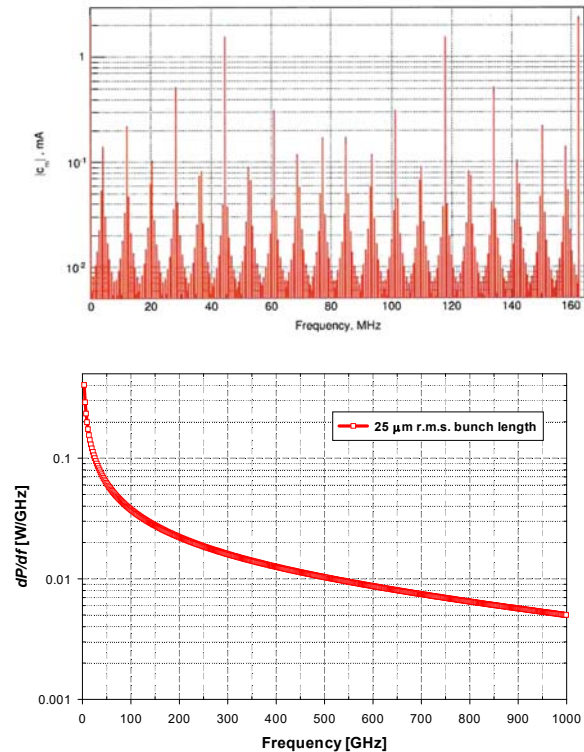

Figure 1: Beam frequency spectrum in the PIP-II linac (up) and power spectrum of wake fields generated by a series of ultra-short bunches in the LCLS-II cryomodule (down)

Due to a nature of SRF cavities they are very good resonance systems with multiple low loss eigenmodes with high intrinsic quality factors. For the coherent excitation one of the beam harmonics must coincide or be close to HOM frequencies. At the same time the HOM spectra in actual cavities will have significant frequency spreads comparing to the cavity with ideal geometry due to mechanical errors. Because of a randomness of mechanical errors, the resonant HOM excitation by the beam is inherently the probabilistic issue. The idea is illustrated on Figure 2, where the left sketch shows overlapping of the beam spectrum line and the HOM frequency spread with a high probability of coherent HOM excitation. The problem is complicated if we consider the propagating HOMs with frequencies above the beam pipe cut off. In this case the probability of mode trapping depends on the frequencies of neighbour cavities and, thus, taking into account the stochastic behaviour of cavity HOMs spectrum is essential for a proper analysis of the HOMs excitation.

Figure 2: Stochastic HOM excitation by the pulsed beam. High excitation probability (left) corresponds to the case of the beam harmonic frequency within the HOM frequency spread and low probability (right) represents the beam spectrum line which is far off the HOM frequency deviation bandwidth.

Recent studies of the HOMs excitation in SRF cavities performed by different research groups was based either on the pre-deterministic approach, when the given spreads of HOM parameters are used for an evaluation of the worst-case scenario, or on the experimental data of frequencies and quality factors measured for most dangerous HOM passbands [7-9]. Both methods give only approximate results since they use rough or limited estimations for HOMs frequencies, tend to overstate quality factors and don't consider possible deviations of HOMs shunt impedances. The accurate evaluation of HOMs coherent effects is important to the design stage of SRF particle accelerators because it determines mechanical, thermal and electromagnetic requirements for expensive accelerator components and might set certain limits on machine operational scenarios. In this paper we propose the method of modelling the eigenmode spectrum in the multi-cell SRF cavities with finite mechanical tolerances defined by the fabrication technology. The method is based on the generation of unique cavity cell geometries with random dimension errors and the instant tuning of individual cells frequencies to preserve cavity operating mode field flatness. By doing the eigenmode spectrum calculation of a series of randomly generated cavities we can accumulate the data for the evaluation the HOM statistics. We used the proposed technique for HOMs analysis in high energy (HE) 650 MHz cavities of the PIP-II linac and 3.9 GHz cavities of the LCLS-II project [10,11]. Finally, we compared our results with available HOMs experimental data and present the procedure for the estimations of cumulative probabilities of resonant HOM losses and the beam emittance dilution in the string of SRF cavities.

