STORAGE RING EDM SIMULATION: METHODS AND RESULTS

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Abstract

The idea of an electric dipole moment (EDM) search using the electrostatic storage ring with polarized beam is based on an accumulation of additional tiny spin rotations, about one billionth of a radian per second, which only occur in the presence of EDM. This method can be realized under conditions of long-time spin coherence ~1000 seconds. During this time, each particle performs about 10^9 turns in the ring moving on different trajectories. Under such conditions, the spin-rotation aberrations associated with various types of space- and time-dependent nonlinearities start playing a crucial role. Computer simulation is necessary to design such a ring, taking into account all the factors affecting the spin. We used COSY Infinity and an integrating program with symplectic Runge-Kutta methods in conjunction with analytical methods and T-BMT differential equation solving. We developed a new lattice based on the alternating spin rotation. As a result, we can achieve a spin coherence time (SCT) of ~5000 seconds. The difficulties of these studies are that aberration growth is observed on the scale of 10^9 turns and a few million particles. For this simulation, we used a supercomputer with a parallel computing process.

METHODS OF SIMULATION

At Forschungszentrum Jülich, two approaches are currently being considered for the EDM search in a storage ring: a method using a resonant RF spin flipper technique in the COSY ring [1] and frozen spin in the "magic" ring specially projected for the EDM search [2,3,4]. The resonance method has the character of preliminary studies of SCT and measuring techniques of spin decoherence arising for various reasons. In this paper, all results were obtained by the second method based on the "magic" ring conception.

The main difficulty in solving the problem of spin-orbital motion simulation together with the EDM signal lies in the fact that the signal expected directly from the EDM is extremely small. In particular, from sufficiently reliable estimations made in [2], it follows that due to EDM the spin in the "magic" ring rotates with the angular velocity of 10^{-9} radians per second. Taking into account the fact that the ring structure contains several hundred elements, and each particle performs about 10^6 revolutions per second, this means that EDM rotates the spin through an angle of approximately $\sim 10^{-18}$ radians per element on average. Accordingly, the EDM signal is expected to change the value of the spin projections on the same tiny scale. Thus, in the EDM search we meet a problem that has not previously been encountered in accelerator physics: the arithmetic coprocessor has a mantissa length of 52 bits and can make a mistake in calculating spin projections after each element. This is a very serious limitation for using programs with the standard mantissa, and must be taken into account. Therefore using a powerful program we take a different approach, where the EDM signal is not implemented, and only the induced error signal is studied. This approach allows us to define the possible low level of the EDM signal and solve the problem from the opposite side.

In our studies, we use the following mathematical tools:

- COSY Infinity program [5], based on map generation using differential algebra and the subsequent calculation of the spin-orbital motion for an arbitrary particle;
- integrating program to study the effects that do not require a long numerical time;
- numerical integration of T-BMT differential equations for a spin in optics with smoothly approximated parameters of orbital motion;
- analytical approach.

Each of these methods is an integral part of our research.

COSY Infinity

COSY Infinity is known as a very powerful instrument for particle tracking in electromagnetic fields. COSY Infinity is a program for the simulation, analysis and design of particle optical systems, based on differential algebraic methods. Full spin-orbital tracking simulations of the entire experiment are absolutely crucial for systematically exploring the feasibility of the planned experiments. In the EDM search, it is the only program which allows the spin-orbit motion of millions of particles to be simulated over a real time scale experiment during n x1000 seconds. At present, we use the MPI (Message Passing Interface) version of the COSY Infinity program installed on a supercomputer with $3{\cdot}10^5$ processors. It is planned to use the COSY Infinity program and to include higher-order nonlinearities, normal form analysis, symplectic tracking and especially spin tracking upon the incorporation of RF-E and RF-B flippers into the program. In order to study subtle effects and simulate the particle and especially spin dynamics during accumulation and build-up of the EDM signal, custom-tailored fast trackers are needed capable of following up to 10–100 billions turns for samples of up to 10^4 -10^6 particles.

At the initial stage of EDM research, we use COSY Infinity to study the behaviour of the spin aberrations for a large number of particles and a long-time calculation.

