MEASURING ANGLE WITH PICO METER RESOLUTION

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Abstract

The kilogram is the only remaining fundamental unit within the SI system that is defined in terms of a material artefact, more specifically a PtIr cylinder kept in Paris. Therefore, one of the major tasks of modern metrology is the redefinition of the kilogram on the basis of a natural quantity or of a fundamental constant. The Institut Laue-Langevin (ILL) and its high-resolution γ-ray spectrometer GAMS with its new optical interferometer can play a crucial role in this attempt.

INTRODUCTION

In the effort of redefining the kilogram one must approach a 10^{-8} relative accuracy in its practical realization. A joint research project amongst the major metrology institutes in Europe has proposed the redefinition of the kilogram based on the mass of the ^{12}C atom. The goal can be achieved by counting in a first step the number of atoms in a macroscopic weighable object and, in a second step, by weighing the atom by means of measuring its Compton frequency ν_C. It is in the second step of the procedure, where the ILL is playing a fundamental role with GAMS, the high-resolution γ-ray spectrometer.

Energies of the γ-rays emitted in the decay of the capture state to the ground state of a daughter nucleus after a neutron capture reaction can be measured with extremely high precision via Laue-diffraction. In order to match the high demand in angle measurement accuracy, a new optical interferometer with 10 pico radian resolution, linearity over a total measurement range of 15° and high stability of about 0.1 nrad/hour has been developed. To drive the interferometer a PLL circuit has been implemented for the heterodyne frequency generation. As the same time a new FPGA based electronics for real time phase measurement and axis-positioning control has been realized.

In the present paper, the basic concepts of the FPGA implementation will be revised and a complete analysis of the performances of the new electronics will be presented.

PHYSICAL MOTIVATION

Two approaches are currently proposed in order to be able to link a new definition of the kilogram to a fundamental constant.

The first project, the so-called "Watt-balance" [1], suggests linking the kilogram to Planck constant h. The second approach, the so-called "crystal density" [2], proposes to measure Avogadro constant N_A and define the kilogram via the mass of a free carbon atom: $1000/12 \cdot N_A \cdot A_r$ (^{12}C). Additionally one could also determine the energy equivalent to the mass of a ^{12}C atom, leading to the fundamental constant $N_A \cdot h$.

A direct measurement of the energy equivalent is difficult since small masses correspond to enormous energies. However, measuring the mass and energy balance in a thermal neutron capture reaction offers an attractive possibility to realize this concept. The key difficulty in the experiment is the precise absolute determinations of γ-ray energies emitted after the thermal neutron capture reaction.

This is done by diffraction of γ-rays at perfect Si crystals and measuring the very small diffraction angles in absolute terms. The aimed uncertainty for the determination of $N_A \cdot h$ is 10^{-8}, which requires an adequate accuracy for the diffraction angles' measurement and the calibration of the apparatus.

A key step towards a new $N_A \cdot h$ measurement is a new optical interferometer with 10 prad precision and linearity over a total measurement range of 15° and high stability of about 0.1 nrad/hour. Figure 1 shows the layout of the interferometer with all the optical elements in place and the various beams' paths.

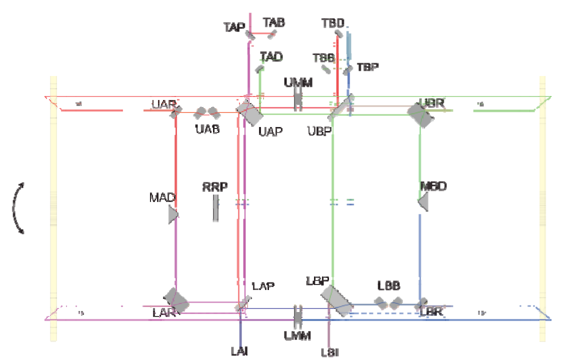

Figure 1: The optical layout of the new GAMS interferometer.

The interferometer follows a Mach-Zehnder layout, which allows a strict separation in space between all beams. This avoids unwanted mixing, which would lead to non-linearity in the angle measurement. A more detailed description of the interferometer can be found in Ref. [3].