EIGENFREQUENCY ANALYSIS OF THE SRF CAVITIES WITH MECHANICAL IMPERFECTIONS

Conventional thin-walled niobium SRF cavities consist of multiple shell components welded together. Mechanical forming of such components and further electron-beam welding introduce significant uncertainty for the final cavity geometry. Typical maximum deviations of cavity profiles in respect to the ideal shape are about ±200 μm and ±100 μm for 1.3 GHz and 3.9 GHz cavities respectively [12,13]. Therefore, cavities get tuned for adjusting operating frequencies and preserving the field flatness in multicell cavities. Since each cavity has a unique geometry, the HOM spectrums vary from cavity to cavity and, then, the

beam to cavity interaction has a probabilistic nature. The ideal geometry of elliptical cavity cells can be characterized by eight parameters and another few parameters are required to describe mechanical imperfections like the cell to cell non-concentricity and the cell transverse deformation. Figure 3 shows ideal geometries of the end and regular cells for a typical multicell elliptical cavity.
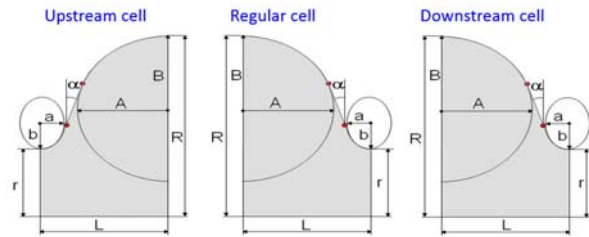

Figure 3: Nominal geometries of the multicell elliptical cavity.

For collecting accurate HOM statistics we perform the eigenmode analysis of HOM spectrum in the cavity with mechanical errors. The procedure is based on adding random components to the geometrical parameters of each cavity cell:

$$P_n^i = P_n^{nom} + |\Delta_{tol}|[2Rnd(1) - 1],$$

where P_n^i is a random dimension of the individual cell, P_n^{nom} is the nominal value of the nth geometrical parameter of the half-cell, i is the cell number, Δ_{tol} is the mechanical tolerance of a cavity fabrication and $Rnd(1)$ is the uniform random function in the range of 0 to 1. For a preliminary tuning of the cavity operating frequency we need to calculate a frequency-dependent sensitivity for each of cavity geometrical parameters. Next frequencies of the half-cells are tuned by adjusting the cell lengths similarly as it happens with real cavities to preserve the field flatness. The half-cell tuning frequency balance is described then:

$$\Delta L^i \frac{\partial f}{\partial L^i} = -\sum_{n=1}^{N}\left[\Delta P_n^i \frac{\partial f}{\partial P_n^i}\right],$$

where ΔL^i is the length compensation of the ith half-cell, $\Delta P_n^i = P_n^i - P_n^{nom}$ is the random error of the ith half-cell dimension, $\partial f/\partial L^i$ and $\partial f/\partial P_n^i$ are frequency-dependent sensitivities of the half-cell length and the nth geometrical parameter. We assume here that small deviations of each parameter won't influence the sensitivity of other parameters, all mechanical tolerances are the same and uncorrelated with each other.
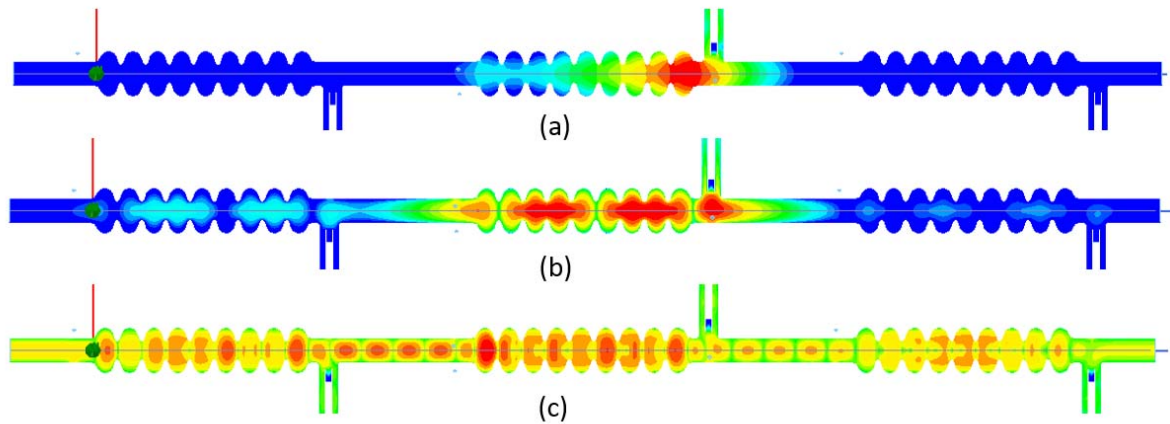
Figure 4: Trapped modes in the infinite chain of SRF cavities with random HOM spectrum: (a) type I high-Q HOM is localized within the cavity volume, (b) type II medium-Q HOM occupies the cavity and adjacent beam pipes and (c) type III low-Q HOM distributed along the cavities chain. All plots show the complex magnitude of electric field, where a blue colour corresponds to the zero amplitude.