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This will allow us to answer the question of whether we can construct a facility in which the EDM signal can be seen against the background of various spin aberrations.

COSY Infinity seeks the solution in the form:

$$\vec{X} = M^1\vec{X}_0 + M^2\vec{X}_0^{[2]} + M^3\vec{X}_0^{[3]} + ... + M^N\vec{X}_0^{[N]}, \quad (1)$$

where $\vec{X}^{[N]} = \underbrace{\vec{X}\otimes...\otimes\vec{X}}_{Ntimes}$ – a Kronecker power of $\vec{X}, \vec{X}^k$ is a vector with C^k_{6+k-1} elements. Matrices M^k have the dimension $6\times C^k_{6+k-1}$. Transfer maps M^k can be generated up to any order. And for spin motion we have initial spin coordinates $\vec{S}_0 = \{S_{x_0}, S_{y_0}, S_{z_0}\}$ under the condition $S^2_{x_0} + S^2_{y_0} + S^2_{z_0} = 1$. After one revolution it is

$$\vec{S} = M_S\vec{S}_0\,, \quad (2)$$

where M_S is a spin rotation matrix.

Symplectic Runge-Kutta Integrating

The integrating Runge-Kutta program is intended to model the spin-orbital motion with fringe fields in elements and including the EDM signal directly in the simulation. The algorithm used in the program is not as fast as COSY Infinity by several orders of magnitude. Therefore, we use it mostly to investigate a short-time phenomenon that does not require long calculation periods.

In the program, the spin-orbital equations are written in the following form [6]:

$$\frac{d}{ds}\vec{Z} = F(s,\vec{Z})\,, \quad (3)$$

where $\vec{Z} = \{x, x', y, y' S_x, S_y, S_z\}$.

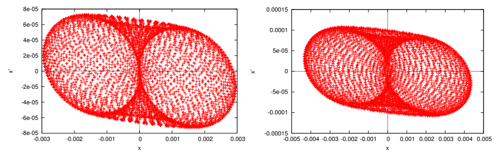
It allows us to use classical step-by-step integration methods to solve this system. As a basic method for the tracking program, a symplectic Runge-Kutta scheme was implemented [7]. According to this scheme, the solution of the spin-orbital equations can be presented in an iterative form:

$$\begin{aligned}\vec{Z}_{n+1} &= \vec{Z}_n + h\textstyle\sum_{j=1}^{2} b_j\vec{F}(s+hc_j, \vec{X}^{(i)})\\ \vec{Z}^{(i)} &= \vec{Z}_n + h\textstyle\sum_{j=1}^{2} a_{ij}\vec{F}(s+hc_j, \vec{X}^{(i)}).\end{aligned} \quad (4)$$

Note that the symplectic scheme imposes the condition of a constant integration step. Moreover, this scheme requires implicit equations to be solved, and appropriate numerical methods can be used.

To compare the computation results of the two programs we used the lattice with cylindrical deflectors described in [4]. Comparing the results of tracking through a single element we found a coincidence with high accuracy in computational models of COSY Infinity and the integrating program.

Figure 1 shows simulation results of both programs for a rather complex phase motion in the radial plane with coupling to the longitudinal motion [6]. The spin behaviour also coincides in both programs.



(a) (b)

Figure 1: x–x′ motion in COSY Infinity (a) and the integrating program (b).

A different choice of the reference orbits and different symplectification methods does not permit absolutely the same numerical results to be obtained, but we can see from figure 1 that the dynamics and behaviour of particles are similar in both programs. At present, the integrating program for introducing the fringe fields and implementing the EDM signal is under development.

The last two methods, the numerical integration of T-BMT differential equations and the analytical formalism, will be discussed in detail later.

SPIN TUNE OF NON-MAGIC PARTICLE

Having the tools for spin-orbital tracking, we now proceed to the problem of formulating a lattice design that meets the requirements of the experiment. At the initial stage, the analytical formalism is the most powerful tool because it allows the issue to be seen as a whole. Moreover, the comparison of analytical estimates in the simplest cases with the program results provides extra computation control.