The interferometer is designed such that we have the possibility to measure the rotation of both spectrometer axes using a single set of optical elements. This highly symmetric layout increases the stability and the accuracy of the angle measurement by self-compensating eventual drift of the apparatus. All the optical elements are high quality fused silica pieces with custom-made surface treatment. All the key optical elements of the interferometer are chemically bonded [4] to the CLEARCERAM base of the spectrometer to avoid the use of glue. The roof-prisms and retro-reflectors used in the realization of the optical path are hollow since in solid elements the path length is not constant with respect to rotation [5].

THE NEW LASER SOURCE

Classical interferometers allow typical displacement measurements around 10^{-2} of the optical wavelength. This resolution can be improved by a factor of 100 by using in each interferometer arm a different wavelength.

This concept is realized in our new interferometer. Starting from a highly stabilized monodyne laser source, the single laser wavelength is split into two beams and each of them is frequency shifted. The two frequencies are obtained by mixing to a beat signal of 100 kHz with a carrier frequency of 40 MHz.

We have developed a Phase-Locked Loop (PLL) circuit that allows mixing the reference signal of a commercial lock-in amplifier with a signal of a precision signal generator. The two frequencies are transferred to the laser beam via Acousto-Optic Modulators (AOM). After frequency shifting the laser beams are injected into the interferometer via glass fibres. The interference signal will be detected as a beat node modulation. The phase of this beat signal is proportional to the length variation of the interferometer paths originated by the rotation of the axis.

The phase gets measured relative to a reference signal generated within the interferometer thanks to a newly developed phase measurement electronics described in more details in the following chapter.

PHASE READ-OUT AND NANO POSITIONG ELECTRONICS

The interferometer delivers for each of the two axes a measurement signal and a reference signal. If one axis is rotated, the phase of the beating of the according signal changes with respect to its reference. In order to determine the absolute displacement of the axis in angular term one has to measure the phase change and count integer periods (where one period is equivalent to one optical fringe or to an angle of about 0.5 µrad). To perform those tasks we have developed a dedicated electronic module based on a VIRTEX-4 FPGA. For compatibility reasons with the existing ILL's electronics and for a maximum of flexibility, the implementation adopts the M-Module (ANSI/VITA 12-1996) [6] mezzanine standard (see Fig. 2). While the back of the mezzanine card contains the FPGA chip and the carrier I/F connector the front part implement all the analogue circuits necessary to treat all signals from the interferometer as well as all the front-panel connectivity.

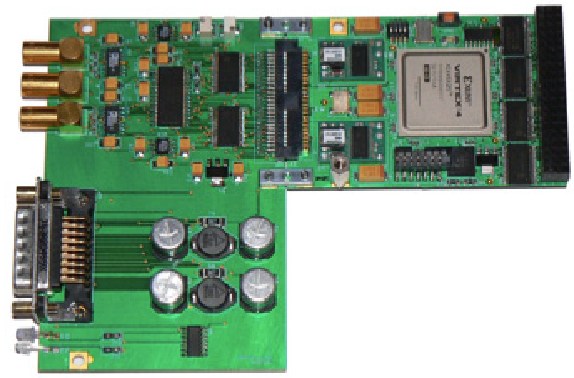

Figure 2: The M-Module responsible for the phase read-out and the fine positioning of the spectrometer's axis.

The beating signals from the photodiodes are first filtered to eliminate the DC offset and the narrow band AC noise due to the detection of ambient light. The signals are then transformed into square waves by high stability differential operational amplifiers. The phase difference between the reference and the interferometer signal is then measured by time interval counting. The module is driven by a master clock running at 100 MHz, which allows the 100 kHz periods to be sampled with an accuracy of ± 10 nsec. This uncertainty is equivalent to 0.0005 interferometer fringes (50 prad). The counting of integer fringes (equivalent to full periods) provides the necessary information for the macroscopic positioning of the spectrometer axes using DC motors coupled to high precision rotation stages. The accuracy of the phase measurement can be pushed up to 0.00005 fringes by means of a sliding average over 100 values, at the price of reducing the sampling frequency down to 1 kHz.

The axis fine positioning is achieved using a piezo actuator controlled by a 0-10 V signal provided by a 14-bit Digital-to-Analogue Converter (DAC). Alternatively, the card offers the possibility of a digital output to directly control the piezo positioning. At present a positioning resolution of 0.01 fringes has been obtained, being limited only by the amplitude of the residual external vibrations transmitted to the axis. To remove any axis drift and to minimize those residual vibrations, the phase information is used as feedback for a close loop real-time regulation mechanism. This regulation is based on hardware Proportional-Integral-Derivative (PID) controller that corrects for any deviation of the measured phase from the desired one.