Conventional accelerating cryomodule consists of a series of superconducting cavities connected by copper plated normal conducting bellows. Because of the unique HOM spectrum of each cavity common modes in the chain of identical ideal cavities will split into modes of individual resonators in the real cryomodule. Following this approach, we can categorize the trapped HOMs in the cryomodule into three types depending on their quality factors. The first kind of trapped HOMs is weakly coupled with the beam pipe and most of the HOM stored energy is localized within the superconducting cavity volume resulting in a high-quality factor. The second kind of trapped HOMs has a good coupling with the beam pipe but the signal is reflected by the neighbouring cavities and a part of the stored energy fills in the volume of adjacent beam pipes causing an additional signal dampening through far upstream and downstream coupler ports and by normal conducting bellows. The third type of HOM represents the case where the HOM signal can propagate through the neighbouring cavities. Then the stored energy is distributed along the cryomodule beam line and damped by many coupler ports and by ohmic losses in interconnecting cavity bellows. Such modes have low quality factors and we don't count them as a dangerous resonant mode. Figure 4 illustrates typical electric field distribution for what is described above, three types of trapped HOMs in the chain of SRF cavities with random HOM spectrums. Evidently the single cavity model is adequate for the HOM type I analysis, while the HOM types II and III require at least the chain of three cavities for accurate simulations. Further increasing the number of cavities might improve results for the low-Q HOMs but at the same time results in unnecessary complication of the eigenmode analysis and a longer simulation time. Thus, we conclude that the chain of three cavities is an optimum choice for the calculation of resonant HOMs spectrum cryomodule and longer chains give a little or no impact to the overall result.
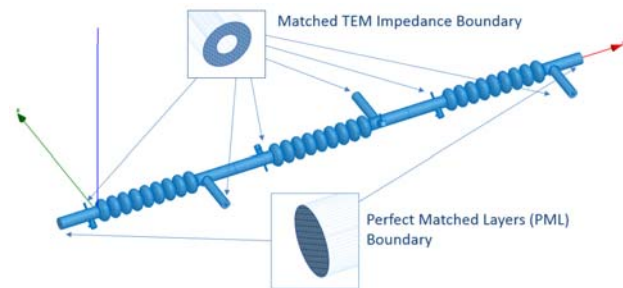

Figure 5: ANSYS HFSS model for the eigenmode analysis of the chain of three SRF cavities.