The spin oscillation equation therefore has the following form:

$$\begin{aligned}\frac{d\vec{S}}{dt} &= \vec{\omega}_G\times\vec{S}\\ \vec{\omega}_G &= -\frac{e}{m_0\gamma c}\left(\frac{1}{\gamma^2-1} - G\right)\cdot(\vec{\beta}\times\vec{E})\end{aligned} \quad (5)$$

As already mentioned, we consider the so-called “magic” [2], purely electrostatic ring for polarized proton, when for the reference “magic” particle $1/(\gamma_m^2-1) - G = 0$. Further spin projection indications will be made in the following line: z is orientated along the momentum, x and y are horizontal and vertical directions, respectively. Taking into account that the vertical and longitudinal electric field components are expected to be small and $\beta_x, \beta_y << \beta_z$ we can obtain an expression for the number of spin oscillations per one turn that is a spin tune:

$$\nu_s = \frac{e}{2\pi m_0 c^2}\cdot L_{orb}E_x\frac{1}{\gamma}\left(\frac{1}{\gamma^2-1} - G\right), \quad (6)$$

where L_{orb} is orbit length. For a particle of different energy from the “magic” value $\gamma \neq \gamma_m$ the factor

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$G-1/(\gamma_m^2-1)\neq 0$ is not equal to zero, and the spin rotates with a tune dependent on particle energy. As we will show later, this leads to the spin tune aberrations. Expanding $G-1/(\gamma^2-1)$ in the Taylor series in the vicinity of $p=p_m$ we have:

$$\left(G-\frac{1}{\gamma^2-1}\right)_{p=p_m+\Delta p}=0\Big|_{p=p_m}+2G\cdot\frac{\Delta p}{p}-\frac{1+3\gamma^2}{\gamma^2}\cdot G\cdot\left(\frac{\Delta p}{p}\right)^2+\ldots \quad (7)$$

In a first approach, an incoherence of the spin tune can be estimated by the simple formula $\delta\nu_s=2\cdot\frac{eE_x\cdot R}{m_0c^2\gamma}\cdot G\frac{\Delta p}{p}$, where $E_x\cdot R$ is the rigidity of the ring depending on energy only and G is the anomalous magnetic moment. If to follow the definition of the spin coherence time (SCT) in [2] as the time during which the rms spread of the orientation spin of all particles in the bunch reaches one radian, then at the momentum spread $\Delta p/p=5\cdot10^{-5}$ ($\Delta W_{kin}/W_{kin}=10^{-4}$) the SCT is less than one millisecond. This disappointing fact perfectly coincides with the numerical simulation of COSY Infinity.

Following the previously proposed method [8], we then used the RF field to average the momentum deviation relative to the "magic" level. Under the RF field, the spin tune is modulated by longitudinal tune ν_z, which is two orders of magnitude higher than the spin tune ν_s, and therefore the spin oscillates with a very small amplitude $\Phi_{\max}\sim(\nu_s/\nu_z)^2$ relative to a central position. However, taking into account $(\Delta p/p)^2$ in (7), the central position of the spin itself drifts very slowly. This drift term averaged over time gives the non-zero contribution:

$$\delta\nu_s=\frac{eE_x\cdot R}{m_0c^2\gamma}\cdot\frac{1+3\gamma^2}{\gamma^2}\cdot\frac{G}{2}\cdot\left(\frac{\Delta p}{p}\right)^2_{\max} \quad (8)$$

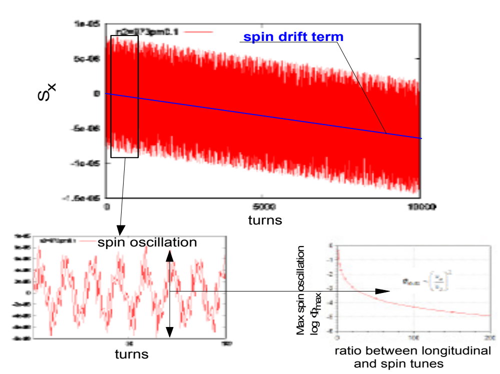

Figure 2: Oscillating and drift terms of spin behaviour.