The phase information is also recorded in coincidence with the γ-events from the HPGe detector in the acquisition card. This supplementary information is fundamental to correctly interpret the measured data since it allows removing the impact of mechanical vibrations that affects the accuracy of the angle measurements due to the incorrect determination of the axis position during the data acquisition.

CARD PERFORMANCES

A series of tests have been carried out to evaluate the overall performances of the ILL's phase read-out electronics in comparison with those provided by the commercial dual phase, wide bandwidth, DSP lock-in amplifier SIGNAL RECOVERY model 7280. The lock-in amplifier uses a different approach to derive the phase information. It uses frequency mixing in combination with a low-pass filter to convert the signal's phase and amplitude to a DC voltage signal. Its main advantage is the capability to handle signals with a high signal-to-noise level.

All tests for which we report the results in the following were performed using a high precision signal generator feeding both the reference and the interferometer signal therefore, the expected phase difference should be equal to zero. Figure 3 shows the evolution of the phase measurement when the beating frequency is increased from 1 kHz up to 120 kHz.

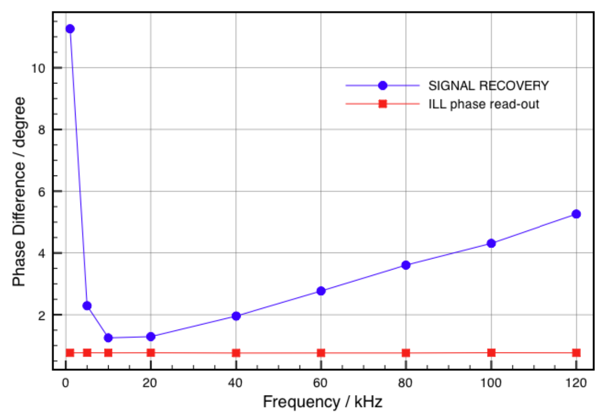

Figure 3: Comparison of the phase offset versus beating frequency.

Having in mind that both inputs are fed with the same beating frequency the expected phase offset should remain constant. With the exception of the first 2 points, the results show clearly a linear correlation between the frequency variation and the phase offset values in the case of the SIGNAL RECOVERY lock-in amplifier. Our explanation is that the observed behaviour is due to the filters and amplifiers that are present at the input stage. For very low frequency, it is evident in the data how the two input channels of the SIGNAL RECOVERY seem to diverge. We do not have a clear explanation for such behaviour. The same measurement repeated using our read-out electronics does not show any appreciable variation of the measured phase as a function of the beating frequency.

A second series of tests has been performed to verify the stability of the phase values with respect to amplitude variations of the input signals. The obtained results are summarized in Fig. 4 where the phase difference has been plotted versus input signal amplitude. As for the previous measurement, the same input signal has been used for both inputs resulting in an expected zero phase difference.

While in the case of our read-out electronics this result is confirmed a part from an initial offset, the SIGNAL RECOVERY lock-in shows a strong dependency between phase value and input signal amplitude.

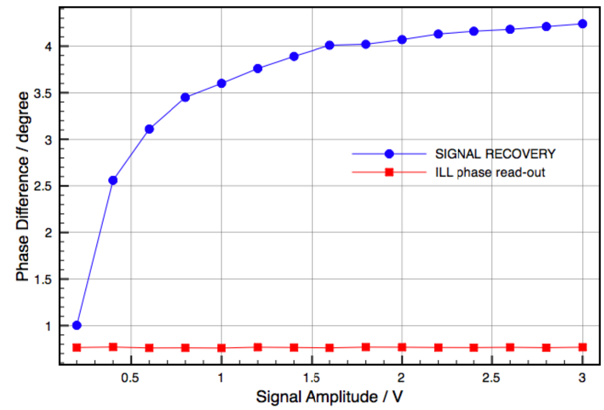

Figure 4: Comparison of the phase offset versus input signal amplitude.