Finally, we create the 3D model of three random cavities with auxiliary coupler ports. For example, Figure 5 shows the ANSYS HFSS model of the chain of three elliptical cavities with matched boundary conditions [14]. All coaxial TEM ports are terminated with the constant free space impedance boundary, while the free radiation to the round beam pipe is ensured by the perfectly matched layers boundary. Since the number of trapped HOMs in the cavity chains are quite large, it is preferable to perform the HOM spectra simulation in series of few tens of eigenmodes starting from the operating passband for achieving better convergence and accuracy. The upper frequency limit is defined by the increasing number of propagating modes in the beam pipe at higher frequencies where the HOM spectrum becomes almost continuous. For mode sorting it is necessary to set up and calculate secondary values during the HOMs analysis: local stored energy in each cavity with adjacent beam pipes, longitudinal and transverse shunt impedances and partial quality factors for coupler ports. By sorting the HOMs compendium we can exclude the end cavities spectra and recognize monopole, dipole and quadrupole HOMs passbands. As the last step we calculated the HOMs statistics, mean and rms values for frequencies, shunt impedances and quality factors. Having in hand
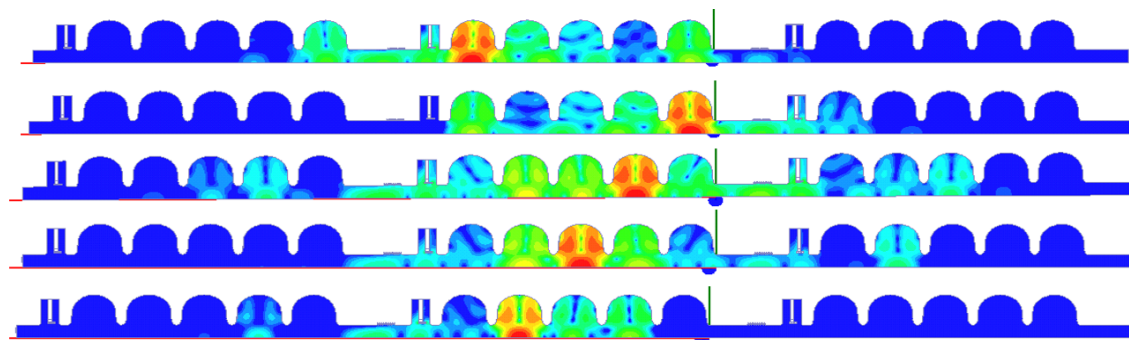
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Figure 6: Stochastic HOM excitation by the pulsed beam. High excitation probability (left) corresponds to the case of the beam harmonic frequency within the HOM frequency spread and low probability (right) represents the beam spectrum line which is far off the HOM frequency deviation bandwidth.

HOMs statistics it is possible to perform a Monte Carlo analysis for a coherent HOMs excitation in the chain of SRF cavities by the beam with any given average current and bunch patterns thus, derive the probabilities for excessive cryogenic losses and beam emittance dilution.

APPLICATION OF THE STOCHASTIC ANALYSIS OF HOM SPECTRUM

Originally, we developed a stochastic approach for the HOMs analysis in the high energy (HE) 650 MHz five cells elliptical structure of the Project-X linac [15]. During the stage of rf design it was found that the 5th monopole band in a cavity with ideal geometry has extremely narrow passband of few tens of kilohertz while the expecting HOMs frequencies deviation due to mechanical errors is at least few megahertz. In this case, when coupling between cells is weak and varies from cell to cell, the usual pass-band structure of N modes of $m\pi/N$-kind, where N is the number of cells and m runs from 1 to N, may change [16]. The field of a cavity mode may be concentrated in a single cell, or two adjacent cells. The calculated distribution of electric fields for the 5th monopole band is presented in Figure 6 for the chain of three random HE 650 MHz cavities. Later we verified field distributions of the 5th monopole band by bead pull measurements on the prototype niobium cavities. Typical measured field distributions are summarized in Figure 7. Evidently the experimental results are in perfect agreement with both theoretical and numerical predictions.
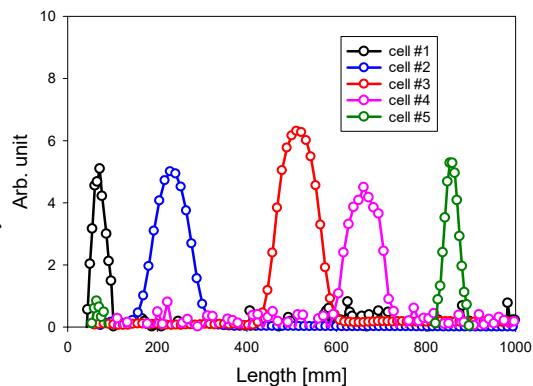

Figure 7: Bead pull measurement of on-axis field profiles of the 5th monopole band in the HE 650 MHz structure