Figure 2 shows the horizontal spin projection S_x behaviour when RF is turned on. Since the oscillating component is always within $\Phi_{\max}$, we will subsequently only be interested in the slow component drift. This depends on the square momentum spread and defines the spin tune incoherence: at the momentum spread $\sim5\cdot10^{-5}$ SCT~ 180 sec. Thus the RF field increases the SCT by five orders. However, this is unfortunately only feasible for particles with zero initial deviation from the axis. Particles with non-zero deviation receive a new equilibrium orbit energy with the momentum shift $\delta p/p$ [9], which inevitably leads to a rapid increase of aberrations $\delta\nu_s=2\cdot\frac{eE_x\cdot R}{m_0c^2\gamma}\cdot G\frac{\delta p}{p}$. In an earlier paper [10] studying this phenomenon, we found a method of introducing additional oscillations on the momentum that gave the averaging of the equilibrium orbit itself. As a result, we can achieve a longer SCT time up to 500 sec at $<\Delta W_{kin}/W_{kin}>=10^{-4}$.

TIME-SPACE SPIN TUNE ABERRATION

The idea of the electric dipole moment search using the storage ring (SrEDM) with polarized beam is realized under conditions of long-time spin coherence of all particles. Following the requirements of the planned SrEDM experiment, the SCT should be more than 1000 seconds. During this time each particle performs about 10^9 turns in the storage ring moving on different trajectories through the optics elements. Under such conditions, the spin-rotation aberrations associated with various types of space and the time-dependent nonlinearities start to play a crucial role. Time-dependent aberration is a spin tune aberration due to the different time of flight of particles in the focusing-deflecting fields. The space-dependent spin aberrations are associated with differences in the focusing-deflecting fields on the trajectory of particles. In the previous section, we considered spin tune dependence on particle energy, which also introduces additional aberrations. Not all of these factors lead to unlimited growth aberrations. In some cases, the aberrations are periodic and remain within a small value that cannot affect the method of searching for EDM. For example, in periodic channels the aberrations oscillate with the betatron tune, while remaining within an acceptable range. However, in the general case it is unfortunately not so and aberrations increase. Assuming "magic" conditions, we define a variation of the spin tune through the finite differences up to the second order:

$$\delta\nu_s=\frac{e}{2\pi m_0c^2}\cdot\delta\left(\frac{1}{\gamma^2-1}-G\right)\cdot L_{orb}E_x\frac{1}{\gamma}\left[1+\frac{\delta L_{orb}}{L_{orb}}+\frac{\delta E_x}{E_x}+\gamma\cdot\delta\left(\frac{1}{\gamma}\right)\right] \quad (9)$$

Representing each of them through the Taylor series expansion in powers of the finite difference $\Delta p/p$ up to second order:

$$\begin{aligned}
&\delta\left(\frac{1}{\gamma^2-1}-G\right)=-2G\frac{\Delta p}{p}+\frac{1+3\gamma^2}{\gamma^2}G\left(\frac{\Delta p}{p}\right)^2+\ldots\\
&\frac{\delta L_{orb}}{L_{orb}}=\alpha_1\cdot\frac{\Delta p}{p}+\alpha_2\cdot\left(\frac{\Delta p}{p}\right)^2+\ldots\\
&\frac{\delta E_x}{E_x}=-k_1\frac{x}{R}+k_2\left(\frac{x}{R}\right)^2+\ldots\\
&\gamma\delta\left(\frac{1}{\gamma}\right)=-\frac{\gamma^2-1}{\gamma^3}\left(\frac{\Delta p}{p}\right)+\frac{(\gamma^2-1)^2}{2\gamma^5}\left(\frac{\Delta p}{p}\right)^2+\ldots
\end{aligned}\quad ,(10)$$

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where α_1 and α_2 are the momentum compaction factor in the first and second approach, respectively; k_1 and k_2 are coefficients of the expansion of the field in the vicinity of the equilibrium orbit. As an example of the cylindrical deflector, the coefficients are $k_1 = 1$ and $k_2 = 1$. After averaging over time, for instance with RF on, the term $\Delta p / p$ makes a zero contribution to the spin tune. Substituting equations (10) to (9), and grouping the $\Delta p / p$ coefficients of powers up to the second order, we obtain:

$$\delta\nu_s = \frac{eL_{orb}E_xG}{2\pi m_0\gamma c^2}\cdot\left[F_2\left(\alpha_1,k_1,k_2,\frac{x}{R}\right)\cdot\left(\frac{\Delta p}{p}\right)^2 + 2F_1\left(k_1,k_2,\frac{x}{R}\right)\cdot\frac{\Delta p}{p}\right] \tag{11}$$

$$F_2\left(\alpha_1,k_1,k_2,\frac{x}{R}\right) = \frac{1+3\gamma^2}{\gamma^2}k_2\left(\frac{x}{R}\right)^2 - \frac{1+3\gamma^2}{\gamma^2}k_1\frac{x}{R} + \frac{5\gamma^2-1}{\gamma^2} - 2\alpha_1$$

$$F_1\left(k_1,k_2,\frac{x}{R}\right) = -k_1\frac{x}{R} + k_2\left(\frac{x}{R}\right)^2$$

Thus, the aberration of the spin is determined by a parabolic equation.

In our consideration, we did not include coefficients k_n with $n > 2$ and α_2 because we only consider the aberration growth up to the second order of $(\Delta p / p)^2$ and $(x / R)^2$. Figure 3 shows the two-dimensional parabolic dependence of spin tune aberration in a 3D representation, where one axis is a momentum spread in units of 10^{-4} and the other axis is a horizontal deviation in *mm*. The spin tune is normalized by a factor $N_F = \frac{eL_{orb}E_xG}{2\pi m_0\gamma c^2}$.

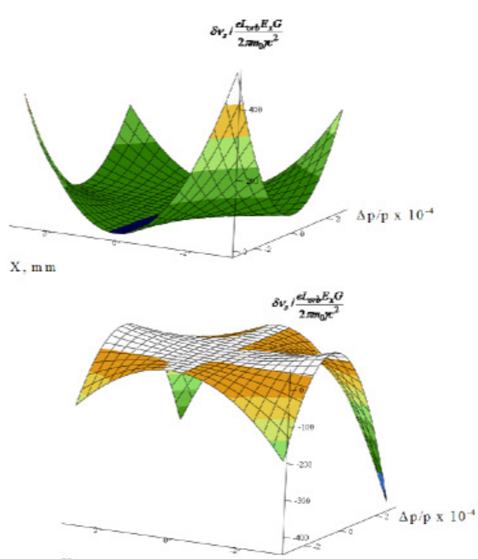

Figure 3: Spin tune aberration dependence on momentum spread and horizontal deviation at different k_1, k_2.

The coefficients k_1, k_2 depend on deflector shape and the momentum compaction factor is defined by the lattice as a whole.

Thus, these results show that it is impossible to exclude the growth aberrations of the tune spin for a non-monochromatic beam with non-zero emittance, that is at $\Delta p / p \neq 0$ and/or $x \neq 0$.

MINIMIZING OF ABERRATION

However, from the formula derived above we can perceive two methods of minimizing the spin aberrations. The first method is a choice of the lattice with compensation of the mutual influence of parameters k_1, k_2, α_1. In other words, we need to make a two-dimensional parabola maximally flat in the workspace of $(\Delta p / p)^2$ and $(x / R)^2$. To verify the analytical results we performed a full-scale simulation using the COSY Infinity program calculating the spin-orbital motion in the purely electrostatic lattice consisting of electrostatic deflectors and electrostatic quadrupoles only. Figure 4 shows the lattice in OptiM format [11].

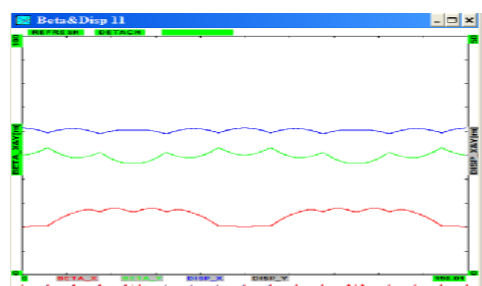

Figure 4: Twiss functions of electrostatic ring for ring and one cell.

The ring consists of two arcs, each arc has 4 FODO cells, and one cell has 4 electrostatic deflectors in each gap between quadrupoles F and D. As an example, the straight section is designed with one FODO cell. The horizontal and vertical tunes have values of 1.3 and 0.635. The electric field between the plates of the deflector is 17 MV/m.