In Fig. 5 and Fig. 6 we compare the power density spectra of the phase difference measured by both the SIGNAL RECOVERY lock-in amplifier and our electronics. The frequency resolution of the SIGNAL RECOVERY spectrum is 0.2143 Hz. The standard deviation of the SIGNAL RECOVERY phase noise (the *rms* value of the signal) - calculated by the square root of the integral spectrum in the 0.1 Hz to 500 Hz frequency interval - is 0.016 degree. The cut-off at about 100 Hz is due to the 1 ms minimum time-constant of the output low-pass filter (6 dB/octave).

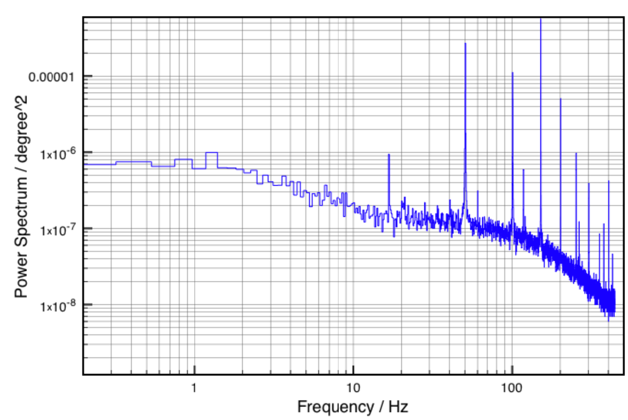

Figure 5: Noise power spectrum obtained with the SIGNAL RECOVERY lock-in amplifier.

The *rms* values are comparable for both the SIGNAL RECOVERY lock-in amplifier and our phase read-out electronics but since for this last one there is no cut-off at 100 Hz, the resulting bandwidth is larger and consequently the total noise lower. At lower frequencies it is evident, especially in the case of the SIGNAL RECOVERY, a slight noise level increase. The fact that the same effect is much less pronounced in our electronics seems to confirm the previous results showing a much higher sensitivity to frequency and amplitude noise in the case of the SIGNAL RECOVERY.

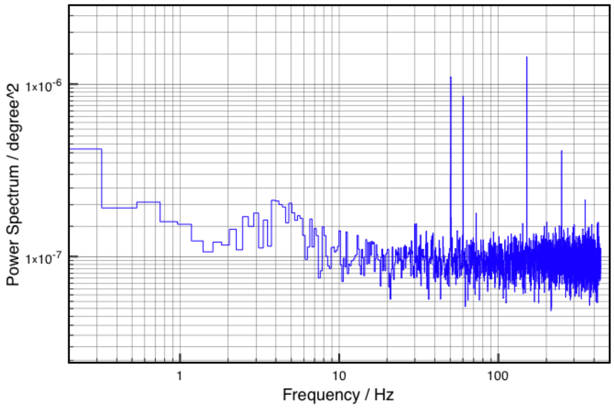

Figure 6: Noise power spectrum obtained with our phase read-out electronics.

From the SIGNAL RECOVERY data we can calculate an the *rms* noise level in the 0.1 Hz to 10 Hz interval of 0.041 degree. This value corresponds to 1.4 mV amplitude noise (*rms* values) of the signal generator in the same frequency band. The remaining noise present in our electronics is probably due the jitter of the quartz (50 ppm accuracy) used to cadence the FPGA and, therefore, to determine the time intervals for the phase measurement. To verify this hypothesis, a new version of the same card implementing a high precision quartz (30 ppb accuracy) is currently under development. Having established the good performance of our electronics, we continued the tests replacing the signal generator inputs with those obtained by a simplified version of our interferometer. The goal was to determine all possible sources of phase noise due to the laser injection and/or the optical fibre as well as to monitor the time stability of those elements.

CONCLUSION

In order to satisfy the specific requirement in terms of high precision phase measurement and positioning accuracy related to the operation of the new ultra high-resolution γ-ray interferometer under development at the ILL, we have specifically developed a new phase read-out electronics. The card adopts the M-Module mezzanine standard for a maximum of flexibility. It is based on a Virtex 4 FPGA that controls both the phase measurement and the fine axis positioning.

An exhaustive series of tests has been performed to evaluate the overall performance of the new electronics. The obtained results are very encouraging, showing a very small, when not absent, sensitivity to frequency and signal amplitude noise. A further step in noise reduction will be achieved with the implementation of the new high-precision quartz.

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