For the second time we calculated HOM statistics for the third harmonic cavity of LCLS-II project [11]. Figure 8 shows frequency standard deviations for modes in the first monopole band. There is a good agreement between calculations and measured 2 K data for the similar 3.9 GHz cavity developed for the XFEL project [17]. The average calculated field flatness of the operating mode is above 80% for both studies, which is close to the conventional specification of minimal 90% field flatness for multicell cavities. Therefore, we conclude that proposed method of stochastic analysis of HOM parameters in the SRF cavities with mechanical imperfections provide a reliable data for the further statistical analysis of the coherent HOM excitation.
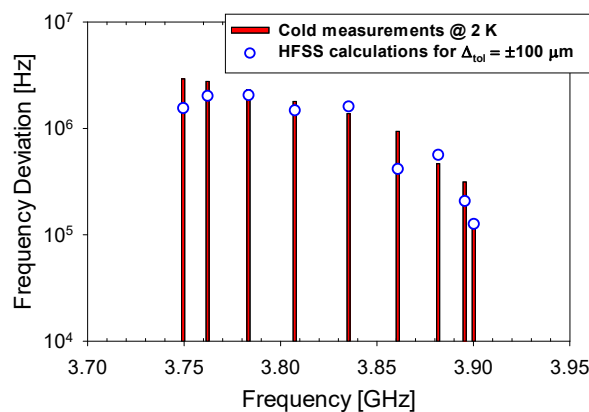

Figure 8: Frequency standard deviations for the 1st monopole band of the 3.9 GHz cavity.

By using the predicted deviations of monopole and dipole HOMs frequencies, shunt impedances and quality factors we can generate the chains of cavities with random HOMs spectra and, thus, estimate probabilities of the rf losses or the beam emittance dilution in real superconducting linacs. As an example, the resulting cumulative probability of rf losses are presented in Figure 9 for the HE 650 MHz PIP-II cryomodule (left) and the LCLS-II third harmonic section (right) respectively.
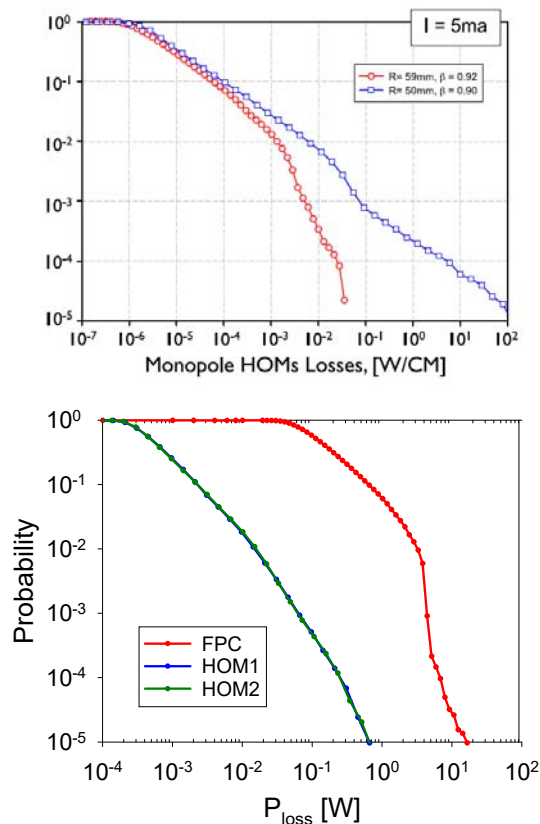
Figure 9: Cumulative probabilities of monopole HOMs RF losses in the HE 650 MHz PIP-II (up) and 3.9 GHz LCLS-II (down) cryomodules.

The accurate quantitative estimation of the negative effects caused by the resonant HOM excitations is important for a comparison of different cavity designs and various regimes of the beam operation. Based on HOMs statistics we chose an optimal design for the HE 650 MHz cavity for PIP-II project and modified the end group in the third harmonic cavity for the LCLS-II linac [18,19]. In summary, we conclude that the proposed method of statistical HOM spectra evaluation in the SRF cavities with mechanical imperfections is straightforward and reliable instrument for the risk analysis of the coherent HOM excitation in superconducting particle accelerators.

CONCULSION

The statistical analysis of the eigenmode spectrum in SRF cavities is reliable and accurate tool for quantitative evaluation of the coherent HOM excitation by the beam with arbitrary time structure. The outcome of HOM analysis resulted in critical decisions for the design of superconducting accelerating cavities. Simplification of the cavity production and operation is a significant part of the overall cost reduction for both machines. The proposed technique can be easily adapted and used for other superconducting particle accelerators operating at high average beam current and high duty factor regimes.

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