The maximum flatness of surface (11) is reached by choosing the parameters of deflector k_1, k_2 and α_1 momentum compaction factor. The requirement for the momentum compaction factor is that it should be as large as possible. This obviously follows from the expressions (11) in the ring with a cylindrical or similar deflector geometry, when the electric field has the coefficients k_1 and k_2 close to unity.

Figure 5 shows the results of a numerical simulation with the optimum parameters of the deflector k_1=0.94 and k_2=0.96 in the whole range of operating parameters of the beam. The red curve is described by a parabola $\Delta\nu_s/NF=0.012\cdot x^2$. At these parameters, the COSY Infinity result for the SCT is ~1000 seconds. Comparing this with purely cylindrical deflectors, we see that the

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flatness in the workspace of the beam has improved by nearly factor ~20.

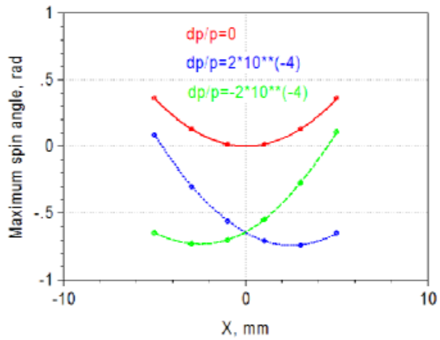

Figure 5: Maximum spin deflection angle after 10^9 turns versus x deviation at $\Delta p/p=0$ and $\pm 2\cdot 10^{-4}$.

ALTERNATING SPIN ABERRATION

The second method is to alternately change the deflector parameters and thereby alternate the sign of the spin aberration growth. In mathematical terms, this means minimizing all the factors F_0, F_1, F_2 by averaging them along an orbit, that is over time. For this purpose, we suggest the alternating spin aberration lattice which rotates spin, for instance, in one direction in even deflectors and in the other direction in odd deflectors. That is, the ring is equipped with two types of deflector having k_1 = const, and $k_2 = k_{av} \pm \Delta k$ changes from deflector to deflector. Figure 6 shows the results of the numerical simulation. We see that by choosing $k_2=0.974\pm 0.1$ we can obtain practically zero aberration for particles with $\Delta p/p = 0$ and the function is described by a parabola $\Delta \nu_s/NF=0.004\cdot x^2$.

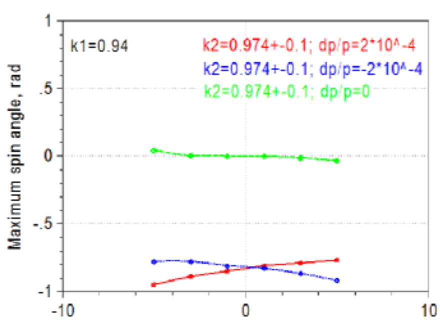

Figure 6: Maximum spin deflection angle after 10^9 turns versus x deviation in mm at $\Delta p/p=0$ and $\pm 2\cdot 10^{-4}$.

Comparing this again with the cylindrical deflector, we can see that the flatness in the workspace of the beam improved nearly ~100 times. However, the particle with non-zero momentum deviation has a finite value of the spin deflection with a parallel shift downward. It is impossible to remove this spread due to the final $\Delta p/p$ using the correct k_1 and k_2. As a result, the total spread of the spin deflection angle does not exceed ±0.5 radian after 10^9 turns, which corresponds to an SCT of about 5000 seconds.

The structure with the alternative geometry of the deflector allows us to tune the desired value of k_1, k_2. Raising the field strength between the plates in even deflectors and reducing it in the odd deflectors effectively adjusts the required coefficients k_1 and k_2. Another possibility is to create the required potential distribution due to potential changes in the stripline placed on the surface of the ceramic plates located at both ends of each deflector.

CONCLUSION

In the present work, we studied the behaviour of spin aberrations in the structure and developed techniques to minimize them. One of the most effective methods is the alternating spin aberration lattice. The analytical model allows us to find the general solution of aberration retention with an SCT of about 5000 seconds confirmed by COSY Infinity spin-orbital tracking.

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